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Fusion cross sections at deep sub-barrier energies

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A recent publication reports that heavy-ion fusion cross sections at extreme sub-barrier energies show a continuous change of their logarithmic slope with decreasing energy, resulting in a much steeper excitation function compared with theoretical predictions. We show that the energy dependence of this slope is partly due to the asymmetric shape of the Coulomb barrier; that is, its deviation from a harmonic shape. We also point out that the large low-energy slope is consistent with the surprisingly large surface diffusenesses required to fit recent high-precision fusion data.

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The primary ingredient in any nuclear reaction calculation is the nucleus-nucleus potential, consisting of the repulsive Coulomb interaction and an attractive nuclear part. Although the Coulomb term $V_C(r)$ is well known, there are large ambiguities in the nucleus-nucleus potential $V_n(r)$, and many attempts have been made to extract information on this quantity from experimental data for heavy-ion reactions. While elastic and inelastic scattering are sensitive mainly to the surface region of the nuclear potential, the fusion reaction is also relatively sensitive to the inner part. They thus provide complementary sources of information.

In heavy-ion reactions, strong channel coupling effects (due to collective inelastic excitations of the colliding nuclei and/or transfer processes) significantly modify the landscape of potential energy surface, replacing the uncoupled single barrier with a distribution of barriers [1–4]. In order to extract the nucleus-nucleus potential from heavy-ion fusion reactions, it is therefore advisable to use either high-energy fusion data where the barrier penetrability is essentially unity for all the distributed barriers, or very low-energy data where only the lowest barrier contributes to the cross section. Of these, the low-energy data probably provide cleaner information since the high-energy data may be complicated by competing reaction processes such as deep-inelastic scattering.

A recent paper [5] has reported on an attempt to measure the fusion cross section σ for the ^{60}Ni+^{89}Y system at deep sub-barrier energies, down to the 10^{-4} mb level. The authors of Ref. [5] used the Wong fusion formula [6] to analyze their data and showed that the experimental cross section exhibited an abrupt decrease at extreme sub-barrier energies. They also analyzed the data in terms of the logarithmic slope, defined by $L(E)=d[\ln(\sigma E)]/dE$, and showed that this quantity exhibited a continuous increase with decreasing energy, in contrast to the theoretical slope that approached a constant value. They also found similar behavior in a few other systems found in the literature, including the ^{58}Ni+^{58}Ni and ^{90}Zr+^{92}Zr reactions.

The main part of the analysis in Ref. [5] relied on the Wong formula as a reference. A natural question is whether this formula, based on a parabolic approximation to the Coulomb barrier, is adequate at deep sub-barrier energies [7]. It was claimed in Ref. [5] that the Wong formula leads to fusion cross sections similar to those obtained with the coupled-channels approach for the ^{58}Ni+^{58}Ni system. However, the former was simply a fit to the latter with parameters that had no physical connection to the potential used in the coupled-channels calculations.

The aim of this paper is to reanalyze critically the ^{58}Ni +^{58}Ni reaction with an exact one-dimensional-potential calculation as well as with coupled-channels calculations [8] and show that the Wong formula is indeed unreliable at very low energies. This is particularly so for a quantity such as the logarithmic slope, which accentuates the energy dependence of the cross section. We also discuss the findings of Ref. [5] in connection with the problem of the large surface diffusenesses of the nuclear potential for sub-barrier fusion, discussed for some time in the literature [4,9,10].

Let us first discuss the validity of the parabolic approximation to the potential. Figure 1 shows the nucleus-nucleus potential for the ^{58}Ni+^{58}Ni system (solid line), along with its parabolic approximation (dashed line)

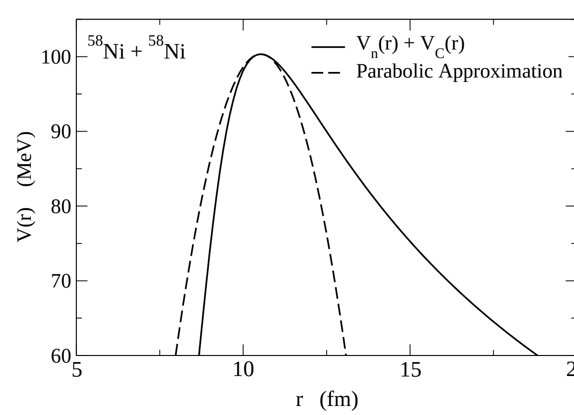

FIG. 1. The nucleus-nucleus potential for the ^{58}Ni+^{58}Ni reaction. The solid line is obtained with a Woods-Saxon nuclear potential with parameters $V_0=160$ MeV, $r_0=1.1$ fm, and $a=0.65$ fm. The dashed line shows the quadratic expansion of the potential around the barrier position.

