Dilaton and moduli fields in D-term inflation

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(Received 12 August 2003; published 30 January 2004)

We investigate the possibility of D-term inflation within the framework of type-I string-inspired models. Although the D-term inflation model has the excellent property that it is free of the so-called η problem, two serious problems appear when we embed D-term inflation in string theory: the magnitude of the FI term and the rolling motion of the dilaton. In the present paper, we analyze the potential of D-term inflation in type-I-inspired models and study the behavior of dilaton and twisted moduli fields. Adopting the nonperturbative superpotential induced by gaugino condensation, the twisted moduli can be stabilized. If the dilaton is in a certain range, it evolves very slowly and does not run away to infinity. Thus D-term-dominated vacuum energy becomes available for driving inflation. By studying the density perturbation generated by the inflation model, we derive the constraints on model parameters and give some implications on D-term inflation in type-I-inspired models.

DOI: 10.1103/PhysRevD.69.023510 PACS number(s): 98.80.Cq, 04.50.+h, 04.65.+e, 11.25.Mj

I. INTRODUCTION

In the success of slow-roll inflation, a sufficient flat potential for the inflaton, which is the inflation driving scalar field, is required. However, it is not so easy to construct a model of inflation where the corresponding scalar field has a very flat potential in the framework of supergravity. Usually, the flatness of the potential is expressed as

$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \tag{1}$$

$$\eta \equiv M_p^2 \frac{V''}{V} \ll 1, \tag{2}$$

in terms of the slow-roll parameters, where the prime denotes a derivative with respect to the inflaton and M_p is the reduced Planck mass. In the case when the vacuum energy is dominated by the F term during inflation, supergravity effects produce the soft mass of the inflaton field whose magnitude is the same order of H and spoil the flatness of the potential. In other words, such a large inflaton mass gives $\eta \sim 1$ and violates the slow-roll condition, which is often referred to as the η problem.

From this point of view, D-term inflation is one of the attractive inflation scenarios within the framework of supersymmetric models [1,2],[1] because the inflaton does not acquire the mass-squared term of the order of H^2 and the flatness of the potential is preserved. In the D-term inflation scenario, the Fayet-Iliopoulos (FI) term is dominant during inflation and this vacuum energy causes an accelerating expansion. The FI term comes from the anomalous U(1) symmetry. In fact, most 4D string models have anomalous U(1) symmetry for both heterotic models [4,5] and type-I models [6]. These anomalies can be canceled by the Green-Schwarz mechanism, where certain fields transform nontrivially under anomalous U(1) symmetries [7]. Such a role is played by the dilaton field S in heterotic models and the twisted moduli fields M in type-I models. That generates the FI term. Thus, D-term inflation seems possible to be realized in string models.

However, there are difficulties in the realization of D-term inflation based on heterotic models. The first problem is the energy scale of inflation. The magnitude of the FI term is given as

$$\xi = \frac{g^2}{192\pi^2}\mathrm{Tr}(Q)M_p^2, \tag{3}$$

where g^2 is the gauge coupling and $\mathrm{Tr}(Q)$ is a model-dependent constant of the order of $O(10)$–$O(10^2)$ [4]. Equation (3) reads $\sqrt{\xi}/M_p = O(10^{-1})$–$O(10^{-2})$. On the other hand, the cosmic microwave background (CMB) anisotropy requires $\xi^{1/2} \simeq 10^{15}$ GeV. We find that the theoretical prediction is too large to meet the observational estimation. This inconsistency is still a serious problem, although several studies to construct a model with effectively suppressed FI terms have been done in heterotic models [8,9]. The second problem is due to the dilaton dependence of the FI term and the anomalous U(1) gauge coupling. The presence of the dilaton S and several types of moduli fields is one of important features in superstring theory. Vacuum expectation values (VEVs) of these fields determine the magnitudes of all the couplings—e.g., gauge couplings and Yukawa couplings. The dilaton field prevents the realization of D-term inflation. Since $g^2 \propto 1/\mathrm{Re}\,S$, the D-term scalar potential $V_D = g^2\xi^2/2$ is in proportion to $(\mathrm{Re}\,S)^{-3}$. Hence the dilaton rolls down the

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[1] See Ref. [3] for an early study of an inflation model relevant to the D term.

potential to infinity, $\mathrm{Re}\, S \rightarrow \infty$, and the D-term potential energy goes to zero, $V_D \rightarrow 0$. This phenomenon is the same as the dilaton runaway problem in generic string models and such runaway behavior of the dilaton and moduli fields prevents a viable inflation [10]. Even if we adopt type-I models rather than heterotic models, the FI term is dependent on the field which plays a role in the Green-Schwarz anomaly cancellation mechanism for the anomalous U(1). Hence, if these fields run away, D-term inflation cannot occur in 4D string models.

