Composite fermion pairing theory in single-layer systems

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We study the pairing state of composite fermions (CF's) at even denominator Landau level fillings. We introduce the composite fermion operators by the Rajaraman-Sondhi nonunitary transformation. The resulting Hamiltonian has a non-Hermitian term. We show that this non-Hermitian term has the effect of destabilizing composite fermions. However, composite fermions are stabilized when the short-range Coulomb interaction is strong enough. Projecting into the Hilbert space where composite fermions are stabilized, we derive the effective Hamiltonian for CF's. Based on this Hamiltonian we discuss the condition for pairing of composite fermions within mean-field theory. We show that the pairing condition is satisfied at $\nu=5/2$ for GaAs/AlGaAs heterojunctions because of the screening effect of the long-range Coulomb interaction induced by the filled Landau levels. We also consider the condition for the pairing state at $\nu=3/2$ and $\nu=1/2$. The absence of the pairing state at half filled high Landau levels is understood as the breakdown of composite fermions because of the reduction of the short-range Coulomb interaction. The instability of the $\nu=5/2$ state against an in-plane magnetic field is also understood as the breakdown of composite fermions. Comparison of the ground state energy reveals the polarization of spins.

I. INTRODUCTION

Two-dimensional electron systems have attracted many theoretical and experimental studies. The fractional quantum Hall system is a typical example.[1] Several years of intensive study reveal that it has very rich structures. The quantum Hall effect is observed in two-dimensional electron systems under a strong magnetic field. It is usually divided into two categories. One is the integral quantum Hall effect and the other is the fractional quantum Hall effect. For the latter, the Coulomb interaction is essential for its occurrence. The nature of the ground state of the fractional quantum Hall state is well captured by the Laughlin wave function.[2] At the Landau level filling fraction $\nu=1/m$, where m is an odd integer, it is given by

$$\Psi(z_1,z_2,\ldots,z_N)=\prod_{i<j}(z_i-z_j)^m\exp\left(-\frac{1}{4l_B^2}\sum_j|z_j|^2\right), \tag{1}$$

where the set z_j $(j=1,2,\ldots,N)$ are the coordinates of the N electrons in complex notation $z=x+iy$ and $l_B=\sqrt{c\hbar/eB}$ is the magnetic length. The Laughlin wave function shows that there exists a strong repulsion between electrons because $(z_i-z_j)^m\to 0$ for $z_i\to z_j$. This strong repulsion comes from the Coulomb interaction and this fact tells us that the Coulomb interaction is important for the fractional quantum Hall effect. We can get some insight into the nature of this Coulomb interaction when we describe it by Haldane's pseudopotential.[3] Haldane decomposed the Coulomb interaction into components according to the relative angular momentum of electron pairs:

$$V=\sum_{i<j}\sum_{m=0}^{\infty}V_m P_m^{ij}, \tag{2}$$

where P_m^{ij} is the projection operator on states with the relative angular momentum of the ith and jth electrons equal to m and V_m are the energies of pairs of particles with relative angular momentum m. For the fractional quantum Hall state with $\nu=1/m$, the components V_j with $j=1,3,\ldots,m-2$, contribute a strong repulsion between electrons, and other components may lead to some unimportant modification to the Laughlin wave function. In fact, the ground state of the Hamiltonian with $V_j\neq 0$ $(j=1,3,\ldots,m-2)$ and $V_j=0$ $(j=m,m+2,\ldots)$ is exactly the Laughlin wave function.[3] Therefore, the component V_j with $j\leqslant m-2$, which is the short-range part of the Coulomb interaction, is essential for the fractional quantum Hall effect.

As an effective theory of the fractional quantum Hall effect, there is the Chern-Simons gauge field theory.[4,5] Since the system is two dimensional, we can transform the electron system into a boson system by a flux attachment. For the case of $\nu=1/m$, we map electrons into composite particles with attached $m\phi_0$ $(\phi_0=ch/e)$ flux. The Aharonov-Bohm phase arising from this fictitious flux is $m\pi$ for an interchange of the positions of two composite particles. To reproduce the Fermi statistics of electrons, the particle should be a boson, which we call a composite boson, for m odd and a fermion, which we call a composite fermion (CF), for m even. At mean-field level, the fictitious fluxes completely cancel the external magnetic fluxes and the Bose condensation is to be expected.[4,5] After including the phase fluctuation of the bose field, we reproduce the Laughlin wave function.[5] Thus, the fractional quantum Hall effect at odd denominator fillings is understood as the Bose condensation of composite bosons.

However, the importance of the short-range Coulomb interaction is not clear in this composite boson theory. We meet the same situation when we consider a system of composite fermions (CF's), because in that case the ground state wave function may contain a factor like the Laughlin wave function Eq. (1) with m even. This is one of the most impor-

tant points that we need to take care of when we study a system of composite particles.

The fractional quantum Hall effect is not limited to the odd denominator fillings. The fractional quantum Hall effect at $\nu=5/2$ was observed by Willett *et al.* in 1987.[6] Immediately after the discovery of the $\nu=5/2$ state, a spin-singlet *d*-wave pairing wave function was proposed.[7] This spin-singlet wave function seemed to explain naturally the fact that the $\nu=5/2$ state is unstable in the presence of an in-plane magnetic field.[8] However, recent numerical work supports a spin-polarized pairing state.[9] When we map the system into that of composite particles, they obey Fermi statistics because we should attach an even number of fluxes to particles according to the denominator of ν. If we apply the condensation scenario to this CF system, we need some pairing mechanism between CF's. Such a pairing interaction was derived by Greiter, Wen, and Wilczek.[10] A Chern-Simons gauge field fluctuation leads to *p*-wave pairing. However, recently Bonesteel showed that other Chern-Simons gauge field fluctuations lead to a pair-breaking effect.[11] Further, the fractional quantum Hall effect at even denominator is observed only at $\nu=5/2$. At $\nu=1/2$ the Hall resistance is linear in a magnetic field and the longitudinal resistivity shows a deep broad minimum,[12] and at $\nu=9/2$ and $\nu=11/2$ anisotropy in the longitudinal resistivity is observed.[13]

In this paper we study a CF system using the Rajaraman-Sondhi nonunitary transformation, which fully takes into account the basic electron correlation for CF's. We show that the Hamiltonian for CF's contains a term that destabilizes CF's as well as the attractive interaction term that leads to the *p*-wave pairing of CF's. The former appears in the Hamiltonian as a non-Hermitian term. We clarify under what condition CF's are stabilized and derive the effective Hamiltonian for CF's. The importance of the short-range Coulomb interaction is stressed. The effect of filled Landau levels is taken into account as the screening of the Coulomb interaction between CF's. Based on this Hamiltonian, we show within mean-field theory that the pairing condition is satisfied at $\nu=5/2$ for GaAs/AlGaAs heterojunctions. We also consider the condition for the pairing state at $\nu=1/2$ and $\nu=3/2$. On the effect of an in-plane magnetic field, we show that such a field destabilizes CF's because of the reduction of the pseudopotential between electrons. The polarization of spins is understood from calculation of the condensation energy combined with the fact that the experimentally observed state at $\nu=1/2$ is spin polarized.

The paper is organized as follows. In Sec. II we introduce the CF operators using the Rajaraman-Sondhi nonunitary transformation. In Sec. III we investigate the nature of the non-Hermitian term and show that this term has the effect of destabilizing CF's. After clarifying the condition for the stability of CF's, we derive the effective Hamiltonian for CF's. In Sec. IV we derive the equations for the mean-field theory. In Sec. V we examine the condition for the pairing state at $\nu=n+\frac{1}{2}$ $(n=0,1,2)$. In Sec. VI we consider the effect of an in-plane magnetic field. In Sec. VII we discuss the spin polarization of the pairing state. Section VIII is devoted to the conclusions. In Appendix A we give the relation between the wave functions of electrons and CF's. In Appendix B we give an analysis of the pairing state in the simplest case.

