The longitudinal stability of intense nonrelativistic particle bunches in resistive structures

Paul J. Channell
Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545

Andrew M. Sessler and Jonathan S. Wurtele
Lawrence Berkeley Laboratory, Berkeley, California 94720

(Received 27 April 1981; accepted for publication 1 June 1981)

The longitudinal stability of intense particle bunches is investigated theoretically in the limit of small wall resistivity compared to total reactance. It is shown that both in the absence of resistivity and to lowest order in the resistance that an intense bunch is stable against longitudinal collective modes. An expression is derived for the lowest order instability rate. Application of these results are made to drivers for heavy ion inertial fusion.

PACS numbers: 41.70. + t, 29.15.Dt, 52.35.Py, 52.35.Fp

Heavy ion fusion is envisioned as having for a driver either an rf linac with storage rings or an induction linac.[1] In the rf linac approach the major current multiplication, so as to reach the requisite power level, is done in the storage rings. The induction linac, on the other hand, must accelerate significant currents directly to the target. Either approach has difficulties (such as the manipulation of beams in and out of storage rings in the rf linac approach), but common to both methods is the need for the stability of intense buches of particles. Much effort has been devoted to this subject.[1,2]

For a bunch in an induction linac an estimate can be obtained by employing the analysis which has been developed for circular machines and modifying it for a linear structure.[3] First, we note that one is "below transition" or in a positive mass regime, so that only in the presence of resistivity is there instability. One finds, for above threshold, that the e-folding length λ is given by

$$\lambda^{-1} = (R/Z_0)\left(\frac{4\pi^2 q^2}{[1+2\ln(b/a)]}\,\frac{M_p}{M}\,\frac{N}{L}\,r_p\right)^{1/2}, \tag{1}$$

where $Z = R + iX$ is the impedance per unit length, N/L is the line density of ions, r_p is the classical proton radius, Z_0 is the free-space impedance (or 377 Ω), q is the degree of ionization of the ions, and M/M_p is the mass of the ions in units of the proton mass.

Putting in $R = 200\ \Omega/\text{m}$, $q = 2$, $M_p/M = 1/200$, $N/L = 10^{15}/20$ m, and $b/a = 1.5$, Eq. (1) yields a length λ of 300 m, which is uncomfortably short for a linac of the length required.

For a storage ring a similar method may be employed and yields growth times which are also uncomfortably short.[4]

It is the purpose of this communication to report on a theoretical analysis which directly applies to the storage ring or the induction linac of heavy ion fusion. We show that to lowest order there is, for any finite bunch, no net resistive instability so that the situation is very much better than estimated above. We also allow in our formalism for an arbitrary impedance of the structure which, at least for the induction linac, is an important effect. Our work is a generalization of that of Kim, who first showed *no* instability for a finite bunch of *uniform* charge, with a *step-function* distribution in momentum, and the impedance of a *uniform* structure.[5] Removing the special assumptions of Kim is important for it allows us to conclude that either in a practical linear induction accelerator or in a realistic storage ring intense particle bunches will not be subject to significant longitudinal instability and, hence, from this very important theoretical point of view heavy ion fusion is a viable and interesting possibility.

The ions, which are collisionless, are described by the nonrelativistic nonlinear Vlasov equation

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z} + \frac{qe}{M}E\frac{\partial}{\partial v}\right) f(z, v, t) = 0, \tag{2}$$

where z is the longitudinal coordinate, v is the velocity associated with the z coordinate, e is the proton charge, t is the time, and the ion distribution function is the unknown f. The longitudinal electric field consists of an applied field E_A and a functional $E_s(n)$ of the line charge density $n(z, t)$, where

$$n(z, t) = \int f(z, v, t)\, dv. \tag{3}$$

We may take moments of the Vlasov equation and close the hierarchy by noting that in our applications the particle thermal velocity is small compared to the collective motion density wave phase velocity. Stopping after two moments we obtain fluid equations which can be combined to yield

$$\frac{\partial^2 n(z, t)}{\partial t^2} + \frac{qe}{M}\frac{\partial}{\partial z}\Big(En(z, t)\Big) = 0. \tag{4}$$

This equation may be linearized about an equilibrium charge distribution $n_0(z)$ which can, in this approximation, be arbitrary; i.e., we can choose an applied field so as to make any $n_0(z)$ stationary. Introduce space and time Fourier transforms by means of

$$\tilde{n}_1(k, \omega) = \int_{-\infty}^{\infty} dt \int_{-L/2}^{L/2} dz n_1(z, t)\, e^{i(kz - \omega t)}, \tag{5}$$

where n_1 is the pertrubed density. We thus obtain

$$-\omega^2 \tilde{n}_1(k, \omega) + \frac{qe}{M} ik \int dz e^{ikz} n_0(z)\, E_s(n_1) = 0. \tag{6}$$