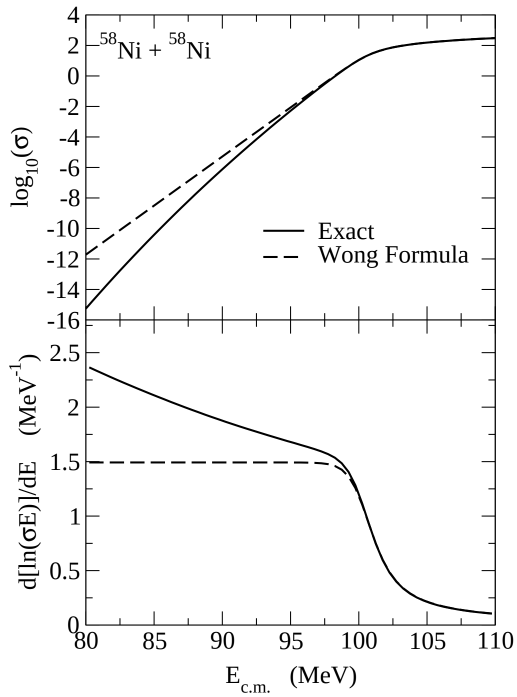

FIG. 2. The validity of the Wong formula (2) for the fusion cross section for the ^{58}Ni+^{58}Ni system. The upper panel shows the fusion cross section σ (in mb) on a logarithmic scale, while the lower panel shows the logarithmic slope $L(E)=d[\ln(\sigma E)]/dE$. The solid and dashed lines denote the exact numerical results and the Wong cross section, respectively.

$$V(r) \sim B - \frac{1}{2}\mu\omega^2(r-R)^2, \tag{1}$$

where B and R are the barrier height and position, respectively. Here μ is the reduced mass of the system and ω is the barrier "curvature" given by $\omega^2 = -V''(R)/\mu$. We use a Woods-Saxon nuclear potential with $V_0 = 160$ MeV, $r_0 = 1.1$ fm, and $a = 0.65$ fm. On the inside, the nuclear potential varies relatively rapidly, while on the outside the Coulomb potential varies slowly, resulting in an asymmetric barrier shape. The deviation from parabolic approximation (1) becomes larger as the energy goes down, and one expects this approximation to break down at energies well below the barrier. It was shown in Ref. [7] that the parabolic approximation is adequate only for $|r-R| \leqslant a$, that is, for incident energies within $\mu\omega^2 a^2/2$ of the barrier height. In the present example $\mu\omega^2 a^2/2 = 2.62$ MeV, and it is evident that the parabolic approximation is valid only in a relatively small range of energies near the barrier top.

An analytic formula for the fusion cross section for parabolic barrier (1) was derived some time ago by Wong [6]:

$$\sigma(E) = \frac{\hbar\omega}{2E} R^2 \ln[1 + e^{2\pi(E-B)/\hbar\omega}]. \tag{2}$$

The upper panel of Fig. 2 compares this formula with the fusion cross section obtained by numerically solving the Schrödinger equation with the true potential. No coupling is included in these calculations. As we saw in Fig. 1, the parabolic approximation underestimates the barrier thickness in the tunneling region, and thus overestimates the penetrability at low energies. The bottom panel of Fig. 2 shows the logarithmic slopes $L(E)$. Equation (2) yields a slope that is constant at low energies, and is given by

$$L(E) \sim \frac{d}{dE} \ln\left[\frac{\hbar\omega}{2} R^2 e^{2\pi(E-B)/\hbar\omega}\right] = \frac{2\pi}{\hbar\omega}. \tag{3}$$

On the other hand, the slope computed from the exact results shows a continuous increase with decreasing incident energy (solid line). This is reminiscent of the experimental findings of Ref. [5].