In this paper we study the behavior of dilaton and moduli fields in the D-term inflation scenario with the above two problems in mind. The purpose of the present paper is to explore a way to avoid these problems at the same time. Actually, studies of the runaway problem have been done for heterotic models in Refs. [11,12]. Now, in particular, we will consider type-I string-inspired models. D-term inflation in type-I-inspired models has been studied in Ref. [13] and it was shown that the magnitude of the FI term is reducible to a desired value. This result arises from the facts that the FI term is determined by the expectation value of the twisted moduli and the string scale M_s is independent of the Planck scale M_p [14] in type-I string models. In fact, the twisted moduli field plays a role in the Green-Schwarz anomaly cancellation in 4D type-I string models. However, in Ref. [13] the stabilization of the twisted moduli and the runway problem have not been discussed. These issues might be not trivial. Indeed, stabilization of the twisted moduli fields were studied and different aspects from those of the dilaton field were revealed [15].[2] As we will show, the twisted moduli field cannot stabilize with nonvanishing vacuum energy in only the D-term potential and any vacuum energy driving inflation does not appear, unless we assume a specific Kähler potential. Therefore, stabilization of the twisted moduli is also an important issue for the inflation in type-I models.

This paper is organized as follows. In the next section, we review briefly D-term inflation. In Sec. III, we study the behavior of dilaton and twisted moduli fields. It will be shown that the twisted moduli field is stabilized during inflation. In Sec. IV, the dynamics of the inflaton and dilaton fields are studied, and we will derive constraints on parameters in our models. Section V is devoted to conclusions and a discussion.

II. D-TERM INFLATION

Here we give a brief review of D-term inflation [1,2]. We consider the $N=1$ supersymmetric model with U(1) gauge symmetry and the nonvanishing FI term ξ. The model includes three matter fields X and $\phi_\pm$. The fields $\phi_\pm$ have U(1) charges ± 1 while X has no U(1) charge. The U(1) D term is written as

$$\mathrm{D} = \xi + |\phi_+|^2 - |\phi_-|^2. \tag{4}$$

Hereafter we take the charge assignment such that $\xi > 0$. Suppose the superpotential

$$W = \lambda X \phi_+ \phi_- . \tag{5}$$

Then, the scalar potential is written as

$$V = \sum_i |\partial_i W|^2 + \frac{g^2}{2} D^2 \tag{6}$$

$$= \lambda^2 |X|^2 (|\phi_-|^2 + |\phi_+|^2) + \lambda^2 |\phi_- \phi_+|^2 + \frac{g^2}{2} (\xi + |\phi_+|^2 - |\phi_-|^2)^2. \tag{7}$$

The true vacuum of this potential corresponds to

$$X = \phi_+ = 0, \quad |\phi_-| = \sqrt{\xi}, \tag{8}$$

and supersymmetry (SUSY) is not broken.

For a value of X fixed, we analyze the minimum of V. We define

$$X_c \equiv \frac{g}{\lambda} \sqrt{\xi}. \tag{9}$$

For $|X| < X_c$, the minimum corresponds to

$$|\phi_-|^2 = \xi - \frac{\lambda^2 X^2}{g^2}, \quad |\phi_+| = 0. \tag{10}$$

On the other hand, for $|X| > X_c$, the minimum corresponds to

$$|\phi_\pm| = 0. \tag{11}$$

In this case, the vacuum energy is obtained as

$$V = \frac{g^2}{2} \xi^2, \tag{12}$$

and the radial part of X is identified with the inflaton. Although the mass of X vanishes at the tree level, since the supersymmetry breaking by the nonvanishing D term generates the mass difference between the masses of $\phi_\pm$,

$$m_\pm^2 = \lambda^2 |X|^2 \pm g^2 \xi, \tag{13}$$

and the masses of those fermionic partners, the one-loop effective potential is given as

$$V_{\text{1-loop}} = \frac{g^2}{2} \xi^2 \left(1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |X|^2}{\Lambda^2} \right), \tag{14}$$

where Λ is the renormalization scale. That generates the mass term of X and the potential is slightly lifted. Then, the inflaton slowly rolls down the potential. From the equation

$$\eta = -\frac{g^2}{8\pi^2} \frac{M_p^2}{|X|^2}, \tag{15}$$

we find that the slow-roll condition is violated ($\eta \sim 1$) when the inflaton reaches at

[2]See also Refs. [16,17].

$$|X_f|^2 = \frac{g^2}{8\pi^2} M_p^2, \tag{16}$$

and the inflation ends.