II. COMPOSITE FERMION OPERATORS

We consider a two-dimensional system of spinless electrons with no impurities and subjected to a magnetic field that is perpendicular to the plane of electrons. For the Landau level filling we consider the case of $\nu=n+1/\tilde{\phi}$ with n an integer and $\tilde{\phi}$ an even integer. The second quantized form of the Hamiltonian for electrons is given by

$$H=K+V^C, \tag{3}$$

where K is the kinetic energy operator

$$K=\int d^2\mathbf{r}\ \ \psi_e^\dagger(\mathbf{r})\frac{1}{2m_b}\left(-i\hbar\boldsymbol{\nabla}+\frac{e}{c}\mathbf{A}\right)^2\psi_e(\mathbf{r}) \tag{4}$$

and V^C is the operator for the Coulomb interaction between electrons. Here the magnetic field is $\boldsymbol{\nabla}\times\mathbf{A}=-B$ $(B>0)$, m_b is the band mass of the electrons, and the operator $\psi_e^\dagger(\mathbf{r})$ $[\psi_e(\mathbf{r})]$ is the creation (annihilation) operator for an electron at point $\mathbf{r}$.

We assume that we can neglect the mixing effect between filled Landau levels and first treat the filled Landau levels $\nu_f=n$ and the partially filled Landau level $\nu_p=1/\tilde{\phi}$ separately. We concentrate on the latter for a while. We take into account the effect of the former in Sec. III. In order to describe the partially filled Landau level, we introduce extended CF operators by Rajaraman and Sondhi's non-unitary transformation,[14,15] which is given by

$$\psi(\mathbf{r})=e^{-J(\mathbf{r})}\psi_e(\mathbf{r}),$$

$$\pi(\mathbf{r})=\psi_e^\dagger(\mathbf{r})e^{J(\mathbf{r})}, \tag{5}$$

where the function $J(\mathbf{r})$ is defined by

$$J(\mathbf{r})=\tilde{\phi}\int d^2\mathbf{r}'\rho(\mathbf{r}')\ln(z-z')-\frac{1}{4l_B^2}|z|^2. \tag{6}$$

Here $\rho(\mathbf{r})=\psi_e^\dagger(\mathbf{r})\psi_e(\mathbf{r})=\pi(\mathbf{r})\psi(\mathbf{r})$ is the density of particles at point $\mathbf{r}$ and $z=x+iy$ is the complex coordinate in the plane. If we retain only the imaginary part of $J(\mathbf{r})$ in Eq. (6), the transformation Eq. (5) gives the usual singular gauge transformation.[4,5] Operators $\psi(\mathbf{r})$ and $\pi(\mathbf{r})$ satisfy the following anticommutation relations:

$$\psi(\mathbf{r})\pi(\mathbf{r}')+\pi(\mathbf{r}')\psi(\mathbf{r})=\delta^{(2)}(\mathbf{r}-\mathbf{r}'),$$

$$\psi(\mathbf{r})\psi(\mathbf{r}')+\psi(\mathbf{r}')\psi(\mathbf{r})=0, \tag{7}$$

$$\pi(\mathbf{r})\pi(\mathbf{r}')+\pi(\mathbf{r}')\pi(\mathbf{r})=0.$$

In deriving these equations we have used $(-1)^{\tilde{\phi}}=1$. In terms of these operators, we may write the kinetic energy operator K in the form

$$K=\int d^2\mathbf{r}\ \ \pi(\mathbf{r})\frac{1}{2m_b}\left[-i\hbar\boldsymbol{\nabla}+\frac{e}{c}(\delta\mathbf{a}+i\hat{\boldsymbol{e}}_z\times\delta\mathbf{a})\right]^2\psi(\mathbf{r}), \tag{8}$$

where $\hat{\boldsymbol{e}}_z$ is a unit vector normal to the layer and $\delta\mathbf{a}$ is the fluctuation of the Chern-Simons gauge field:

$$\delta\mathbf{a}(\mathbf{r})=\frac{\phi_0}{2\pi}\tilde{\phi}\int d^2\mathbf{r}'\,\delta\rho(\mathbf{r}')\boldsymbol{\nabla}\,\mathrm{Im}\ln(z-z'), \tag{9}$$

with $\delta\rho(\mathbf{r})=\pi(\mathbf{r})\psi(\mathbf{r})-\bar{\rho}$ the fluctuation of the density ($\bar{\rho}$ is the average particle density). From Eq. (8), we obtain

$$K=K^0+V^H+V^{NH}, \tag{10}$$

where K^0 is the kinetic energy operator for CF's and V^H and V^{NH} describe interactions between the CF fields and the Chern-Simons gauge fields:

$$K^0=\int d^2\mathbf{r}\;\;\pi(\mathbf{r})\left(-\frac{\hbar^2}{2m_b}\boldsymbol{\nabla}^2\right)\psi(\mathbf{r}), \tag{11}$$

$$V^H=\int d^2\mathbf{r}\;\;\frac{e}{c}\,\delta\mathbf{a}\cdot\mathbf{j}_{\mathrm{CF}}, \tag{12}$$

$$V^{NH}=\int d^2\mathbf{r}\;\;\frac{e}{c}(i\hat{\boldsymbol{e}}_z\times\delta\mathbf{a})\cdot\mathbf{j}_{\mathrm{CF}}. \tag{13}$$

Here we have introduced the current operator for CF's,

$$\mathbf{j}_{\mathrm{CF}}=\frac{\hbar}{2m_b i}[\pi(\mathbf{r})\boldsymbol{\nabla}\psi(\mathbf{r})-[\boldsymbol{\nabla}\pi(\mathbf{r})]\psi(\mathbf{r})]. \tag{14}$$

V^H has the form of minimal coupling between CF's and the Chern-Simons gauge field fluctuation and it leads to the p-wave pairing of CF's (see Appendix B).[10] V^{NH} describes the minimal coupling between CF's and an imaginary vector potential $i\hat{\boldsymbol{e}}_z\times\delta\mathbf{a}$. This term is non-Hermitian. In the next section we study the effect of it.

By Rajaraman-Sondhi's nonunitary transformation we can fully take into account the Laughlin type correlation between electrons. This is clearly understood from the relation between the wave function for CF's and that for electrons, which is derived in Appendix A.

III. STABILITY OF COMPOSITE FERMIONS AND THE EFFECTIVE HAMILTONIAN

In order to understand the quantum mechanical effect of V^{NH}, we consider a two-electron problem. Since the center-of-mass motion is unimportant for our purpose, we concentrate on their relative motion. The Hamiltonian is given by

$$H_2=\frac{1}{m_b}\boldsymbol{\pi}^2, \tag{15}$$

with

$$\boldsymbol{\pi}=-i\hbar\boldsymbol{\nabla}-\frac{eB}{4c}\hat{\boldsymbol{e}}_z\times\mathbf{r}. \tag{16}$$

Here $\mathbf{r}$ is the relative coordinate for two electrons. We neglect the Coulomb interaction for a while. The problem is solved exactly and the wave function with relative angular momentum m in the lowest Landau level is given by $\psi_m(z)=z^m\exp(-|z|^2/8l_B^2)$. The first quantized form of the Rajaraman-Sondhi transformation for two electrons is given by

$$H_2'=[\psi_m(z)]^{-1}H_2\psi_m(z), \tag{17}$$

where H_2' describes the relative motion of two CF's,

$$H_2'=-\frac{\hbar^2}{m_b}\nabla^2+\frac{\hbar\omega_c}{2}+\left(\frac{ml_B^2}{r^2}-\frac{1}{4}\right)\omega_c(L_z-i\mathbf{r}\cdot\mathbf{p}). \tag{18}$$

The term proportional to $L_z=-i\hbar\partial_\theta$ ($\theta=\tan^{-1}y/x$), is Hermitian and corresponds to V^H. The term proportional to $i\mathbf{r}\cdot\mathbf{p}$ is non-Hermitian (anti-Hermitian) and corresponds to V^{NH}. Since the operator $\mathbf{r}\cdot\mathbf{p}$ may have small matrix elements for motion with nonzero angular momentum, we expect that V^{NH} is unimportant for such motion.

