Exotic Hadrons in s-Wave Chiral Dynamics

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We study s-wave scattering of a hadron and a Nambu-Goldstone boson induced by the model-independent low energy interaction in the flavor SU(3) symmetric limit. Establishing the general structure of the interaction based on group theoretical arguments, we find that the interaction in the exotic channels are in most cases repulsive, and that for possible attractive channels the coupling strengths are weak and uniquely given independent of channel. Solving the scattering problem, we show that the attraction in the exotic channels is not strong enough to generate a bound state.

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Chiral symmetry is one of the fundamental symmetries in QCD with massless quarks and is spontaneously broken down to the diagonal flavor symmetry. The low energy dynamics of the Nambu-Goldstone (NG) boson with other hadrons are essentially governed by chiral symmetry and its spontaneous breaking [1,2]. Many recent studies of hadron resonances and exotic hadrons are based on the underlying QCD with the aid of chiral symmetry.

Conventionally, hadron resonances have been studied in the S-matrix theory. There the scattering amplitude is obtained by considering the analyticity and unitarity together with a partial wave decomposition valid in a limited range of energies [3]. One of the classical applications is the description of $\Lambda(1405)$ [4], where $\Lambda(1405)$ emerges as a Feshbach resonance in the s-wave coupled channel problem in the $\pi\Sigma$ and $\bar{K}N$ channels. The recent development on this line is to determine the low energy scattering amplitude by chiral symmetry, reproducing various baryon resonances as dynamically generated states [5–7]. Since it was shown that the resonances obtained in the coupled channel approach became bound states of single channels in the flavor SU(3) limit [8–12], the origin of the physical resonances may be studied by the bound states in the SU(3) limit.

The exotic hadrons have the flavor which cannot be reached by $\bar{q}q$ or qqq, and must have more valence quarks than the ordinary hadrons. The pentaquark Θ^+ [13] is one of the possible candidates. In spite of continuous experimental efforts to search for exotic hadrons, existence of such a state has not been clearly established. Almost complete absence of exotic states in hadron spectrum is, however, highly nontrivial. Theoretically, there has been no simple way to exclude the exotic states in effective models describing the ordinary hadrons well, even more the fundamental theory of QCD does not forbid the exotic states.

In this Letter, we propose an attempt to approach the problem of exotic hadrons based on the symmetries of the underlying QCD, where we study s-wave scattering problem of the NG boson from a target hadron in the SU(3) limit. First, based on group theoretical arguments, we show that interactions in most of exotic channels are repulsive and that attractive interactions appear only in some limited number of channels with a universal strength. After establishing the general features of the low energy interaction, we examine whether the attraction is strong enough to generate an exotic state as a bound state of the NG boson.

Let us consider the interaction of the NG boson with a target hadron (T). According to the current algebra of the three flavor chiral symmetry, the scattering amplitude of the NG boson from a hadron in the low energy limit is unambiguously expressed by the hadron matrix element of the conserved vector current of the flavor symmetry. Thus the invariant amplitude $\mathcal{M}$ can be written in a model-independent way as

$$\mathcal{M}_\alpha = \frac{1}{f^2}\frac{p\cdot q}{2M_T}\langle \boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha + \mathcal{O}((m/M_T)^2), \quad (1)$$

with the decay constant f, the masses of the NG boson m and of target hadron M_T, and the momenta p and q for the hadron and the NG boson. $\boldsymbol{F}_T$ and $\boldsymbol{F}_{\mathrm{Ad}}$ stand for the flavor SU(3) generators in the representations of the hadron T and the NG boson of the adjoint (Ad) representation, respectively. Finally, due to flavor SU(3) symmetry, the matrix element $\mathcal{M}$ is diagonal in the SU(3) representation α of the s channel hadron-NG boson system; $\langle\ldots\rangle_\alpha \equiv \langle\alpha|\ldots|\alpha\rangle$. All the above is the essence of the celebrated Weinberg-Tomozawa theorem [2].