We have reviewed on the simplest D-term inflation model. However, as noted in the Introduction, the magnitude of the FI term ξ in such a simple model is required to be much smaller than the values of ξ derived from 4D heterotic models. Reference [8] has proposed a mechanism in heterotic models to reduce effectively the FI term compared with its original value. Here we also give a brief review of such models. Instead of the single field X, we consider two fields $X_\pm$. Generic string models have several $U(1)_a$ symmetries. Some of them are anomalous and others are anomaly free. In Ref. [8], 4D heterotic models were studied and the FI term was considered for only one of the U(1) symmetries. That is because only one U(1) symmetry can be anomalous in a 4D heterotic model. However, we will study inflation models inspired by type-I string models, where two or more U(1) symmetries can be anomalous. Thus, here we extend the model of Ref. [8] into the case with many FI terms ξ_a, while $\xi_a = 0$ for anomaly-free $U(1)_a$. Suppose that $X_\pm$ have $U(1)_a$ charges $\pm Q_a$. Then the D term is written as

$$D_a = \xi_a + Q_a(|X_+|^2 - |X_-|^2) + \cdots, \tag{17}$$

where the ellipsis denotes contributions due to other chiral matter fields. We consider large field values for $X_\pm$, $X_\pm \gg \sqrt{\xi_a}$ as before and assume that the other fields gain mass terms—e.g.,

$$W = \lambda \frac{X_+ X_-}{M} \phi_+ \phi_-. \tag{18}$$

Then, the matter fields other than $X_\pm$ vanish for $X_\pm$, $X_\pm \gg \sqrt{\xi_a}$, and the D term is dominated by $X_\pm$. In this case, the scalar potential is written as

$$V_D = \sum_a \frac{g_a^2}{2} [\xi_a + Q_a(|X_+|^2 - |X_-|^2)]^2. \tag{19}$$

The direction $|X_+| = |X_-|$ corresponds to the inflaton. The minimum of V_D is obtained as

$$(V_D)_{min} \equiv \frac{g_{eff}^2}{2} \xi_{eff}^2 = \sum_a \frac{g_a^2}{2} \xi_a^2 - \frac{\left(\sum_a g^2 Q_a \xi_a\right)^2}{\sum_a g_a^2 Q_a^2} \tag{20}$$

for

$$|X_+|^2 - |X_-|^2 = -\frac{\sum_a g_a^2 Q_a \xi_a}{\sum_a g_a^2 Q_a^2}. \tag{21}$$

In a certain situation (maybe with fine-tuning), this vacuum energy $(V_D)_{min}$ becomes much smaller than $O(g_a^2 \xi_a^2)$. Furthermore, in Ref. [9] more fields X_i relevant to the inflaton have been introduced and in such a case an effective value of the FI term can be reduced.

III. BEHAVIOR OF THE DILATON AND TWISTED MODULI FIELDS

In this section we study the behavior of dilaton and twisted moduli fields in type-I string-inspired D-term inflation models. We consider the case that the anomalous U(1) originates from the D9-brane. In this case the gauge kinetic function is obtained as [18]

$$f_9 = S + \sigma \frac{M}{M_s}, \tag{22}$$

where σ is a model-dependent constant and we define $\tilde{\sigma} = \sigma / M_s$. The corresponding gauge coupling g_9 is obtained as $g_9^2 = 1/\mathrm{Re}(f_9)$. The Kähler potential of the dilaton field S is written as

$$K(S, \bar{S}) = -\ln(S + \bar{S}). \tag{23}$$

On the other hand, the Kähler potential of the twisted moduli field $K(M, \bar{M})$ is not clear. For small values of M, it can be expanded as [19]

$$K(M, \bar{M}) = \frac{1}{2}(M + \bar{M})^2 + \cdots. \tag{24}$$

However, the reliability of this form is not clear for large values of M—i.e., $M \gg O(1)$.

The twisted moduli field plays a role in the 4D Green-Schwarz (GS) anomaly cancellation mechanism, and the FI term is obtained as

$$\xi = \delta_{GS} K'(M, \bar{M}), \tag{25}$$

where δ_{GS} is a model-dependent constant. Thus the D-term scalar potential is obtained during inflation,

$$V_D = \frac{[\delta_{GS} K'(M, \bar{M})]^2}{S + \bar{S} + \tilde{\sigma}(M + \bar{M})}. \tag{26}$$

First, let us study behavior of S during inflation. If S runs away to infinity, D-term inflation cannot succeed. Here, we introduce the canonically normalized dilaton field ϕ, which is defined as

$$\phi = M_p \frac{1}{\sqrt{2}} \ln \frac{S + \bar{S}}{2}. \tag{27}$$

The first and second derivatives of V_D by ϕ are obtained as

$$\frac{\partial V_D}{\partial \phi} = -\frac{\sqrt{2}(S + \bar{S})}{S + \bar{S} + \tilde{\sigma}(M + \bar{M})} \frac{V_D}{M_p}, \tag{28}$$

$$\frac{\partial^2 V_D}{\partial \phi^2} = -\frac{2\sqrt{2}(S+\bar{S})}{S+\bar{S}+\tilde{\sigma}(M+\bar{M})}\frac{V_D}{M_p^2} \times \left(1-\frac{2(S+\bar{S})}{S+\bar{S}+\tilde{\sigma}(M+\bar{M})}\right). \tag{29}$$