In order to reveal the effect of V^{NH} on CF's, we describe it by the operators for *electrons*. We introduce the creation and annihilation operators for the Landau level by

$$a^\dagger=\frac{1}{\hbar l_B}(\pi_x-i\pi_y), \tag{19}$$

$$a=\frac{1}{\hbar l_B}(\pi_x+i\pi_y), \tag{20}$$

and those for the relative angular momentum by

$$b=\frac{1}{2l_B}(iX+Y), \tag{21}$$

$$b^\dagger=\frac{1}{2l_B}(-iX+Y). \tag{22}$$

Here $(X,Y)=\mathbf{r}-2l_B^2\hat{\mathbf{e}}_z\times\boldsymbol{\pi}/\hbar$ is the operator for the coordinate of the center of mass. In terms of these operators we obtain

$$L_z/\hbar=-a^\dagger a+b^\dagger b, \tag{23}$$

$$i\mathbf{r}\cdot\mathbf{p}/\hbar=a^\dagger b-ab^\dagger. \tag{24}$$

Note that the former does not affect the relative angular momentum m of *electrons*. Meanwhile, the latter changes the value of m to $m\pm1$. Since m is related to the electron correlation which gives rise to CF's, we may say that the non-Hermitian term has the effect of destabilizing CF's.

The presence of the term that destroys CF's can be understood by a simple argument. Even if we consider a system that completely lacks the Coulomb interaction, there is no obstruction to performing the transformation from the electron system into the CF system. However, in this case CF's would not be stable quasiparticles because of the absence of the short-range Coulomb interaction, which stabilizes composite fermions. Therefore, the Hamiltonian for CF's must contain a term that destroy CF's. Such a term is the counter-term to the short-range Coulomb interaction because it stabilizes CF's.

In fact, if we take into account the Coulomb interaction, the energy becomes m dependent and decrease of m increases the short-range part of the Coulomb energy. In Haldane's pseudopotential description, the decrease of m is prevented by the gap $V_{m-2}-V_m$. On the other hand, increase of m has the gap $\hbar\omega_c$ for the density fluctuation (Kohn's theorem[16]). Therefore, we can say that the change of m has the gap $\Delta_{\mathrm{CF}}\equiv\min\{\hbar\omega_c,V_{m-2}-V_m\}$. Recalling the fact

that the correlation, which is described by Eq. (1) with even m, between electrons consists of the two-body correlation only and the number of fluxes attached to each CF corresponds to the relative angular momentum of the electron pair, we may say that the mode to destroy CF's has the gap $\Delta_{\rm CF}$.

However, if we are concerned with an energy scale much lower than $\Delta_{\rm CF}$, we can make the projection into the subspace where CF's are stable quasiparticles. In that case, we can neglect V^{NH} and the effective Hamiltonian for CF's is given by

$$H=H^0+\frac{1}{2\Omega}\sum_{\mathbf{k}_1\neq\mathbf{k}_2}\sum_{\mathbf{q}} V_{\mathbf{k}_1,\mathbf{k}_2}\pi_{\mathbf{k}_1+\mathbf{q}/2}\pi_{-\mathbf{k}_1+\mathbf{q}/2} \times\psi_{-\mathbf{k}_2+\mathbf{q}/2}\psi_{\mathbf{k}_2+\mathbf{q}/2}, \tag{25}$$

with Ω the area of the system. Here $V_{\mathbf{k}_1\mathbf{k}_2}=V^H_{\mathbf{k}_1\mathbf{k}_2}+V^{\rm LC}_{\mathbf{k}_1\mathbf{k}_2}$, where from Eqs. (9) and (12) $V^H_{\mathbf{k}_1\mathbf{k}_2}$ is given by

$$V^H_{\mathbf{k}_1\mathbf{k}_2}=\frac{4\pi i}{m_b}\tilde{\phi}\frac{\mathbf{k}_1\times\mathbf{k}_2}{|\mathbf{k}_1-\mathbf{k}_2|^2}, \tag{26}$$

and $V^{\rm LC}_{\mathbf{k}_1\mathbf{k}_2}$ denotes the long-range part of the Coulomb interaction. As long as we fix the filling fraction, we do not need the short-range part of it. We need it when we consider quantum Hall systems of CF's. In the presence of filled Landau levels, we take $V^{\rm LC}_{\mathbf{k}_1\mathbf{k}_2}$ as the form given by Aleiner and Glazman, which takes into account the screening effect by electrons in filled Landau levels.[17] They derived the effective interaction in a partially filled Landau level by integrating out electron fields in filled Landau levels. The static dielectric function of filled Landau levels is given by[17]

$$\epsilon_\sigma(q)=1+\frac{4\sigma\lambda}{ql_B}e^{-q^2l_B^2/2}f\left(\frac{q^2l_B^2}{2}\right), \tag{27}$$

where $f(z)=\int_0^z dx(e^x-1)/x$ and we set $\sigma=1$ for $\nu=3/2$ and $\sigma=2$ for $\nu=5/2$ because there is one $\nu=1$ filled Landau level in the case of $\nu=3/2$ and there are two $\nu=1$ filled Landau levels for $\uparrow$ spin and $\downarrow$ spin in the case of $\nu=5/2$.[18] Here λ is given by $\lambda=(e^2/\epsilon l_B)/\hbar\omega_c=38(m_b/m_e)/\sqrt{B}$, where ω_c is the cyclotron frequency, m_e is the electron mass in the vacuum, and the applied external magnetic field B is measured in units of teslas. Using Eq. (27), the long-range part of the Coulomb interaction is given by

$$V^{\rm LC}_{\mathbf{k}_1\mathbf{k}_2}=\frac{V^C_{\mathbf{k}_1-\mathbf{k}_2}}{\epsilon_\sigma(|\mathbf{k}_1-\mathbf{k}_2|)}, \tag{28}$$

where $V^C_{\mathbf{q}}=2\pi e^2/\epsilon q$ is the bare Coulomb interaction in the absence of screening by electrons in filled Landau levels.

IV. MEAN-FIELD THEORY

We study the possibility of the pairing of CF's based on the Hamiltonian (25) within mean-field theory. We take as the mean fields $\langle\pi_{\mathbf{k}}\pi_{\mathbf{k}'}\rangle$, $\langle\psi_{\mathbf{k}}\psi_{\mathbf{k}'}\rangle$, and $\langle\pi_{\mathbf{k}}\psi_{\mathbf{k}'}\rangle$. Because of the constraint $\mathbf{k}_1\neq\mathbf{k}_2$ for the summation in the interaction term, the mean fields $\langle\pi_{\mathbf{k}_1+\mathbf{q}/2}\psi_{\mathbf{k}_2+\mathbf{q}/2}\rangle$ and $\langle\pi_{-\mathbf{k}_1+\mathbf{q}/2}\psi_{-\mathbf{k}_2+\mathbf{q}/2}\rangle$ are absent. We consider an equilibrium state and set $\mathbf{q}=\mathbf{0}$ in these mean-fields. Introducing the gap functions $\Delta_{\mathbf{k}}$ and $\overline{\Delta}_{\mathbf{k}}$,

$$\Delta_{\mathbf{k}}=-\frac{1}{\Omega}\sum_{\mathbf{k}'(\neq\mathbf{k})}V_{\mathbf{k}\mathbf{k}'}\langle\psi_{-\mathbf{k}'}\psi_{\mathbf{k}'}\rangle, \tag{29}$$