For a baryon target, we obtain the s-wave scattering amplitude as

$$V_\alpha = -\frac{\omega}{2f^2}C_{\alpha,T}, \quad (2)$$

where $C_{\alpha,T} \equiv -\langle 2\boldsymbol{F}_T\cdot\boldsymbol{F}_{\mathrm{Ad}}\rangle_\alpha$ and ω denotes the energy of the NG boson. In this equation, the spin averaged sum has been taken as $V \equiv \frac{1}{2}\sum_\sigma \bar{u}\,\mathcal{M}u$ with the spinor normalization $\bar{u}u = 1$. This is equivalent to the leading term of the chiral perturbation theory and known as the Weinberg-Tomozawa (WT) term. It is found that the scattering amplitude (2) can be applied also to the meson target in the heavy mass approximation [14].

Apart from the common factor in Eq. (2), the coupling strength $C_{\alpha,T}$ is determined only by specifying the representations of the system α and the target T:

$$C_{\alpha,T} = -\langle 2\boldsymbol{F}_T \cdot \boldsymbol{F}_{\rm Ad}\rangle_\alpha = C_2(T) - C_2(\alpha) + 3, \qquad (3)$$

where $C_2(R)$ is the quadratic Casimir of SU(3) for the representation R, and we use $C_2(\mathrm{Ad}) = 3$ for the adjoint representation of the NG boson. In our notation, a negative $C_{\alpha,T}$ leads to a repulsive interaction, whereas a positive $C_{\alpha,T}$ gives an attractive interaction.

Let the target hadron belong to an arbitrary SU(3) representation $[p, q]$ in the tensor notation. Possible representations α for the scattering channels are obtained in the irreducible decomposition of direct product of $[p, q]$ and the adjoint representation [1,1] of the NG boson:

$$[p,q]\otimes[1,1] = [p+1,q+1]\oplus[p+2,q-1]\oplus[p-1,q+2]\oplus[p,q]\oplus[p,q]\oplus[p+1,q-2] \oplus[p-2,q+1]\oplus[p-1,q-1], \qquad (4)$$

where the labels of representations $[a, b]$ satisfies $a, b \geq 0$, and one of the two $[p, q]$ representations on the right hand side satisfies $p \geq 1$ and the other $q \geq 1$.

We evaluate the coupling strength $C_{\alpha,T}$ for the channel α using Eq. (3) and the quadratic Casimir $C_2([p, q]) = [p^2 + q^2 + pq + 3(p + q)]/3$. We show a general expression for the coupling strength of the WT interaction for arbitrary representations in the second column of Table I. Since p and q are nonnegative integer, $C_{\alpha,T}$ takes an integer value whose sign is determined for a given α except for $[p + 2, q - 1]$ and $[p - 1, q + 2]$.

We are interested in exotic channels in the scattering of the NG boson and a target hadron. To specify the exotic channels, it is convenient to define the exoticness E of $[p, q]$ as in Refs. [14,15] in terms of the number of valence quark-antiquark pairs to compose the given flavor multiplet $[p, q]$ and the baryon number B carried by the u, d, and s quarks (the number of the heavy quarks is not included in this definition of the baryon number). We consider the light baryons (qqq), heavy baryons (qqQ), and heavy mesons ($q\bar{Q}$) as the target hadrons. For $B > 0$ the exoticness E is given by [14]

$$E = \epsilon\theta(\epsilon) + \nu\theta(\nu), \qquad (5)$$

TABLE I. Properties of the WT interaction in the channel α of the NG boson scattering on the target hadron with the $T = [p, q]$ representation. The coupling strengths of the WT term is denoted as $C_{\alpha,T}$, ΔE is the differences of the exoticness E between the channel α and the target hadron T, and $C_{\alpha,T}(N_c)$ denotes the coupling strengths for arbitrary N_c.