Suppose the twisted moduli M is stabilized somehow such that $S+\bar{S} \ll \tilde{\sigma}(M+\bar{M})$. Then, we obtain

$$\left(\frac{1}{V_D}\frac{\partial V_D}{\partial \phi}\right)^2 \simeq 2\left(\frac{S+\bar{S}}{\tilde{\sigma}(M+\bar{M})}\right) \ll 1, \tag{30}$$

$$\frac{1}{V_D}\frac{\partial^2 V_D}{\partial \phi^2} \simeq -\frac{2\sqrt{2}(S+\bar{S})}{\tilde{\sigma}(M+\bar{M})} \ll 1; \tag{31}$$

that is, the slow-roll condition is satisfied for the dilaton field and it does not run away. Note that this result is independent of the form of $K(M,\bar{M})$.

We have considered the case that the U(1) gauge multiplet originates from the D9-brane. We can easily extend the above analysis into another case: for example, when the U(1) gauge multiplet originates from the D5-brane, wrapping a 2D torus of the 6D compact space. In this case, the gauge kinetic function is obtained as

$$f_{5i} = T_i + \tilde{\sigma}(M+\bar{M}), \tag{32}$$

where T_i is the moduli field whose VEV determines the volume of the 2D torus. Its Kähler potential is the same as the dilaton field—i.e.,

$$K(T_i,\bar{T_i}) = -\ln(T_i+\bar{T_i}). \tag{33}$$

Hence, the D-term scalar potential during D-term inflation is written as

$$V_D = \frac{[\delta_{GS}K'(M,\bar{M})]^2}{T_i+\bar{T_i}+\tilde{\sigma}(M+\bar{M})}. \tag{34}$$

In a similar way to the dilaton field, the moduli field T_i does not run away if M is stabilized such that $T_i+\bar{T_i} \ll \tilde{\sigma}(M+\bar{M})$.

Next, we study the stabilization of M in the model with V_D, Eq. (26), that U(1) originates from the D9-brane. For $S+\bar{S} \ll \tilde{\sigma}(M+\bar{M})$, the potential reduces to

$$V_D = \frac{[\delta_{GS}K'(M,\bar{M})]^2}{\tilde{\sigma}(M+\bar{M})}. \tag{35}$$

Unfortunately, only the term V_D does not stabilize the twisted moduli M at a finite value with nonvanishing vacuum energy, unless we choose a special form of $K(M,\bar{M})$. In fact, the minimum in this potential is located at $M+\bar{M}=0$ and $V_D|_{M+\bar{M}=0}=0$ for the Kähler potential, $K(M,\bar{M})=(M+\bar{M})^2/2$. Hence, another term is necessary for stabilization of M such that D-term inflation can be realized. We assume that gaugino condensation of another non-Abelian gauge group generates the nonperturbative superpotential

$$W_{np} = de^{-\Delta(S+\tilde{\sigma}'M)}, \tag{36}$$

where $\Delta = 24\pi^2/(-b)$ and b is the one-loop beta-function coefficient of the gauge coupling. In general, the constant σ' is different from σ. Anyway one needs a stabilization mechanism of S as well as M in the true vacuum after inflation ends. It is the usual approach to stabilize S by this type of nonperturbative superpotential. Thus, it is natural to assume the above superpotential. For stabilization of S in the true vacuum, double or more gaugino condensations are often used in the so-called racetrack model. Here, for simplicity, we consider the case that only the single gaugino condensation potential is dominant during inflation. As we did in estimation of V_D, we assume $S+\bar{S} \ll \tilde{\sigma}(M+\bar{M})$ during inflation; that is, we have $W_{np}=de^{-\Delta\tilde{\sigma}'M}$. In this case, the scalar potential in the global SUSY is written as[3]

$$V = (\Delta\tilde{\sigma}')^2 d^2 e^{-\Delta\tilde{\sigma}'m} + \frac{\delta_{GS}^2}{\tilde{\sigma}}m, \tag{37}$$

where $m \equiv M+\bar{M}$. We need an explicit form of $K(M,\bar{M})$ to discuss the stabilization of M. Here and hereafter we take

$$K(M,\bar{M}) = \frac{1}{2}(M+\bar{M})^2. \tag{38}$$

The stationary condition $\partial V/\partial m = 0$ is satisfied by

$$\langle M+\bar{M}\rangle_{inf} = \frac{1}{\Delta\tilde{\sigma}'}\ln\left(\frac{\tilde{\sigma}d^2(\Delta\tilde{\sigma}')^3}{\delta_{GS}^2}\right), \tag{39}$$