$$\overline{\Delta}_{\mathbf{k}}=-\frac{1}{\Omega}\sum_{\mathbf{k}'(\neq\mathbf{k})}V_{\mathbf{k}'\mathbf{k}}\langle\pi_{\mathbf{k}'}\pi_{-\mathbf{k}'}\rangle, \tag{30}$$

the mean-field Hamiltonian reads

$$H_{\rm MF}\simeq\sum\ \xi^*_k\pi_{\mathbf{k}}\psi_{\mathbf{k}}-\frac{1}{2}\sum_{\mathbf{k}}(\pi_{\mathbf{k}}\pi_{-\mathbf{k}}\Delta_{\mathbf{k}}+\overline{\Delta}_{\mathbf{k}}\psi_{-\mathbf{k}}\psi_{\mathbf{k}})+\text{const}, \tag{31}$$

with ξ^*_k the renormalized kinetic energy,

$$\xi^*_k=\xi_k-\frac{1}{\Omega}\sum_{\mathbf{k}'(\neq\mathbf{k})}V_{(\mathbf{k}-\mathbf{k}')/2,(\mathbf{k}'-\mathbf{k})/2}\langle\pi_{\mathbf{k}'}\psi_{\mathbf{k}'}\rangle. \tag{32}$$

Note that both V^H and V^{NH} have no contribution to ξ^*_k because $V^{(N,NH)}_{\mathbf{k},-\mathbf{k}}=0$. Only the Coulomb interaction term contributes to ξ^*_k. Introducing the two-component description

$${}^t\pi^{\mathbf{k}}=[\pi_{\mathbf{k}}\ \psi_{-\mathbf{k}}],\quad \psi^{\mathbf{k}}=\begin{bmatrix}\psi_{\mathbf{k}}\\ \pi_{-\mathbf{k}}\end{bmatrix}, \tag{33}$$

Eq. (31) reads

$$H_{\rm MF}=\sum_{\mathbf{k}}{}'\ {}^t\pi^{\mathbf{k}}\mathcal{E}^{\mathbf{k}}\psi^{\mathbf{k}}+\text{const}, \tag{34}$$

where $\Sigma'_{\mathbf{k}}$ denotes $\Sigma_{k_x>0,k_y}$ and

$$\mathcal{E}^{\mathbf{k}}=\begin{bmatrix}\xi^*_{\mathbf{k}} & -\Delta_{\mathbf{k}}\\ -\overline{\Delta}_{\mathbf{k}} & -\xi^*_{\mathbf{k}}\end{bmatrix}. \tag{35}$$

We define quasiparticle field operators $q_{\mathbf{k}}$ and $p_{\mathbf{k}}$,

$$q^{\mathbf{k}}=\begin{bmatrix}q_{\mathbf{k}}\\ p_{-\mathbf{k}}\end{bmatrix}=\overline{U}^{\mathbf{k}}\psi^{\mathbf{k}}, \tag{36}$$

$${}^tp^{\mathbf{k}}=[p_{\mathbf{k}}\quad q_{-\mathbf{k}}]={}^t\pi^{\mathbf{k}}\overline{U}^{\mathbf{k}}, \tag{37}$$

where

$$U^{\mathbf{k}}=\overline{U}^{\mathbf{k}}=\frac{1}{\sqrt{2E_{\mathbf{k}}(E_{\mathbf{k}}+\xi^*_k)}}\begin{bmatrix}-\xi^*_k-E_{\mathbf{k}} & \Delta_{\mathbf{k}}\\ \overline{\Delta}_{\mathbf{k}} & \xi^*_k+E_{\mathbf{k}}\end{bmatrix}. \tag{38}$$

In terms of these quasiparticle fields, the Hamiltonian is diagonalized as

$$H_{\rm MF}=\sum_{\mathbf{k}}{}'\ {}^tp^{\mathbf{k}}\begin{bmatrix}E_{\mathbf{k}} & 0\\ 0 & -E_{\mathbf{k}}\end{bmatrix}q^{\mathbf{k}}+\text{const}. \tag{39}$$

Here $E_{\mathbf{k}}\equiv\sqrt{\xi^{*2}_k+\overline{\Delta}_{\mathbf{k}}\Delta_{\mathbf{k}}}$ is the quasiparticle energy. Since $p_{\mathbf{k}}$ and $q_{\mathbf{k}}$ satisfy the anticommutation relations, we obtain

$$\langle p_{\mathbf{k}}q_{\mathbf{k}}\rangle=\frac{1}{\exp(\beta E_{\mathbf{k}})+1}, \tag{40}$$

$$\langle q_{-\mathbf{k}} p_{-\mathbf{k}} \rangle = \frac{1}{\exp(-\beta E_{\mathbf{k}})+1}, \tag{41}$$

with $\beta = 1/k_B T$.

In the following analysis we concentrate on the ground-state properties. From Eqs. (29) and (30), we obtain the gap equations

$$\Delta_{\mathbf{k}} = -\frac{1}{2\Omega} \sum_{\mathbf{k}'(\neq \mathbf{k})} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{E_{\mathbf{k}'}}, \tag{42}$$

$$\overline{\Delta}_{\mathbf{k}} = -\frac{1}{2\Omega} \sum_{\mathbf{k}'(\neq \mathbf{k})} V_{\mathbf{k}'\mathbf{k}} \frac{\overline{\Delta}_{\mathbf{k}'}}{E_{\mathbf{k}'}}. \tag{43}$$

From Eq. (32), we obtain

$$\xi_k^* = \xi_k - \frac{1}{2\Omega} \sum_{\mathbf{k}'(\neq \mathbf{k})} V^{\mathrm{LC}}_{(\mathbf{k}-\mathbf{k}')/2,(\mathbf{k}'-\mathbf{k})/2} \left(1 - \frac{\xi^*_{k'}}{E_{\mathbf{k}'}}\right). \tag{44}$$

Within the mean-field approximation, the ground-state energy E_{gs} is given by

$$E_{\mathrm{gs}} \simeq \frac{1}{2} \sum_{\mathbf{k}} \xi_k^* \left(1 - \frac{\xi_k^*}{E_{\mathbf{k}}}\right) - \frac{1}{4} \sum_{\mathbf{k}} \frac{\overline{\Delta}_{\mathbf{k}} \Delta_{\mathbf{k}}}{E_{\mathbf{k}}}. \tag{45}$$

Next we derive the ground-state wave function of CF's. In the ground-state there are no quasiparticles. Therefore, the ground-state $|\mathrm{gs}\rangle$ satisfies $q_{\mathbf{k}}|\mathrm{gs}\rangle = 0$ and $q_{-\mathbf{k}}|\mathrm{gs}\rangle = 0$. Using the CF operators, these equations become

$$(\xi_k^* + E_{\mathbf{k}}) \psi_{\mathbf{k}} |\mathrm{gs}\rangle = \Delta_{\mathbf{k}} \pi_{-\mathbf{k}} |\mathrm{gs}\rangle, \tag{46}$$

$$\Delta_{\mathbf{k}} \pi_{\mathbf{k}} |\mathrm{gs}\rangle = -(\xi_k^* + E_{\mathbf{k}}) \psi_{-\mathbf{k}} |\mathrm{gs}\rangle. \tag{47}$$

Since $\pi_{\mathbf{k}}$ and $\psi_{\mathbf{k}}$ satisfy the anticommutation relations, we can replace $\psi_{\mathbf{k}}$ by $\partial/\partial \pi_{\mathbf{k}}$.[19] Applying this replacement, Eqs. (46) and (47) become differential equations with respect to $\pi_{\mathbf{k}}$. Solving these equations, we obtain

$$|\mathrm{gs}\rangle = \exp\left[\sum_{\mathbf{k}}{}' \frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}} + \xi_k^*} \pi_{\mathbf{k}} \pi_{-\mathbf{k}} \right] |0\rangle. \tag{48}$$