α	$C_{\alpha,T}$	ΔE	$C_{\alpha,T}(N_c)$
$[p+1, q+1]$	$-p-q$	1 or 0	$\frac{3-N_c}{2} - p - q$
$[p+2, q-1]$	$1-p$	1 or 0	$1-p$
$[p-1, q+2]$	$1-q$	1 or 0	$\frac{5-N_c}{2} - q$
$[p, q]$	3	0	3
$[p+1, q-2]$	$3+q$	0 or -1	$\frac{3+N_c}{2} + q$
$[p-2, q+1]$	$3+p$	0 or -1	$3+p$
$[p-1, q-1]$	$4+p+q$	0 or -1	$\frac{5+N_c}{2} + p + q$

with the step function $\theta(x)$ and the quantities ϵ and ν defined by

$$\epsilon \equiv \frac{p+2q}{3} - B, \qquad \nu \equiv \frac{p-q}{3} - B.$$

The detailed derivations are given in Ref. [14].

The more exotic channel is characterized by $\Delta E = 1$ where ΔE denotes the difference between the exoticness parameters E of the channel α and that of the target hadron T. ΔE can take the values $+1$, 0, and -1 when the quark-antiquark pair $q\bar{q}$ of the NG boson is added to the target hadron. The possible values of ΔE for all channels are given in the third column of Table I. We find that $\Delta E = 1$ is achieved when one of the following conditions is satisfied: (i) $\Delta\epsilon = 1$, $\Delta\nu = 0$, $\epsilon_T \geq 0$, (ii) $\Delta\epsilon = 0$, $\Delta\nu = 1$, $\nu_T \geq 0$, (iii) $\Delta\epsilon = 1$, $\Delta\nu = -1$, $\nu_T \leq 0$, with $\Delta\epsilon = \epsilon_\alpha - \epsilon_T$ and $\Delta\nu = \nu_\alpha - \nu_T$. The case (i) is satisfied for $\alpha = [p + 1, q + 1]$ but the interaction is always repulsive for this channel. The case (ii) is satisfied for $\alpha = [p + 2, q - 1]$ and $\nu_T \geq 0$, but the attraction is realized with $p = 0$. This leads to a negative baryon number $B \leq -q/3$ because of $\nu_T \geq 0$. The case (iii) is satisfied for $\alpha = [p - 1, q + 2]$, where the interaction can be attractive only when $q = 0$. In this case, the strength is always $C_{\alpha,T} = 1$ and the condition $\nu_T \leq 0$ gives $p \geq 3B$.

Thus, to summarize, we find that most of exotic channels are repulsive and the attractive interaction is realized only in the channel $\alpha = [p - 1, 2]$ of the NG boson scattering on the target hadron $T = [p, 0]$ ($p \geq 3B$) with the universal strength

$$C_{\rm exotic} = 1. \qquad (6)$$

It is interesting that the attractive interaction for the exotic channel takes place with the smallest strength of the WT term, only when the target hadron belongs to the symmetric representation $[p, 0]$.

The next question is whether the attractive interaction Eq. (6) is strong enough to provide an exotic state. For this purpose, we solve scattering problem with the WT interaction treated beyond the perturbation. The scattering equation is solved as a single channel problem for each α in the SU(3) limit. There, enough attraction generates a bound state with the flavor quantum number α.

In order to obtain the relevant scattering amplitude we impose the elastic unitarity condition in the N/D method. This procedure has been shown to be equivalent to solving the scattering equation with the WT amplitude as the kernel interaction [6]. Then we obtain the scattering amplitude of the NG boson and the target hadron in the channel α as

$$t_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s})G(\sqrt{s})} V_\alpha(\sqrt{s}), \tag{7}$$

as a function of the center-of-mass energy $\sqrt{s}$. Here $V_\alpha(\sqrt{s})$ denotes the WT interaction (2), and $G(\sqrt{s})$ is given by the once-subtracted dispersion integral

$$G(\sqrt{s}) = -\tilde{a}(s_0) - \frac{1}{2\pi}\int_{s^+}^{\infty} ds'\left(\frac{\rho(s')}{s' - s} - \frac{\rho(s')}{s' - s_0}\right), \tag{8}$$

with the phase space integrand $\rho(s) = 2M_T\sqrt{(s - s^+)(s - s^-)}/(8\pi s)$ and $s^\pm = (m \pm M_T)^2$. The subtraction constant $\tilde{a}(s_0)$ is not determined within the N/D method.