 0003-6951/81/160359-03$00.50 © 1981 American Institute of Physics

Downloaded 11 Mar 2008 to 130.54.110.22. Redistribution subject to AIP license or copyright; see http://apl.aip.org/apl/copyright.jsp

The self-electric field, $E_s(n)$, can be related to n_1 by means of an impedance function

$$\tilde{E}_s(k,\omega) = -Z(k,\omega)\,\tilde{n}_1(k,\omega). \tag{7}$$

The entire effect of the storage ring or the induction linac is contained in the function $Z(k,\omega)$, which describes the reaction of the structure to a disturbance of laboratory frequency ω'. The laboratory frequency ω' is $\omega' \simeq kv_b + \omega$, where v_b is the beam velocity and the term in ω is due to motion of the disturbance in the beam frame. To good approximation the term in ω can be neglected, and then $Z(k,\omega)$ is a function of k alone. We use this approximation in most of our work, but take the ω in to account in $Z(k,\omega)$ when we calculate to second order [Eq. (17)].

We combine Eqs. (6) and (7) and the approximation for $Z(k,\omega)$ to obtain

$$-\omega^2\tilde{n}_1(k,\omega) + \frac{qe}{M}\,\frac{ik}{2\pi}\int dk_1\,\tilde{n}_0(k-k_1) \times Z(k_1)\tilde{n}_1(k_1,\omega) = 0, \tag{8}$$

where clearly $n_0(k)$ is related to the equilibrium density $n_0(z)$ by an equation analogous to Eq. (5).

It is easy to see that Eq. (8) yields a growth rate as given by Eq. (1) under the same circumstances. For a long wavelength disturbance, on a beam of radius b in a structure of radius a, the impedance is

$$Z(k) = -qe\,ik(1 + 2\ln b/a) - qev_bR, \tag{9}$$

where v_b is the beam velocity. For a uniform beam $n_0(z)$ is a constant and $n_1(z,t)$ varies sinusoidally in space and time. For a long wavelength disturbance and for a uniform beam, Eq. (8) and Eq. (9) yield Eq. (1).

For a bunched beam, however, we can show *two* consequences of Eq. (8); namely there is no instability if the impedance is purely imaginary (i.e., purely reactive) and furthermore that there is no instability if the resistance is small. *First*, consider the case in which there is no resistance, so that we may write

$$Z(k) = iX(k), \tag{10}$$

where the reactance X is odd. We assume X is negative for positive k, in order to be in the positive mass regime, and it is under this condition that there is stability.

Multiplying Eq. (8) by $\tilde{n}_1^*(k)\,Z^*(k)/k$, and integrating we obtain

$$-\omega^2\int_{-\infty}^{\infty}|\tilde{n}_1(k)|^2\,\frac{Z^*(k)}{k}\,dk + \frac{qei}{M2\pi}\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}dk_1\,Z^*(k) \times Z(k_1)\,\tilde{n}_1^*(k)\,\tilde{n}_0(k-k_1)\,\tilde{n}_1(k_1) = 0. \tag{11}$$

We use Eq. (10) and the theorem that

$$\int_{-\infty}^{\infty}\tilde{F}(k)\,\tilde{G}^*(k)dk = \int_{-\infty}^{\infty}F(x)\,G^*(x)\,dx, \tag{12}$$

with

$$\tilde{F}(k) = \int_{-\infty}^{\infty}dk_1\,\tilde{n}_0(k-k_1)\,Z(k_1)\,\tilde{n}_1(k_1),$$

$$\tilde{G}^*(k) = Z^*(k)\,\tilde{n}_1^*(k), \tag{13}$$

to write Eq. (11) in the form:

$$-\omega^2\int_{-\infty}^{\infty}|n_1(k)|^2\left|\frac{X(k)}{k}\right|dk + \frac{qe}{M}\int_{-\infty}^{\infty}n_0(z)\,|G(z)|^2dz = 0. \tag{14}$$

In this form it is clear that ω^2 is real and positive and, hence, there is stability.