The real space form of the ground state $|\mathrm{gs}\rangle$ is given by[20]

$$\Psi_{\mathrm{CF}}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{2N}) = \langle 0 | \psi(\mathbf{r}_{2N}) \cdots \psi(\mathbf{r}_2) \psi(\mathbf{r}_1) |\mathrm{gs}\rangle = \mathrm{Pf}\, \phi_{\mathrm{orb}}(\mathbf{r}_i - \mathbf{r}_j), \tag{49}$$

where $\mathrm{Pf} M_{ij} = \mathcal{A}(M_{12} M_{34} \cdots M_{2N-1,2N})$ with $\mathcal{A}$ the antisymmetrization operator of the entire function, and the orbital wave function $\phi_{\mathrm{orb}}(\mathbf{r})$ is given by

$$\phi_{\mathrm{orb}}(\mathbf{r}) = \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}} + \xi_k^*} \exp(i\mathbf{k}\cdot\mathbf{r}). \tag{50}$$

In Appendix B we give an analysis of the pairing state for the case of $V_{\mathbf{k}_1\mathbf{k}_2} = V^H_{\mathbf{k}_1\mathbf{k}_2}$. In that case we find that the ground state is the p-wave pairing state of CF's,[10] and the ground-state wave function is the so-called Pfaffian state [see Eq. (B13)].

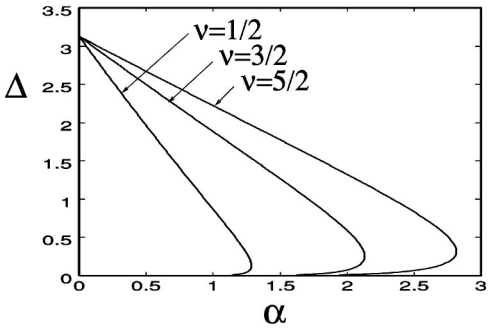

FIG. 1. The Coulomb energy α [$\equiv (e^2/\epsilon l_B)/\epsilon_F$] dependence of the gap Δ. In evaluating the screening effect, we set $\lambda = 1.2$, which is the value at $B = 5$ T.

V. POSSIBILITY OF THE PAIRING STATE

In this section we investigate the possibility of pairing at $\nu = n + \frac{1}{2}$ with n integer. Since V^H leads to the p-wave pairing state[10] as we show in Appendix B, and the long-range part of the Coulomb interaction V^{LC} has a pair-breaking effect, we consider the stability of the p-wave pairing state in the presence of V^{LC}. We take the same form of the gap function $\Delta_{\mathbf{k}}$ as in Appendix B [Eqs. (B1) and (B8) with $l = 1$].

In Fig. 1, we show the $\alpha \equiv (e^2/\epsilon l_B)/\epsilon_F$ dependence of the gap Δ. At each filling, the qualitative behavior of Δ is almost the same. The gap Δ goes to zero around a critical value α_c. When $\alpha < \alpha_c$ is satisfied, pairing occurs. The behavior near $\Delta \sim 0$ is not important. It may arise from the failure of the approximation Eq. (B8) in solving the gap equation because of the presence of V^{LC}. Extrapolating the behavior of Δ from the region where Δ is a monotonically decreasing function with respect to α, we roughly estimate the critical value α_c. From Fig. 1, we estimate α_c for each filling, and obtain $\alpha_c^{(\nu=1/2)} \sim 1.3$, $\alpha_c^{(\nu=3/2)} \sim 2.2$, and $\alpha_c^{(\nu=5/2)} \sim 3.0$. Note that the critical value of α increases with additional filled Landau levels. Therefore, the pairing state at $\nu = 5/2$ is more stable than that at $\nu = 1/2$ or $\nu = 3/2$.

To find whether pairing occurs or not, we estimate the value of α for GaAs/AlGaAs heterojunctions. By definition, we obtain $\alpha = 75.6(m^*/m_e)/\sqrt{B}$, where m^* is the effective mass of CF's, B is measured in units of teslas, and we have set $\epsilon = 13$. Since we assume the irrelevance of V^{NH}, m^* approaches $m_b = 0.07 m_e$ in the limit $\Delta \to +0$. In that case, α is given by $\alpha = 5.3/\sqrt{B}$. Substituting $B = 5$ T into this, we obtain $\alpha = 2.4$. This value of α is lower than $\alpha_c^{(\nu=5/2)}$. Since the magnetic field used by Willett *et al.*[6] was ~ 5 T, the condition for the pairing of CF's was satisfied there. Thus, we can understand the existence of the pairing state at $\nu = 5/2$.

For the case of $\nu = 3/2$, the pairing condition $\alpha < \alpha_c^{(\nu=3/2)}$ is satisfied when the applied magnetic field is larger than 6 T. For the electron charge density n, this condition corresponds to $n > 2.2 \times 10^{11}$ cm^{-2}. A large magnetic field is a more plausible cause for the pairing state of CF's. Therefore, the pairing state at $\nu = 3/2$ may be realized at sufficiently large but still realistic magnetic field.

Now we discuss the possibility of a pairing state at $\nu = 1/2$. We solve Eqs. (42) and (44) self-consistently. In order to set $\xi^*_{k=k_F} = 0$, we change Eq. (44) to

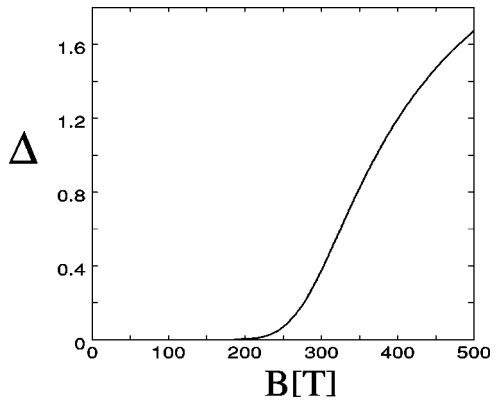

FIG. 2. The magnetic field dependence of the gap Δ at $\nu=1/2$.

$$\xi_k^* = \xi_k - (F_k - F_{k=k_F}), \tag{51}$$

where F_k is given by

$$F_k = \frac{1}{\Omega}\sum_{\mathbf{k}'\neq\mathbf{k}} V^C_{\mathbf{k}-\mathbf{k}'}\left[\left(1-\frac{\xi^*_{k'}}{E_{\mathbf{k}'}}\right)-\left(1-\frac{\xi^*_{k'}}{E_{\mathbf{k}'}}\right)_{\bar{\Delta}_{\mathbf{k}'}\Delta_{\mathbf{k}'}=0}\right]. \tag{52}$$

Here we define Eq.(52) as the difference between the pairing state and the no-pairing state because the mass renormalization effect, which comes from the exchange interaction term of the Coulomb interaction, has already been included in the band mass m_b. Equation (51) is a nonlinear equation with respect to ξ_k^*. To solve this nonlinear equation, we assume that ξ_k^* has the form $\xi_k^* = (k^2 - k_F^2)/2m^*$. We evaluate m^* on the Fermi surface of the CF's:

$$\frac{1}{m^*} = \frac{1}{m_b} - \frac{1}{k_F}\left[\frac{dF_k}{dk}\right]_{k=k_F}. \tag{53}$$