At low energies, the logarithmic slope is related to the s-wave barrier penetrability P_0 by $L(E) = d \ln[P_0(E)]/dE$. In the WKB approximation, the penetrability is given by

$$P_0(E) = e^{-2S(E)/\hbar} = \exp\left[-2\int_{r_1}^{r_2} dr \sqrt{2\mu[V(r)-E]/\hbar^2}\right] \tag{4}$$

at energies well below the barrier. Here, r_1 and r_2 are the inner and the outer turning points, respectively. Defining $\Delta(E)$ as the difference between the true action integral $S(E)$ and its value in the quadratic approximation, we have (ingoring an unimportant constant factor)

$$S(E) = \int_{r_1}^{r_2} dr \sqrt{2\mu[V(r)-E]} = \frac{\pi}{\omega}(B-E) + \frac{\hbar}{2}\Delta(E). \tag{5}$$

Since the Coulomb barrier $V(r)$ has a nonsymmetric shape, $\Delta(E)$ increases as the energy decreases, and the logarithmic slope $L(E) = 2\pi/\hbar\omega - d\Delta(E)/dE$ is always larger than $2\pi/\hbar\omega$. Furthermore, one can show that the second derivative of this action integral is a positive quantity and thus $L(E)$ is a decreasing function of E. For example, this is the case for the sharp-cut potential, $V_n(r) = [-V_C(r) - V_0]\theta(R_0 - r)$, for which the action integral can be evaluated analytically [11]. These facts are consistent with the numerical result shown in the lower panel of Fig. 2 as well as with the experimental findings discussed in Ref. [5]. We thus conclude that the continuous increase of the logarithmic slope with decreasing energy is not in itself evidence of anomalous behavior of the fusion cross section at very low energies, as claimed in Ref. [5].

We now discuss the relation between the logarithmic slope $L(E)$ and the surface property of the nuclear potential. For scattering processes, it seems well accepted that the surface diffuseness parameter a should be around 0.63 fm if V_n is parametrized by a Woods-Saxon form [12–14]. In marked contrast, recent high-precision fusion data suggest that a much larger diffuseness, between 0.8 and 1.4 fm, is needed to fit the data [9]. This is not just for particular systems but seems to be a rather general result [4,10,15–18]. Note that fusion depends strongly on the potential on both sides of the barrier, in contrast to the elastic scattering which depends mainly on the potential on the outside. At high energies, the fusion cross section changes with the diffuseness due to the way the position and height of the *l*-dependent barrier change with increasing *l*. At lower energies, the main effect comes from the overall width of the barrier. A large diffuseness seems to be desirable in both these respects [9].

For a fixed value of the barrier height *B*, the barrier curvature $\hbar\omega$ is approximately proportional to $a^{-1/2}$ [7]. Equation (3) then indicates that the logarithmic slope $L(E)$ is roughly proportional to $a^{1/2}$. The large experimental slope found in Ref. [5] may therefore be another indication of the large surface diffusenesses already noted in heavy-ion fusion. In order to assess this, we perform the exact coupled-channels calculations for the ^{58}Ni+^{58}Ni reaction using the computer code CCFULL [8] with different values of the surface diffuseness. This code uses the isocentrifugal approximation to reduce the dimensionality of the coupled-channels equations (see Ref. [8] for details), but we have checked that this is still valid at energies well below the Coulomb barrier. In the calculations, we include the double quadrupole-phonon excitations in both the projectile and target nuclei. A similar coupling scheme successfully explained the experimental fusion cross section and barrier distribution for the very similar ^{58}Ni+^{60}Ni system [17]. The dynamical quadrupole deformation parameter β_2 for the Coulomb coupling is estimated to be 0.177 from the experimental $B(E2)$ [19] with the radius parameter $r_{\rm coup}=1.2$ fm. We require a somewhat larger value of $\beta_2=0.261$ (with $r_{\rm coup}=1.06$ fm) for the nuclear coupling in order to fit the data. The fusion reaction often requires a radius parameter of around 1.06 fm, smaller than the usual value of around 1.2 fm, used to extract a deformation parameter from the electromagnetic transition probability. This results in a larger deformation parameter as well as in a larger deformation length $\beta r_{\rm coup}$. Although the Coulomb-coupling Hamiltonian is independent of the value of the radius parameter to be used, the nuclear coupling term depends on it through the combination $\beta r_{\rm coup}$. Therefore, this problem may also be related to the parametrization of the nucleus-nucleus potential, and thus to the large surface diffuseness problem, though the value of $r_{\rm coup}=1.06$ fm should be reasonable for finite nuclei with a diffuse surface [20].

