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AUTHOR(S):
Kiwamoto, Y; Soga, Y; Aoki, J

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Radial transport in magnetized non-neutral plasma driven by rotating wave

Y. Kiwamoto, a) Y. Soga, and J. Aoki
Graduate School of Human and Environmental Studies, Kyoto University, Yoshida Nihonmatsu-cho, Sakyo-ku, Kyoto 606-8501, Japan

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Radial transport in non-neutral plasmas driven by a rotating wave field is examined in terms of the drift-kinetic Vlasov equation. It is shown that the radial flux is generated by the \( \mathbf{E} \times \mathbf{B} \) drift of resonant particles subject to Landau damping in the axial dynamics. The rate of change in the canonical angular momentum associated with the radial flux is equal to the torque resonantly exerted by the azimuthal component \( E_\phi \) of the wave. The absorbed wave energy is shared between the axial kinetic energy of the particles and potential energy of the charged particle system. The basic idea of this model may be extended to a scenario that the radial flux is generated by any other dissipative processes that shift the phase relation between the wave and the particle orbits. © 2005 American Institute of Physics. [DOI: 10.1063/1.2035427]

An appreciable number of observations have been accumulated that indicate the significant contributions rotating electric fields make to the particle transport and to modifications of the density distribution in non-neutral plasmas.1–4 While a theoretical model has been proposed for interpretation of the radial transport, more efforts may be necessary toward constructing a clear model for coherent understanding of the processes under the observation.5–7 In this Brief Communication we study the transport process in terms of the interaction between the waves and particles starting from the drift-kinetic Vlasov equation and quantitatively examine relations among involved processes.

The present study will reveal that the radial particle flux is generated by the \( \mathbf{E} \times \mathbf{B} \) drift of resonant particles that absorb the energy and momentum of the azimuthally traveling waves through Landau damping. The rate of change in the canonical angular momentum associated with the radial flux is shown to be equal to the torque exerted by the azimuthal wave field to the resonant particles. A conservation law holds in the process of the conversion of the wave energy to the resonant particles. A conservation law holds in the process of the conversion of the wave energy to the resonant particles. A conservation law holds in the process of the conversion of the wave energy to the resonant particles. A conservation law holds in the process of the conversion of the wave energy to the resonant particles.

The configuration we study here consists of a cylindrically symmetric plasma of electrons, with charge \( -e \) and mass \( m \), immersed in a strong homogeneous magnetic field \( \mathbf{B}=B \hat{z} \) and a small-amplitude wave traveling helically with the azimuthal wave number \( \ell \) and the axial wave vector \( k_z \). Conversion to the ion plasma case is easily made by replacing the mass and charge. The plasma is infinitely long axially but bounded radially by a co-axial conductor wall placed at \( r=w \). We allow arbitrary dependence of the equilibrium density profile \( n_0(r) \) on the radial coordinate \( r \). We assume that the magnetic field is so strong that the transverse motion of the particles is described by the guiding-center approximation:

\[
\frac{\partial f}{\partial t} + \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla f + v_\parallel \frac{\partial f}{\partial z} - \frac{e}{m} E_\phi \frac{\partial f}{\partial \theta} = 0. \tag{1}
\]

The equilibrium state is determined by the zeroth order parameters included in the second term, that requests an azimuthal component 

\[
\mathbf{E}_0 \cdot \mathbf{B} = \frac{E_\phi}{B} \hat{\theta} = \hat{\theta} \cdot \frac{1}{\varepsilon_0 B r} \int_0^r dr \, r n_0(r) = r \omega_r(r) \hat{\theta}. \tag{2}
\]

Here \( \hat{\theta} \) stands for the unit vector in the azimuthal direction, and \( \omega_r(r) \) represents the frequency of the azimuthal rotation that depends on the radial coordinate \( r \).