Given the value of Δ, we obtain the value of α from the gap equation Eq. (42) and the value of m^*/m_b from Eq. (53). Setting the former to $\alpha_c(\Delta)$ the condition for the pairing state with a gap larger than Δ is written as $\alpha_c(\Delta) > \alpha_r$, where α_r is the value of α for a real sample. For GaAs/AlGaAs heterojunctions, we obtain

$$\alpha = 5.3\frac{m^*}{m_b}B^{-1/2}. \tag{54}$$

Substituting Eq. (53) into Eq. (54) and after some algebra, we see that B is a monotonically decreasing function with respect to α. Therefore, there is a critical value of the magnetic field above which a pairing state with a gap larger than Δ occurs. This value of the magnetic field is calculated by setting $\alpha = \alpha_c(\Delta)$ in Eq. (54). In Fig. 2, we show the gap Δ versus the magnetic field at $\nu=1/2$. Within our approximation, it seems that pairing occurs when the magnetic field is larger than $B_c \sim 200$ T. It seems that the quantum Hall effect at $\nu=1/2$ might not be impossible but hard to realize for GaAs/AlGaAs heterojunctions.[21]

Here we remark on the weak magnetic field limit. At half-filled high Landau levels, such as $\nu=9/2$ and $\nu=11/2$, the fractional quantum Hall effect is not observed.[13] To deal with these states, the Hamiltonian Eq. (25) may not be useful. In a weak magnetic field, the short-range part of the Coulomb interaction is not strong enough to stabilize CF's. At these fillings states based on CF's may not be stabilized by the effect of the non-Hermitian term. A charge density wave state or a Wigner crystal may be more plausible[22] than states based on CF's.

VI. EFFECT OF IN-PLANE MAGNETIC FIELD

Now we discuss the effect of tilting the magnetic field. Pan *et al.* observed that at $\nu=5/2$ an in-plane magnetic field induces a strong electrical anisotropy, which is similar to the behavior at half-filled high Landau levels.[23] Since we expect CF's not to be stable objects at half-filled high Landau levels, the collapse of the $\nu=5/2$ quantum Hall state may be understood as the breakdown of CF's rather than pair breaking of the pairing state of CF's. When we denote Haldane's pseudopotential as V_m^n for the nth Landau level, the gap $V_{m=0}^n - V_{m=2}^n \equiv \delta_n$ stabilizes CF's for short-range correlation. With increasing n, δ_n decreases as $\delta_0/(e^2/\epsilon l_B) = 0.5539$, $\delta_1/(e^2/\epsilon l_B) = 0.1592$, and $\delta_2/(e^2/\epsilon l_B) = 0.1330$.

When we apply an in-plane magnetic field, the value of δ_n decreases because the energy of an electron pair with $m=2$ is larger than that of an electron pair with $m=0$ in the presence of the in-plane magnetic field. Let us calculate the change of δ_n. For simplicity we assume a harmonic potential $V(z) = \frac{1}{2}m_b\Omega_c^2 z^2$ for the confining potential and that the angular momentum of the relative motion of electron pairs is parallel to the direction of the total magnetic field. In a tilted magnetic field with the axis of tilting parallel to the x axis, electron pairs experience the potential $V(y,z,\theta) = \frac{1}{2}m_b\Omega_c^2(z\cos\theta + y\sin\theta)^2$. When we set $\epsilon_m(\theta)$ as the energy of the electron pair with relative angular momentum m under a tilted magnetic field with tilting angle θ, calculation of the first order perturbation with respect to $V(y,z,\theta)$ yields $\epsilon_m(\theta) = (m+1)/2(\Omega_c/\omega_c)\hbar\Omega_c\sin^2(\theta/2)$. The point of collapse of the CF's may be determined by solving the equation $\delta_2 - \delta_1 = \epsilon_0(\theta) - \epsilon_2(\theta)$. For GaAs/AlGaAs heterojunctions $\hbar\omega_c \sim 8.5$ meV and $e^2/\epsilon l_B \sim 10$ meV at $B \sim 5$ T and the energy gap for the motion of the z direction is of the order of 10 meV. Substituting these values into the above equation and setting $\Omega_c \sim 10$ meV, we obtain $\theta \sim 10°$. Although the estimation is crude the order of magnitude of this value seems reasonable. Thus, we may understand the collapse of the spin-polarized pairing state as the breakdown of CF's.

VII. REAL SPIN DEGREES OF FREEDOM

In this section we discuss the effect of the real spin degrees of freedom and the Zeeman energy. To begin with, we discuss the former in the absence of the latter. There is the possibility of a spin-unpolarized pairing state. For electrons with spin, we can also perform the Rajaraman-Sondhi transformation.[15] In that case V^H becomes

$$V^H = \frac{1}{2\Omega}\sum_{\mathbf{k}_1\neq\mathbf{k}_2}\sum_{\mathbf{q}} \times K_{\alpha\beta}V^H_{\mathbf{k}_1,\mathbf{k}_2}\pi_{\mathbf{k}_1+\mathbf{q}/2}\pi_{-\mathbf{k}_1+\mathbf{q}/2}\psi_{-\mathbf{k}_2+\mathbf{q}/2}\psi_{\mathbf{k}_2+\mathbf{q}/2}, \tag{55}$$

where α and β, which take values $\uparrow$ or $\downarrow$, are indices for spins and the matrix K is given by

$$K=\begin{bmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 \\ \tilde{\phi}_2 & \tilde{\phi}_1 \end{bmatrix}$$

with $\tilde{\phi}_1$ and $\tilde{\phi}_2$ the number of fluxes attached to composite particles. The flux number $\tilde{\phi}_1$ is for particles with the same spin and the flux number $\tilde{\phi}_2$ is for particles with opposite spin. The Landau level filling fraction ν is related to $\tilde{\phi}_1$ and $\tilde{\phi}_2$ as $\nu=2/(\tilde{\phi}_1+\tilde{\phi}_2)$ up to filled Landau levels.

For the case of $\nu=1/2$, there are several choices for $\tilde{\phi}_1$ and $\tilde{\phi}_2$. We consider a system of CF's and set $\tilde{\phi}_1$ and $\tilde{\phi}_2$ as even integers. The case of $\tilde{\phi}_2=0$ should be excluded because the pairing state with this flux attachment is two independent $\nu=1/4$ pairing states. Also, the case of $\tilde{\phi}_1=0$ should be excluded because the cost in Coulomb energy is larger than for other flux attachments. From these arguments, we set $\tilde{\phi}_1=\tilde{\phi}_2=2$.[25]

Essentially the pairing interaction is the same as in the case of spinless CF's. Therefore, we may apply the discussion of the spinless CF's to multicomponent CF's. Energetically, the *p*-wave pairing state is also the most plausible one for multicomponent CF's. The *s*-wave pairing is impossible for CF's because V^H has no effect on the *s*-wave pairing. Since we consider the case of $\tilde{\phi}_1=\tilde{\phi}_2$, the pairing state that has the lowest ground-state energy is the $S_z=0$ pairing state.[25] For that pairing state the energy difference between the pairing state and the no-pairing state is given by

$$\delta E^{(2)}=-\frac{\Omega}{8\pi m^*}k_F^4 G(\Delta), \tag{56}$$

where $G(\Delta)$ is a function of Δ. For the spin-polarized pairing state Eq. (56) changes to

$$\delta E^{(1)}=-\frac{\Omega}{16\pi m^*}k_F^4 G(\Delta). \tag{57}$$

Comparing Eq. (56) with Eq. (57), $\delta E^{(1)}$ is half of $\delta E^{(2)}$. However, the Fermi wave number k_F is different for the spin-unpolarized pairing state and the spin-polarized pairing state. The former has $k_F^{(2)}=(\sqrt{2}l_B)^{-1}$ and the latter has $k_F^{(1)}=l_B^{-1}$. Substituting these equations into Eqs. (56) and (57), respectively, we obtain

$$\delta E^{(2)}=-\frac{\Omega}{32\pi m^*}l_B^{-4} G(\Delta), \tag{58}$$

$$\delta E^{(1)}=-\frac{\Omega}{16\pi m^*}l_B^{-4} G(\Delta). \tag{59}$$

Since l_B is the same for both the spin-unpolarized case and the spin-polarized case, $\delta E^{(1)}$ is twice $\delta E^{(2)}$. Therefore, for the spin-polarized pairing state the energy gain of being the pairing state is larger than that for the spin-unpolarized pairing state. Meanwhile, it is observed that a compressible liquid of $\nu=1/2$ has $k_F=l_B^{-1}$.[24] From this fact and $\delta E^{(1)}<\delta E^{(2)}<0$, we may conclude that if a pairing state is realized at the half-filled Landau level then it is the spin-polarized pairing state. The above scenario is also applicable for the case of the half-filled Landau level with filled Landau levels. The pairing state at $\nu=5/2$ may be the *p*-wave spin-polarized pairing state. For spin-polarized pairing states the effect of the Zeeman energy is just to shift the chemical potential of CF's. Such an effect may not cause any qualitative change to the pairing state.