where $\langle\cdots\rangle_{inf}$ denotes the expectation value during inflation. If the effective mass squared during inflation,

$$\left.\frac{\partial^2 V}{\partial m^2}\right|_{m=\langle M+\bar{M}\rangle_{inf}} \simeq \frac{\Delta\tilde{\sigma}}{\langle M+\bar{M}\rangle_{inf}}V_D = 3\frac{\Delta\tilde{\sigma}}{\langle M+\bar{M}\rangle_{inf}}M_p^2 H^2, \tag{40}$$

is larger enough than H^2 around this stationary point, the twisted moduli can be stabilized. The condition is satisfied in the present model because of $\Delta\tilde{\sigma}M_s^2/\langle M+\bar{M}\rangle_{inf} \gg 1$ and $M_p \sim M_s$,[4] as we will see soon. Note that the key point for stabilization is the polynomial form of the Kähler potential, in particular the canonical form. When we consider additional higher terms $(M+\bar{M})^n$ $(n>2)$ in the Kähler potential, we obtain qualitatively a similar result for stabilization.

[3]When we study the scalar potential within the framework of supergravity theory, we obtain qualitatively the same result.

[4]This relation is derived in the next section.

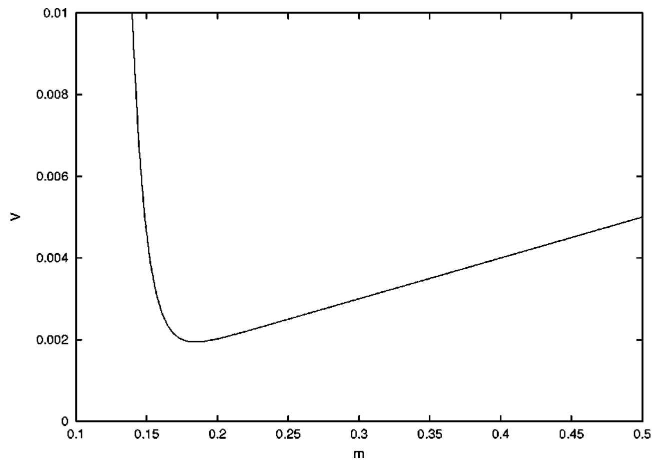

FIG. 1. The scalar potential (37) of m for $\Delta\sigma'=100$ and $\delta_{GS}^2/\sigma d^2=0.01$.

However, if the twisted moduli field has the logarithmic form of the Kähler potential, this type of stabilization cannot be realized.

Now let us estimate $\langle M+\bar{M}\rangle_{inf}$. If $b/\sigma'=O(1)$, we estimate $(\Delta\sigma')^{-1}=O(10^{-3})$–$O(10^{-2})$, because of $(\Delta\sigma')^{-1}=0.0004\times b/\sigma'$. For such a small value of $(\Delta\sigma')^{-1}=O(10^{-3})$–$O(10^{-2})$, the factor $(\Delta\sigma')^3$ inside of the logarithmic function enlarges $\langle M+\bar{M}\rangle_{inf}$ by a factor of $O(10)$. Also, the GS coefficient δ_{GS} may enlarge $\langle M+\bar{M}\rangle_{inf}$ by a few factor for $\delta_{GS}\ll 1$. Then we can estimate

$$\frac{\langle M+\bar{M}\rangle_{inf}}{M_s}=O(10^{-2})\text{–}O(10^{-1}). \tag{41}$$

Figure 1 shows V against m for $\Delta\sigma'=100$ and $\delta_{GS}^2/\sigma d^2=0.01$. The form of the Kähler potential, Eq. (38), is not reliable for a large value of $\langle M+\bar{M}\rangle_{inf}$—i.e., $\langle M+\bar{M}\rangle_{inf}/M_s\geq O(1)$. From this point, the above result that M is stabilized as a small value is favorable. Another nice point of the result is that the stabilized value satisfies

$$\langle M+\bar{M}\rangle_{inf}\gg\frac{1}{\Delta\tilde{\sigma}'}, \tag{42}$$

when the value inside of the logarithmic function is large. Note that the F-term and D-term parts in the scalar potential satisfy

$$\frac{V_D}{V_F}\simeq\Delta\tilde{\sigma}'\langle M+\bar{M}\rangle_{inf}. \tag{43}$$

Equations (42) and (43) imply the inequality

$$V_F\ll V_D; \tag{44}$$

that is, with this stabilized value of $\langle M+\bar{M}\rangle_{inf}$, the D-term scalar potential is dominant. That allows us to realize D-term inflation. Therefore, the vacuum energy during inflation is estimated as

$$V=\frac{\delta_{GS}^2}{\tilde{\sigma}}\langle M+\bar{M}\rangle_{inf}. \tag{45}$$