To fix $\tilde{a}(s_0)$, we take the prescription given in Refs. [7,16]:

$$G(M_T) = 0, \tag{9}$$

which is equivalent to $t_\alpha(\sqrt{s}) = V_\alpha(\sqrt{s})$ at $\sqrt{s} = M_T$. Since the WT term V_α has the crossing symmetry, this prescription restores the crossing symmetry approximately in the scattering amplitude (7) at energies around $\sqrt{s} = M_T$. This is enough for our purpose, since the bound state energy M_b is calculated within $M_T < M_b < M_T + m$. The prescription (9) is realized in Eq. (8) by choosing $\tilde{a}(s_0) = 0$ at $s_0 = M_T^2$.

The bound state is expressed as a pole of the scattering amplitude (7). To find the bound state, we define a function D as the denominator of the amplitude:

$$D(\sqrt{s}) \equiv 1 - V_\alpha(\sqrt{s})G(\sqrt{s}). \tag{10}$$

The bound state energy M_b should be within the energies between the target mass and the scattering threshold. Thus if bound states exist, the bound state energies are obtained by

$$D(M_b) = 0 \quad \text{with} \quad M_T < M_b < M_T + m. \tag{11}$$

Now we can show that there is at most one solution to the above equation as follows. The renormalization condition (9) requires $D(M_T) = 1 > 0$. Observing that both $V_\alpha(\sqrt{s})$ and $G(\sqrt{s})$ shown in Eqs. (2) and (8) are decreasing functions of $\sqrt{s}$ for attractive interaction $C_{\alpha,T} > 0$ in the energy region $M_T < \sqrt{s} < M_T + m$, we find that $D = 1 - V_\alpha G$ is also monotonically decreasing in this region. This means that, if $D(M_T + m) < 0$ is satisfied, we have a unique solution of the Eq. (11).

Thus, the smallest attractive coupling strength $C_{\rm crit}$ to produce a bound state of the NG boson in the target hadron is calculated so as to satisfy $D(M_T + m) = 0$:

$$C_{\rm crit} = \frac{2f^2}{m[-G(M_T + m)]}. \tag{12}$$

If the coupling strength is smaller than $C_{\rm crit}$ for a given NG boson mass m and the target mass M_T, there is no bound state. Shown in Fig. 1 is $C_{\rm crit}$ as functions of M_T with $f = 93$ MeV. We use the average mass of the octet mesons (π, K, η) $m = 368$, since the observed pseudoscalar meson masses can be reproduced well by the averaged mass with the linear order of SU(3) breaking. We also plot $C_{\rm exotic} = 1$, which is the universal strength of the attraction in exotic channels.

As seen in Fig. 1, it is clear that $C_{\rm exotic} = 1$ is not enough to bind the NG boson in the target hadron with the mass $M_T < 6$ GeV, where possible target hadrons have been observed in experiments. The critical coupling strength $C_{\rm crit}$ is monotonically decreasing as the target mass M_T increases. Therefore, it will become smaller than 1 at sufficiently large M_T. Quantitatively, to have a bound state for the exotic channel $C_{\rm exotic} = 1$, the mass of the target hadron M_T should be larger than 14 GeV for $m = 368$ MeV and $f = 93$ MeV. Exotic states can appear as

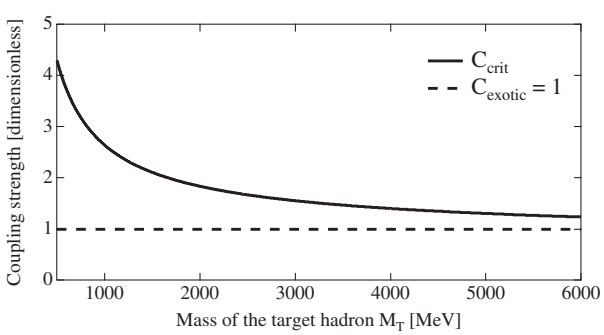

FIG. 1. Critical coupling strength $C_{\rm crit}$ for the averaged mass over the pseudoscalar octet mesons, $m = 368$ MeV (solid line). The dashed line denotes the universal attractive coupling strength in exotic channels $C_{\rm exotic} = 1$.