VIII. CONCLUSIONS

In this paper, we have investigated the condition for a pairing state of CF's. We have introduced the CF operator by performing the Rajaraman-Sondhi nonunitary transformation. The Hamiltonian for CF's contains not only the attractive interaction that leads to the *p*-wave pairing state but also a term that destabilizes CF's. The latter appears in the Hamiltonian as a non-Hermitian term. When the short-range Coulomb interaction is strong enough, CF's may be stable and we can project the system into the subspace of states based on CF's.

For the long-range Coulomb interaction, this gives rise to a pair-breaking effect. In the presence of filled Landau levels, the long-range Coulomb interaction is screened. At $\nu=5/2$, this screening effect is enough to satisfy the pairing condition within the analysis of mean-field theory. At $\nu=3/2$, the necessary condition for pairing is $B>6$ T. At $\nu=1/2$, there is a critical magnetic field above which pairing occurs. This critical magnetic field is about 200 T for GaAs/AlGaAs heterojunctions. The pairing state at $\nu=1/2$ might not be impossible but will be hard to realize for GaAs/AlGaAs heterojunctions. At $\nu=9/2$, $\nu=11/2$, and other half-filled high Landau levels, CFs may not be stabilized because of the reduction of the short-range Coulomb interaction.

The instability of the $\nu=5/2$ state against an in-plane magnetic field is understood as a breakdown of CF's because it decreases the gap produced by the short-range Coulomb interaction. The resulting state may be a similar state to that observed at $\nu=9/2$ and $\nu=11/2$.

On the spin polarization of the pairing state we have compared the ground-state energy of the spin-unpolarized pairing state with that of the spin-polarized pairing state. Since the former is larger than the latter and an experimentally observed compressible liquid at $\nu=1/2$ is spin polarized, the pairing state may be spin polarized.

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APPENDIX A: WAVE FUNCTION FOR COMPOSITE FERMIONS

In this Appendix we derive the relation between the electron wave function and the CF wave function. Suppose a state of N particles and denote it as $|\Psi\rangle_N$. If we use the field operators of electrons, $|\Psi\rangle_N$ is described by

$$|\Psi\rangle_N=\frac{1}{N!}\int d^2\mathbf{r}_1 d^2\mathbf{r}_2\cdots d^2\mathbf{r}_N\Psi_{\rm el}(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_N)\times\psi_e^\dagger(\mathbf{r}_1)\psi_e^\dagger(\mathbf{r}_2)\cdots\psi_e^\dagger(\mathbf{r}_N)|0\rangle, \tag{A1}$$

where $\Psi_{\rm el}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)$ is the first quantized wave function of electrons and $|0\rangle$ is the vacuum state. If we use the field operators of extended CF's, the $|\Psi\rangle_N$ is described by

$$|\Psi\rangle_N=\frac{1}{N!}\int d^2\mathbf{r}_1 d^2\mathbf{r}_2\cdots d^2\mathbf{r}_N\Psi_{\rm CF}(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_N)\times\pi(\mathbf{r}_1)\pi(\mathbf{r}_2)\cdots\pi(\mathbf{r}_N)|0\rangle, \tag{A2}$$

where $\Psi_{\rm CF}(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_N)$ is the first quantized wave function of extended CF's. Let us find the relation between $\Psi_{\rm el}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)$ and $\Psi_{\rm CF}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)$.[14] Using the second equation in Eqs. (5), we obtain

$$\begin{aligned}
&\pi(\mathbf{r}_1)\pi(\mathbf{r}_2)\cdots\pi(\mathbf{r}_N)|0\rangle\\
&\quad=\psi_e^\dagger(\mathbf{r}_1)e^{J(\mathbf{r}_1)}\psi_e^\dagger(\mathbf{r}_2)e^{J(\mathbf{r}_2)}\cdots\psi_e^\dagger(\mathbf{r}_N)e^{J(\mathbf{r}_N)}|0\rangle\\
&\quad=\prod_{i<j}(z_i-z_j)^{\tilde\phi}\psi_e^\dagger(\mathbf{r}_1)\\
&\qquad\times\psi_e^\dagger(\mathbf{r}_2)\cdots\psi_e^\dagger(\mathbf{r}_N)e^{J(\mathbf{r}_1)}e^{J(\mathbf{r}_2)}\cdots e^{J(\mathbf{r}_N)}|0\rangle\\
&\quad=\prod_{i<j}(z_i-z_j)^{\tilde\phi}\exp\left(-\frac{1}{4l_B^2}\sum_{j=1}^N|z_j|^2\right)\\
&\qquad\times\psi_e^\dagger(\mathbf{r}_1)\psi_e^\dagger(\mathbf{r}_2)\cdots\psi_e^\dagger(\mathbf{r}_N)|0\rangle,
\end{aligned}\tag{A3}$$

where we have used $e^{J(\mathbf{r}_1)}e^{J(\mathbf{r}_2)}\cdots e^{J(\mathbf{r}_N)}|0\rangle=\exp(-1/4l_B^2\Sigma_{j=1}^N|z_j|^2)|0\rangle$. Substituting Eq. (A3) into Eq. (A2), we obtain

$$\begin{aligned}
|\Psi\rangle_N=&\frac{1}{N!}\int d^2\mathbf{r}_1 d^2\mathbf{r}_2\cdots d^2\mathbf{r}_N\prod_{i<j}(z_i-z_j)^{\tilde\phi}\\
&\times\exp\left(-\frac{1}{4l_B^2}\sum_{j=1}^N|z_j|^2\right)\\
&\times\Psi_{\rm CF}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)\psi_e^\dagger(\mathbf{r}_1)\psi_e^\dagger(\mathbf{r}_2)\cdots\psi_e^\dagger(\mathbf{r}_N)|0\rangle.
\end{aligned}\tag{A4}$$

Comparing Eq. (A1) with Eq. (A4), we find

$$\begin{aligned}
&\Psi_{\rm el}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)\\
&\quad=\prod_{i<j}(z_i-z_j)^{\tilde\phi}\exp\left(-\frac{1}{4l_B^2}\sum_{j=1}^N|z_j|^2\right)\\
&\qquad\times\Psi_{\rm CF}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N).
\end{aligned}\tag{A5}$$

Note that the correlation effect described by the Jastrow factor is completely factorized out of the wave function of electrons $\Psi_{\rm el}(\mathbf{r}_1,\ldots,\mathbf{r}_N)$. Therefore, the wave function of CF's is not blurred by the two-body correlation of the Laughlin wave function. When we perform the usual singular gauge transformation, we need to include the fluctuation from the ground state of the CF's to obtain Eq. (A5).