Selecting a Fourier component of \( \phi(r)e^{i(\ell \theta + k_z z - \omega t)} \) from the potential perturbation in Eq. (1), we obtain the perturbation of the velocity distribution function as

\[
\hat{\mathbf{f}} = \frac{e}{m} \frac{\partial f_0}{\partial v} - \frac{\ell}{\omega_k} \frac{\partial n_0}{\partial r} \int_0^r dr \, r n_0(r) \hat{\mathbf{r}} \phi(r) = - \frac{e}{m} n_0 \hat{\mathbf{r}} \phi(r), \tag{3}
\]

Here \( f_0 \) is the zeroth order velocity distribution function normalized to unity on integration over the axial velocity \( v \). Introduction of the density perturbation associated with \( \hat{\mathbf{f}} \) into the Poisson’s equation leads to the differential equation

\[
\nabla \cdot \hat{\mathbf{e}} \cdot \nabla \phi = 0. \tag{4}
\]

Here \( \hat{\mathbf{e}} \) is the dielectric tensor given as,

\[
\hat{\mathbf{e}} = \varepsilon_0 \begin{pmatrix} \varepsilon_\perp & -i e_X & 0 \\ i e_X & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{5}
\]

where \( \varepsilon_0 \) is the dielectric constant in vacuum. The transverse components of the tensor are
\[\begin{align*}
\epsilon_\perp &= 1 + \frac{\omega_p^2}{\omega_C^2}, \\
\epsilon_X &= \frac{\omega_p^2}{\omega_C (\omega - \ell \omega_o)}, \\
\text{and the axial component } \epsilon_\parallel &= \frac{1}{k^2} \int dv \dot{L}_j, \\
\epsilon_\parallel &= 1 - \frac{\omega_p^2}{k^2} \int dv \dot{L}_j, \\
\epsilon_\parallel &= 1 - \frac{\omega_p^2}{k^2} \int dv \frac{\hat{L}_j}{r - (\omega - \ell \omega_o)k},
\end{align*}\]

where \(\omega_p\) is the local plasma frequency, and \(\omega_C = eB/m\) is the gyrofrequency.

The explicit form of the Poisson’s equation is written as

\[1 \frac{\partial}{\partial r} \left( \epsilon_\parallel \frac{\partial \phi}{\partial r} \right) + \frac{\ell}{r} \frac{\partial \epsilon_x}{\partial r} \frac{\partial \phi}{\partial r} - \epsilon_\parallel \frac{\partial^2 \phi}{\partial \epsilon_\parallel \phi} - k^2 \epsilon_\parallel \phi = 0. \]

In a non-neutral plasma trapped in a strong magnetic field, we usually see the condition \(\omega_C \gg \omega_p > \omega_o\). If the imaginary part \(\partial \epsilon_x/\partial r\) coming from damping effects in \(\omega\) is negligibly small compared to \(\epsilon_\parallel = \text{Im} \{\epsilon_\parallel\}\), the imaginary part of \(\epsilon_\parallel\), we obtain

\[\int_0^w dr (r_\parallel - \epsilon_\parallel^*) |\phi|^2 = 0. \]

This relation is derived mathematically by multiplying the complex conjugate of \(\phi\) to Eq. (10) and integrating over the radial coordinate from the axis \(r=0\) to the wall \(r=w\) after subtracting it from the complex conjugate of its counterpart. By separating the wave frequency \(\omega\) into the real part \(\omega^0\) and the imaginary part \(\gamma\) and introducing it into Eq. (11), we obtain the growth rate of the wave as

\[\gamma = \int_0^w dr r_\parallel |\phi|^2 = \int_0^w dr r_\parallel \epsilon_\perp \phi. \]

Here \(\epsilon_\parallel^*\) is the real part of \(\epsilon_\parallel\).