Secondly, consider the case in which

$$Z(k) = iX(k) + R(k), \tag{15}$$

and $R(k)$ is real and symmetric and small; i.e., $R(k) \ll X(k)$. For a nonrelativistic bunch this will generally be true, since the self-term in the reactance is non-negligible.[6] Employing perturbation theory we find that $R(k)$ creates a frequency shift $\delta\omega_n$ of the nth mode frequency ω_n^0 given by

$$\delta\omega_n = \frac{qei\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dkdk_1X(k)\tilde{n}_n^*(k)\,\tilde{n}_n(k_1)\,\tilde{n}_0(k-k_1)\,R(k_1)}{2\omega_n^0\int_{-\infty}^{\infty}2\pi M\,dk\,|\tilde{n}_n(k)|^2\,[X(k)/k]}, \tag{16}$$

where $\tilde{n}_n(k)$ is the eigenfunction of the nth mode.[7] For a symmetric unperturbed bunch, and provided the modes ω_n are nondegenerate, it is easy to show that $\delta\omega_n$ is zero.

With these results we conclude that the growth distance (or time) is greatly increased over that given by Eq. (1). Explicit calculation must employ improved equations and a particular model for $n_0(z)$ [and hence $\tilde{n}_n(k)$ and ω_n].[8] This work, which will be described elsewhere, yields

$$\lambda^{-1} = \lambda^{-1}_{\text{uniform beam}}\,(v_p/v_b)\,g(n), \tag{17}$$

where $\lambda^{-1}_{\text{uniform beam}}$ is given by Eq. (1), $g(n)$ is a dimensionless function of mode number n and

$$v_p = qe\,[n_0(1 + 2\ln b/a)/M]^{1/2}. \tag{18}$$

For typical parameters the additional factor in Eq. (17), over Eq. (1), is $\simeq$500.

We have shown that to lowest order there is no instability, whereas Wang and Pellegrini have shown that, under certain circumstances, bunches are unstable.[9] An important difference between our work and theirs is that they, working with relativistic particles, take $Z(k)$ to have a broad resonance and no self-term, so that an expansion in $R(k)/X(k)$ would be invalid. On the other hand, for the nonrelativistic particles of heavy ion fusion an expansion in R/X—and hence a very different conclusion—is valid.

Although we have shown that to lowest order there is neither an absolute or a convective instability, there is still the possibility of transient spatial amplification. We have estimated this effect, using uniform beam theory, the impe-

Downloaded 11 Mar 2008 to 130.54.110.22. Redistribution subject to AIP license or copyright; see http://apl.aip.org/apl/copyright.jsp

dance one expects in practice, and the time for a disturbance to reach a bunch end.[6] We find that less than one e folding occurs.

Finally, we have been concerned that our results depend upon reflection of disturbances at bunch ends, which is exactly where our analysis is invalid because we have linearized about an unperturbed distribution which is, in fact, going to zero at the bunch end. We have, consequently, examined a more realistic model for the impedance than Eq. (9); namely a model in which (neglecting resistance)

$$Z(k)=c_1 k/(c_2+k^2), \tag{19}$$

where c_1 and c_2 are constants. For small k this model can be matched to Eq. (9), but for short wavelengths Eq. (19) converts to a plasma oscillation in which the structure is not important. For an impedance given by Eq. (19) we are able to reduce Eq. (8) to a second-order differential equation and show that a wave reflects *before* it reaches a bunch end; i.e., before the linear approximation becomes invalid. This work, which will be described elsewhere, lays to rest our concern about the validity of the results reported herein.

The authors are grateful to K. J. Kim for helpful discussions and to E. Hameiri for assistance with the mathematics.

This work was supported by the Office of Inertial Confinement Fusion of the Department of Energy and performed under Contract No. W-7405-ENG-48.

[1]D. Keefe and A. M. Sessler, *Proceedings of the XI International Conference on High Energy Accelerators* (Birkhauser, Basel, 1980), p. 201.
[2]I. Haber, Proceedings of the Heavy Ion Fusion Workshop, Argonne National Laboratory Report ANL-79-41 (1978); G. Lambertson, L. J. Laslett, and L. Smith, IEEE Trans. Nucl. Sci. **NS-24**, 993 (1977).
[3]V. K. Neil and A. M. Sessler, Rev. Sci. Instrum. **36**, 429 (1965).
[4]D. Mohl, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 315.
[5]Kwang Je Kim, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 187.
[6]A. Faltens, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 182.
[7]P. Morse and H. Feshback, *Methods in Theoretical Physics, Part II* (McGraw-Hill, New York, 1953), p. 1015.
[8]D. Neuffer, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-1031 (1980), p. 245.
[9]J. M. Wang and C. Pellegrini, *Proceedings of the XI International Conference on High Energy Accelerators* (Birkhauser, Basel, 1980), p. 554.

Downloaded 11 Mar 2008 to 130.54.110.22. Redistribution subject to AIP license or copyright; see http://apl.aip.org/apl/copyright.jsp