In explicit type-I string models through type-IIB orientifold construction, the GS coefficient δ_{GS} is calculated in units of M_s as $\delta_{GS}/M_s=O(10^{-1})$–$O(10^{-3})$ [20]. It is not completely clear what is a natural value of δ_{GS} from the viewpoint of generic type-I models except type-IIB construction. Furthermore, as reviewed in the previous section, the original value of the FI term appears as the inflation vacuum energy in the simplest model, but it can be effectively reduced in models with more fields and U(1) symmetries. As another remark, if d is suppressed by any mechanism, the stabilized value $\langle M+\bar{M}\rangle_{inf}/M_s$ becomes small like $O(10^{-2})$–$O(10^{-3})$.

IV. DYNAMICS AND DENSITY PERTURBATION

Now let us discuss the dynamics in this inflation model. The relevant potential including one-loop correction during inflaton is given by

$$V=\frac{\delta_{GS}^2\langle M+\bar{M}\rangle_{inf}^2}{S+\bar{S}+\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\times\left[1+\frac{1}{16\pi^2}\frac{2}{S+\bar{S}+\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\ln\frac{\lambda^2|X|^2}{(S+\bar{S})\Lambda^2}\right]. \tag{46}$$

Here the F-term contribution is negligibly smaller than the D term in the potential, as already mentioned. On the other hand, one should notice that the stabilization of twisted moduli is achieved by the F-term potential and we can replace $M+\bar{M}$ with $\langle M+\bar{M}\rangle_{inf}$. Furthermore, hereafter we will consider only the region $S+\bar{S}\ll\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}$ in order to avoid runway motion of the dilaton and obtain a successful inflation. In this sense, the initial value problem of the dilaton, why the dilaton takes such a initial value, remains unsolved.

From

$$\eta_{XX}=-\frac{1}{8\pi^2}\frac{2}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\frac{M_p^2}{X^2}, \tag{47}$$

we find that when the inflaton[5] X reaches the critical point X_f, where

$$X_f^2=\frac{1}{8\pi^2}\frac{2M_p^2}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}, \tag{48}$$

the slow-roll condition is violated. On the other hand, X_c is expressed as

[5] Hereafter, we omit the absolute value symbol $||$.

$$X_c^2 \simeq \frac{2}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}} \frac{\delta_{GS}\langle M+\bar{M}\rangle_{inf}}{\lambda^2}. \tag{49}$$

Hence, if the inequality

$$\frac{M_p^2}{8\pi^2} > \frac{\delta_{GS}\langle M+\bar{M}\rangle_{inf}}{\lambda^2} \tag{50}$$

holds, inflationary expansion terminates when the inflaton reaches X_f rather than X_c. This inequality holds for $\lambda \sim 1$.

Then the equations of motion for the homogeneous part of the scalar fields X and ϕ are, respectively, given by

$$\begin{aligned}\ddot{X}+3H\dot{X} &= -\partial_X V \\ &\simeq -\frac{\delta_{GS}^2}{\tilde{\sigma}^2}\frac{1}{8\pi^2}\frac{2}{X}\end{aligned} \tag{51}$$

and

$$\begin{aligned}\ddot{\phi}+3H\dot{\phi} &= -\partial_\phi V \\ &\simeq \frac{\delta_{GS}^2}{\tilde{\sigma}^2}\frac{2^{3/2}}{M_p}e^{\sqrt{2}\phi/M_p},\end{aligned} \tag{52}$$

where we ignored the dilaton dependence in the radiative correction part because of the weakness of the dependence. The solutions under the slow-roll approximation are

$$X^2 = X_e^2 + \frac{\delta_{GS}^2}{\tilde{\sigma}^2}\frac{1}{2\pi^2}\frac{H(t_e-t)}{3H^2}, \tag{53}$$

$$e^{-\sqrt{2}\phi/M_p} = e^{-\sqrt{2}\phi_e/M_p} + \frac{4\delta_{GS}^2}{\tilde{\sigma}^2}\frac{H(t_e-t)}{3M_p^2H^2}, \tag{54}$$

with $X_e = X(t_e)$ and $\phi_e = \phi(t_e)$, where t_e denotes the time of the end of inflation. By using the definition of the number of e-folds, $-dN \equiv Hdt$, Eq. (54) is rewritten as

$$\left.\frac{S+\bar{S}}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\right|_{t_e} = \frac{1}{\dfrac{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}{S+\bar{S}} - 2N}. \tag{55}$$