APPENDIX B: ANALYSIS OF V^H

In this Appendix, we solve the mean-field equations derived in Sec. IV for the case of $V_{\mathbf{k},\mathbf{k}'}=V^H_{\mathbf{k},\mathbf{k}'}$, and show that it leads to the p-wave pairing state following the analysis of Ref. 10. In order to solve the gap equations we set

$$\Delta_{\mathbf{k}}=\Delta_k\exp(-il\theta_{\mathbf{k}}), \tag{B1}$$

for the l-wave pairing state. Here l must be chosen as an odd integer because we consider a system of spinless CF's. Obviously, Eq. (B1) is not general. However, as we will see below, the attractive interaction arises only in the case of $l>0$. Therefore, we neglect the possibility of combinations of $l>0$ components and $l<0$ components.

Since $V^H_{\mathbf{k}\mathbf{k}'}$ satisfy $(V^H_{\mathbf{k}\mathbf{k}'})^*=V^H_{\mathbf{k}'\mathbf{k}}$, $\overline{\Delta_{\mathbf{k}}}$ is the complex conjugate of $\Delta_{\mathbf{k}}$. Therefore, it is enough to consider the gap equation (42) only. Furthermore we need not consider Eq. (44) because we neglect the Coulomb interaction here. Therefore, we set $m^*=m_b$ in this Appendix. Substituting Eq. (B1) into Eq. (42), we obtain

$$\Delta_k=\frac{\tilde\phi}{2m_b}\int_0^\infty dk'\frac{k'\Delta_{k'}}{E_{k'}}I_l^H\left(\frac{k^2+k'^2}{2kk'}\right), \tag{B2}$$

where the function $I_l^H(\lambda)$ is given by

$$I_l^H(\lambda)=-\int_0^{2\pi}\frac{d\theta}{2\pi i}\frac{\sin\theta}{\lambda-\cos\theta}\exp(-il\theta). \tag{B3}$$

In the case of $l>0$, we calculate $I_l^H(\lambda)$ from the contour integral setting $\exp(-il\theta)=z$. After some algebra, we obtain

$$I_l^H(\lambda)=\left(\frac{k_<}{k_>}\right)^l, \tag{B4}$$

where $k_<$ ($k_>$) denotes a smaller (larger) value of k and k'. Since the right hand side of Eq. (B4) is larger than zero, an attractive interaction is induced between CF's. On the other hand, for $l<0$ we obtain $I_l^H(\lambda)=-[I_{|l|}^H(\lambda)]^*=-I_{|l|}^H(\lambda)$. In that case a repulsive interaction arises between them.

We set $l>0$ in the following. Substituting Eq. (B4) into Eq. (B2), we obtain

$$\Delta_k=\frac{\tilde\phi}{2m_b}\int_0^k dk'\frac{k'\Delta_{k'}}{E_{k'}}\left(\frac{k'}{k}\right)^l+\frac{\tilde\phi}{2m_b}\int_k^\infty dk'\frac{k'\Delta_{k'}}{E_{k'}}\left(\frac{k}{k'}\right)^l. \tag{B5}$$

To solve the nonlinear equation (B5), we introduce an approximation for Δ_k.[10] From Eq. (B5) we find the asymptotic form of Δ_k:

$$\Delta_k\propto k^l\quad\text{for}\quad k\to+0, \tag{B6}$$

$$\Delta_k\propto k^{-l}\quad\text{for}\quad k\to+\infty. \tag{B7}$$

From Eqs. (B6) and (B7), we assume the form of Δ_k to be

$$\Delta_k=\begin{cases}\epsilon_F\Delta(k/k_F)^l & \text{for}\quad k<k_F\\ \epsilon_F\Delta(k_F/k)^l & \text{for}\quad k>k_F.\end{cases} \tag{B8}$$

The remaining parameter Δ is determined by the following equation:

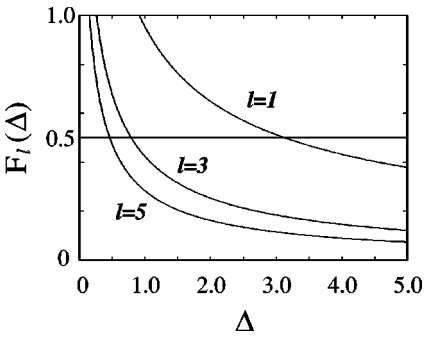

FIG. 3. The gap Δ dependence of the function $F_l(\Delta)$ in Eq. (B9) for $l=1,3,5$. The gap Δ is determined from the point where the curve crosses the horizontal line given by $F_l(\Delta)=1/\tilde{\phi}$. The figure shows the case of $\tilde{\phi}=2$.

$$F_l(\Delta)\equiv\int_0^1 dx\,\frac{x^{2l+1}}{\sqrt{(x^2-1)^2+\Delta^2x^{2l}}}+\int_1^\infty dx\,\frac{x^{1-2l}}{\sqrt{(x^2-1)^2+\Delta^2x^{-2l}}}=\frac{1}{\tilde{\phi}}. \tag{B9}$$

In Fig. 3, we show the gap Δ dependence of $F_l(\Delta)$ for the case of $\tilde{\phi}=2$ ($\nu=1/2$). The largest value of Δ is obtained for the case of $l=1$. The gaps for $l\geqslant 7$ are smaller than the gap for $l=5$. For any $\tilde{\phi}$, the largest gap Δ is obtained at $l=1$. In Fig. 4, we show the gap dependence of the condensation energy δE for $l=1,3,5$. We see that δE always has a negative value and the decrease of the energy is the largest for $l=1$. Furthermore, the ground-state energy is a monotonically decreasing function with respect to Δ. Therefore, the ground state is the *p*-wave pairing state.

Let us calculate the ground-state wave function for the pairing state. Substituting Eq. (B8) into Eq. (50), we obtain

$$\phi_{\rm orb}(\mathbf{r})=\frac{i}{2\pi}e^{i\theta_{\mathbf{r}}}\left[\int_0^{k_F}dk\,\frac{k\Delta\epsilon_F(k/k_F)}{\sqrt{\xi_k^2+\Delta^2\epsilon_F^2(k/k_F)^2}+\xi_k}J_1(kr)+\int_{k_F}^\infty dk\,\frac{k\Delta\epsilon_F(k_F/k)}{\sqrt{\xi_k^2+\Delta^2\epsilon_F^2(k_F/k)^2}+\xi_k}J_1(kr)\right], \tag{B10}$$

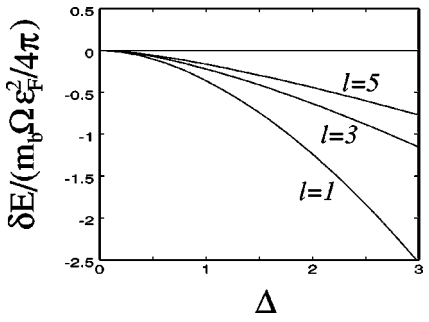

FIG. 4. The condensation energy versus the gap Δ.

where we have used the formula $\int_0^\pi d\theta e^{ix\cos\theta}\cos\theta=i\pi J_1(x)$ with $J_1(x)$ the Bessel function of first order. Equation (B10) has a simple form when the condition $\Delta=2$ is satisfied and we take the limit $rk_F\propto r/l_B\to\infty$. In that case, we obtain

$$\phi_{\rm orb}(\mathbf{r})\propto\frac{e^{-i\theta_{\mathbf{r}}}}{r}=\frac{1}{z}. \tag{B11}$$

From Eqs. (49) and (B11) the ground-state wave function for $2N$ CF's is given by

$$\Psi_{\rm CF}(z_1,z_2,\ldots,z_{2N})={\rm Pf}\left(\frac{1}{z_i-z_j}\right). \tag{B12}$$

From Eqs. (A5) and (B12) the ground-state wave function for $2N$ electrons is given by

$$\Psi_{\rm el}(z_1,z_2,\ldots,z_{2N})={\rm Pf}\left(\frac{1}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2\times\exp\left(-\frac{1}{4l_B^2}\sum_{j=1}^{2N}|z_j|^2\right). \tag{B13}$$

The wave function (B13) is the so-called Pfaffian state.[26]

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