In Fig. 3, we show the dependence of the fusion cross section (upper panel) and of the logarithmic slope (lower panel) on the surface diffuseness parameter *a* for the ^{58}Ni+^{58}Ni reaction. The figure also includes the experimental data [21] for comparison. The experimental slope was computed using point-difference formulas with both two and three successive data points. The dotted line is the result with the nuclear potential shown in Fig. 1, that is, with $a=0.65$ fm, while the dashed line is obtained with the potential parameters $V_0=195$ MeV, $r_0=0.94$ fm, and $a=1.0$ fm. The former leads to a cross section whose slope is not steep enough to account for the experimental data at energies below the barrier. As a consequence, the logarithmic slope $L(E)$ is underestimated at these energies, as in Ref. [5]. On the other hand, the potential with $a=1.0$ fm improves the agreement considerably both for the cross section and the logarithmic slope. We also include in the figure a calculation with $a=1.3$ fm (solid line). This further improves the fit to the logarithmic slope, although it somewhat worsens the fit to the cross section itself at incident energies around 97 MeV. (We have confirmed that none of these results depends on the value of r_0 as long as V_0 is adjusted, so that the barrier height remains unchanged.) Clearly, the experimental data favor a large value of the surface diffuseness, as in many other systems in the literature.

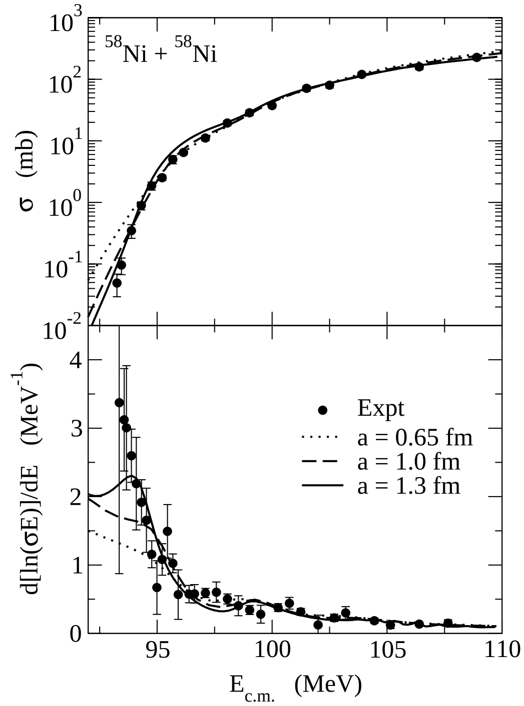

FIG. 3. Dependence of the fusion cross section (upper panel) and the logarithmic slope (lower panel) on the surface diffuseness parameter *a* for the ^{58}Ni+^{58}Ni reaction. The dotted, dashed, and solid lines are coupled-channels results using diffuseness parameters of 0.65 fm, 1.0 fm, and 1.3 fm, respectively. The double quadrupole-phonon exitations in both the projectile and target are taken into account. Experimental data are from Ref. [21].

In summary, the "unexpected" behavior of heavy-ion fusion cross sections at extreme sub-barrier energies claimed in Ref. [5] has two causes. One is the use of the Wong formula, which is inadequate at energies far below the barrier. The exact numerical calculation is vital in discussing the fusion cross section and especially the logarithmic slope $L(E)$ at low energies. We pointed out that the exact calculation shows a similar energy dependence of the logarithmic slope as in the experimental data even without coupling. The other reason for this apparent anomaly is the use of a diffuseness parameter that is widely used in calculations for scattering processes, that is, $a\approx 0.63$ fm. This potential leads to fusion cross sections whose logarithmic slope is much smaller than for the experimental data at deep sub-barrier energies. If such a calculation is used as a reference, the experimental data may appear to fall much more steeply than expected [5]. However, if one uses a larger value of the diffuseness parameter in the phenomenological potential, the data can be reproduced within the present coupled-channels framework. The need for a large diffuseness to describe the fusion process has also been found consistently in other systems. However, the reason for the large differences in diffuseness parameters extracted from scattering and from fusion analyses remains an open problem. In particular, it is still not clear whether a large surface diffuseness reflects the true nature of the potential or simply mocks up other effects that cause a rapid decrease of fusion at low energies. In this context, we mention that neither the double-folding potential [9] (which is usually much deeper and narrower than the Woods-Saxon one) nor the geometrical corrections to the coupling potential [22] seem to resolve this problem.

More experimental and theoretical studies of fusion at deep sub-barrier energies are needed to improve our understanding of this process, which may be especially important in astrophysical fusion reactions. Isotopic dependences may also be of interest, particularly for exotic nuclei whose surface properties may be modified by the presence of weakly bound nucleons.

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