Since the left-hand side (LHS) must be less than the order of 10^{-1}, we find that $\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}/(S+\bar{S}) \sim 10^2$ for the present horizon scale. Similarly, from Eq. (53) we obtain

$$X^2 = X_e^2 + \frac{N}{2\pi^2\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}M_p^2; \tag{56}$$

we unfortunately found that the somewhat large field value region $X = O(1)M_p$ is used in this inflation model. This is the undesirable feature of this model. Here we estimate the quantity

$$\begin{aligned}\left(\frac{\dot{\phi}}{\dot{X}}\right)^2 &= (8\pi^2\tilde{\sigma}\langle M+\bar{M}\rangle_{inf})^2\left(\frac{S+\bar{S}}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\right)^2\frac{X^2}{2M_p^2} \\ &\simeq 10^{-1}\left(\frac{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}{10^{-1}}\right)\left(\frac{\left.\dfrac{S+\bar{S}}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\right|_{t_e}}{10^{-1}}\right)^2 \\ &\quad\times\frac{1+2H(t_e-t)}{\left(1+\left.\dfrac{S+\bar{S}}{\tilde{\sigma}\langle M+\bar{M}\rangle_{inf}}\right|_{t_e}2H(t_e-t)\right)^2},\end{aligned} \tag{57}$$

where we adopted X_f as X_e and suppose $\sigma \sim 1$. Thus we find that the ratio of kinetic energies of these scalar fields takes a value of $O(10^{-1})$–$O(10^{-2})$, because the final part in this expression is almost unity during inflation.

Next, we turn to the density perturbation generated by this inflation model. The WMAP data show that the adiabatic fluctuation is favored [21,22]. On the other hand, in general, an inflation model which contains several evolving scalar fields produces also the isocurvature fluctuation. Accordingly, we estimate the contribution from the isocurvature mode and confirm this model to be consistent with observations. The perturbed Einstein equation in Fourier space reads

$$\begin{aligned}\delta\ddot{\phi}+3H\delta\dot{\phi}+\left(\frac{k^2}{a^2}+V_{,\phi\phi}\right)\delta\phi+V_{,\phi X}\delta X \\ = -4\dot{\phi}\dot{\Phi}+2V_{,\phi}\Phi,\end{aligned} \tag{58}$$

$$\begin{aligned}\delta\ddot{X}+3H\delta\dot{X}+\left(\frac{k^2}{a^2}+V_{,XX}\right)\delta X+V_{,X\phi}\delta\phi \\ = -4\dot{X}\dot{\Phi}+2V_{,X}\Phi,\end{aligned} \tag{59}$$

$$\dot{\Phi}+H\Phi = -\frac{1}{2M_p^2}(\dot{\phi}\delta\phi+\dot{X}\delta X), \tag{60}$$

where k is the wave number, a is the scale factor, the comma denotes a derivative of V with respect to the fields (X,ϕ), $\delta\phi$ and δX represent perturbations of the scalar fields ϕ and X, respectively, and Φ is the curvature perturbation variable in the notation of Ref. [23]. These equations have analytic solutions for a long wavelength under the slow-roll approximation [24]:

$$\delta X = (\ln V)_{,X}\left[Q_1+Q_3\int_{t_k}^{t}(\ln(\ln V)_{,X})_{,\phi}Jd\phi\right], \tag{61}$$

$$\delta\phi = (\ln V)_{,\phi}\left[Q_1+Q_3-Q_3\int_{t_k}^{t}(\ln(\ln V)_{,\phi})_{,X}JdX\right], \tag{62}$$

with

$$J=\exp\left(-\int_{t_k}^{t}[(\ln(\ln V),_X),_\phi d\phi+(\ln(\ln V),_\phi),_X dX]\right). \tag{63}$$

Here t_k denotes the horizon crossing time for a wave number k, $k=a(t_k)H$. In addition, Q_1 and Q_2 are integration constants and expressed as

$$Q_1=-\frac{H^2}{\sqrt{2k^3}\dot{X}}e_X(\mathbf{k})\bigg|_{t_k}, \tag{64}$$

$$Q_3+Q_1=-\frac{H^2}{\sqrt{2k^3}\dot{\phi}}e_\phi(\mathbf{k})\bigg|_{t_k}, \tag{65}$$

where $e_X(\mathbf{k})$ and $e_\phi(\mathbf{k})$ are classical random quantities which are normalized as

$$\langle e_X(\mathbf{k})e_X^*(\mathbf{k}')\rangle=\langle e_\phi(\mathbf{k})e_\phi^*(\mathbf{k}')\rangle=\delta^{(3)}(\mathbf{k}-\mathbf{k}'). \tag{66}$$