TABLE II. Masses of the dynamically generated states in the SU(3) limit. The values of the masses M_T and M_b are given in units of MeV. The SU(3) representations are written in their dimensions. Examples of the corresponding resonances with SU(3) breaking are shown in the last column.

Target hadron	T	M_T	α	$C_{\alpha,T}$	M_b	Examples
Light baryon	8	1151	1	6	1450	$\Lambda(1405)$ [8]
			8	3	1513	$\Lambda(1670)$ [8]
	10	1382	8	6	1668	$\Lambda(1520)$ [10]
			10	3	1737	$\Xi(1820)$ [10]
Charmed	$\bar{3}$	2408	$\bar{3}$	3	2736	$\Lambda_c(2880)$ [11]
baryon	6	2534	$\bar{3}$	5	2804	$\Lambda_c(2593)$ [11]
			6	3	2860	-
D meson	3_c	1900	3	3	2240	$D_s(2317)$ [12]
B meson	3_b	5309	3	3	5600	-

the bound states if a stable heavy hadron exists in this energy region.

If we use a heavier NG boson mass, the critical coupling $C_{\rm crit}$ is smaller than the present case and the NG boson could be bound in the exotic channel. For instance, with $m = 500$ MeV an exotic bound state appears with $M_T \simeq 2500$ MeV as shown in Ref. [11].

In the present approach, exotic states are treated as quasibound states of a NG boson and a hadron on the same footing as the nonexotic resonances. We find bound states in some nonexotic channels with enough strength of attractions in the SU(3) limit as listed in Table II. The target mass is taken as the averaged mass of the observed hadrons in the multiplets. We also show in Table II examples of resonances found in those channels, referring to the literature in which realistic calculations are performed with the SU(3) breaking. Considering the fact that several known resonances have been properly generated by the chiral unitary approaches, our conclusion on the exotic states should be of great relevance.

Our discussion on the coupling strength of the WT term can be extended to baryons with the arbitrary number of color N_c. It is known that the WT term scales as $1/N_c$ in the large-N_c limit, since it contains $1/f^2$ and $f \propto \sqrt{N_c}$. This is the case that the coupling strength $C_{\alpha,T}$ has no N_c dependence. Here we show that $C_{\alpha,T}$ does have the N_c dependence in the case of the baryon target. Also in Ref. [17], nontrivial N_c dependence was reported for the spin-flavor SU(6) extended WT term. An interesting discussion was recently made on interplay of the chiral and $1/N_c$ expansions [18]. For arbitrary N_c, a baryon is constructed by N_c quarks. The flavor multiplet of baryon $[p, q]$ for $N_c = 3$ is conventionally extended to $[p, q + (N_c - 3)/2]$ for arbitrary N_c so as to fix the baryon spin [19], while the SU(3) representation of the meson does not change. Using Eq. (3), we obtain the coupling strength for arbitrary N_c, which are summarized in the last column of Table I. We find that the possible attraction in exotic channels (6) for $N_c = 3$ changes into repulsive in the large-N_c limit. This means that there is no way to provide bound states in exotic channels in the large-N_c limit, independently of the masses of the NG boson and target baryons.

To conclude, we have investigated possibility to have exotic states in chiral dynamics, focusing on the study of an s-wave bound state of a Nambu-Goldstone boson from a target hadron in the SU(3) limit. The chiral dynamics supplies the model-independent coupling strength for the s-wave scattering of the NG boson and the target hadron at low energies. Based on the group theoretical arguments, we have shown that most of exotic channels are repulsive and possible attractive channels are weak with a universal strength $C_{\rm exotic} = 1$. Exploiting the scattering theory with the WT term as the kernel interaction, we have proved that the strength of attraction in the exotic channel of the WT term is not enough to generate bound states of the NG boson with the experimentally observed hadrons. We have also found that there are no attractive exotic channels in the large-N_c limit. It should be worth noting that the present analysis does not exclude the existence of the exotic states formed by other mechanisms, for instance the genuine quark states.

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