Under a strong magnetic field \(B\) particles drift radially due to the azimuthal component \(E_\theta\) of the wave, and the radial drift velocity is given as

\[\dot{\epsilon}_\parallel = \frac{\epsilon_\parallel}{B} = - \frac{e \ell \phi}{r B}. \]

The radial flux density of the particles is obtained by extracting the time-averaged microflux \(\langle \epsilon_\perp \cdot \dot{r} \rangle\) and integrating it over the axial velocity distribution as

\[\Gamma = \int dv \langle \dot{\epsilon}_\parallel \cdot \dot{r} \rangle = \frac{e n_0 \ell}{2 \epsilon B r} |\phi|^2 \int dv \text{Im} \{\dot{L}_j\}, \]

\[= n_0 \epsilon \ell k^2 |\phi|^2 \text{Im} \{\epsilon_\parallel\}. \]

The propagating wave exerts a torque to the plasma contained in the shell as

\[\dot{L}_j = \pi \ell k^2 \epsilon_0 \int_0^w dr r |\phi|^2 \text{Im} \{\epsilon_\parallel\}. \]
Resorting to the same procedure as for Eq. (15), this expression may be rewritten as

$$dN = \pi r \, dr \, k^2 \omega_0 |\phi|^2 \text{Im}|\epsilon|.$$

Comparison of Eqs. (22) and (24) leads to the relation

$$dL_z = dN.$$

This equation clearly indicates that the canonical angular momentum of the plasma is driven by the torque of the azimuthal electric field of the propagating wave and that the relevant mechanism is the Landau resonance with the particles streaming in the axial direction.

The modification of the density distribution $n_0(r)$ due to the radial flux results in a change in the potential energy $U$ of the particle system. On recalling the potential distribution $\phi_0$ of the zeroth order, the increasing rate of $U$ may be written as

$$U = \int_0^r \, dr \, 2\pi r(-e\phi_0) \frac{\partial n_0}{\partial t} = \int_0^r \, dr \, 2\pi r(-e\phi_0) \times \left(-\frac{1}{r} \frac{\partial}{\partial r} (r\Gamma) \right) = -\int_0^r \, dr \, 2\pi r e \frac{\partial \phi_0}{\partial r} \Gamma.$$

Therefore the contribution from the shell bounded within $r$ and $r+dr$ is

$$dU = -2\pi r \, dr \, e \frac{\partial \phi_0}{\partial r} \Gamma,$$

$$= -\pi r \, dr \, e_0 \, \omega \, \omega_0 k^2 |\phi|^2 \int d\nu \, \text{Im}|\hat{\mathcal{J}}_f|,$$

$$= \pi r \, dr \, e_0 \, \omega \, k^2 |\phi|^2 \text{Im}|\epsilon|.$$

Here we have used the relation $(\partial \phi_0/\partial r)/rB = \omega_0$ that corresponds to Eq. (2). By combining Eq. (28) with Eq. (24) we obtain the relation between the energy increment and the torque power,

$$dU = \omega_0 dN.$$

On the other hand the resonant wave-particle interaction in the axial direction should be associated with particle acceleration. The increasing rate of the kinetic energy $dK$ in the shell is evaluated as follows:

$$dK = 2\pi r \, dr \int d\nu (-e) v(\delta w) = \pi r \, dr \, \frac{ie \omega}{2} \int d\nu \, v(\phi^* \delta f - \phi \delta \phi^*)$$

$$= \pi r \, dr \, e_0 \omega_0 \omega_0 |\phi|^2 \int d\nu \, v \text{Im}|\hat{\mathcal{J}}_f|,$$

$$= \pi r \, dr \, e_0 (\omega - \ell \omega) \omega_0 \omega_0 |\phi|^2 \text{Im}|\epsilon|.$$

By comparing Eqs. (28) and (31) we obtain an interesting relation:

$$\frac{dK}{dU} = \frac{\omega - \ell \omega_r}{\ell \omega_r},$$

and, on integration over $r$, we further obtain

$$K + U = -2\gamma \int_0^r \, 2\pi r \, dr \, e_0 \omega_0 \omega_0 \frac{\partial \phi_0'}{\partial \omega'} \frac{k^2}{4} |\phi|^2 = -\hat{W}.$$
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14R. C. Davidson, Physics of Nonneutral Plasmas (Addison-Wesley, Redwood City, 1990), p. 248.
15This part is stimulated by the referee’s comment.