The appearance of Q_3 is due to the existence of isocurvature perturbation. Furthermore, as we already mentioned, since the dilaton dependence in the radiative correction part is negligible, the potential, Eq. (46), seems to be a separate form $V=V_\phi(\phi)V_X(X)$. Since $J=1$ in this case, Eqs. (61) and (62) can be rewritten as

$$\delta X=\frac{V,_X}{V}Q_1, \tag{67}$$

$$\delta\phi=\frac{V,_\phi}{V}(Q_1+Q_3). \tag{68}$$

Then, the curvature perturbation is expressed as

$$\Phi=-\frac{\dot{H}}{H^2}Q_1+\frac{(V,_\phi)^2}{2V^2}Q_3 \tag{69}$$

$$=\frac{\dot{H}}{H^2}\frac{H^2}{\sqrt{2k^3}\dot{X}}e_X(\mathbf{k})\bigg|_{t_k}+\frac{(V,_\phi)^2}{2V^2}\times\left(\frac{H^2}{\sqrt{2k^3}\dot{X}}e_X(\mathbf{k})-\frac{H^2}{\sqrt{2k^3}\dot{\phi}}e_\phi(\mathbf{k})\right)\bigg|_{t_k}. \tag{70}$$

From $\dot{X}^2\gg\dot{\phi}^2$, we find that the first term is much larger than the second term and the primary contribution to the density perturbation comes from the adiabatic fluctuation generated by inflaton. Hence, we find that the density perturbation in this inflation model is almost adiabatic with small isocurvature fluctuations.

Here we shall introduce the so-called Bardeen parameter which is defined by

$$\zeta\equiv\Phi+\frac{2}{3}\frac{\Phi+H^{-1}\dot{\Phi}}{1+w}, \tag{71}$$

which is conserved in the superhorizon scale $k\ll aH$ if the fluctuation is adiabatic [25]. Here w denotes the ratio of pressure to energy density. However, the present model contains two slow-rolling scalar fields and may produce the isocurvature fluctuation. In this case, the time variation of ζ is caused by the isocurvature fluctuation and the Bardeen parameter is no longer a conserved quantity on superhorizon scales. Indeed, the time variation of ζ is expressed as

$$\frac{3}{2}H(1+w)\dot{\zeta}=-c_s^2\frac{k^2}{a^2}\Phi-\frac{\rho}{2}w\Gamma, \tag{72}$$

where $c_s^2=\dot{p}/\dot{\rho}$ and the quantity

$$\Gamma=\frac{\delta p}{p}-\frac{c_s^2}{w}\frac{\delta\rho}{\rho} \tag{73}$$

represents the amplitude of an entropy perturbation. In the case that the universe is filled with two component scalar fields (X,ϕ), Eq. (72) is rewritten as [26]

$$\dot{\zeta}=\frac{H}{\dot{H}}\frac{k^2}{a^2}\Phi+\frac{H}{2}\frac{\partial}{\partial t}\left(\frac{\dot{X}^2-\dot{\phi}^2}{\dot{X}^2+\dot{\phi}^2}\right)\left(\frac{\delta X}{\dot{X}}-\frac{\delta\phi}{\dot{\phi}}\right). \tag{74}$$

In a single field inflation model, either $\dot{X}$ or $\dot{\phi}$ goes to zero and the second term on RHS of Eq. (74) vanishes. Thus, ζ becomes a conserved quantity on superhorizon scales in that case. However, we need to take the variation of ζ into account, because there are two evolving scalar fields in the present model. The spectrum of the density perturbations ζ at the horizon crossing time is

$$\mathcal{P}_\zeta|_{t_k}=\left(\frac{H^2}{2\pi|\dot{X}|}\right)^2\left[1-\frac{1}{2}\frac{\epsilon_\phi}{\epsilon_X}\right]\bigg|_{t_k}, \tag{75}$$

up to the first order of $\dot{\phi}^2/\dot{X}^2$, where $\epsilon_\phi=\dot{\phi}^2/(2V)$ and $\epsilon_X=\dot{X}^2/(2V)$ are slow-roll parameters. From Eq. (74), the variation of the Bardeen parameter after the horizon crossing is estimated as

$$\Delta\zeta\equiv\int_{t_k}^{t_e}\dot{\zeta}dt=Q_3\left(\frac{\dot{\phi}^2}{\dot{X}^2}\bigg|_{t_k}-\frac{\dot{\phi}^2}{\dot{X}^2}\bigg|_{t_e}\right). \tag{76}$$

Furthermore, the power spectrum of the density perturbations at the end of inflation [27] is estimated as

$$\mathcal{P}_\zeta|_{t_e}=\left(\frac{H^2}{2\pi|\dot{X}|}\right)^2\bigg|_{t_k}\left[1-2\frac{\epsilon_\phi}{\epsilon_X}\bigg|_{t_e}+\frac{(\epsilon_\phi/\epsilon_X|_{t_e})^2}{\epsilon_\phi/\epsilon_X|_{t_k}}\right]. \tag{77}$$

We find that the variation of ζ on superhorizon scales by the isocurvature perturbation is suppressed by the power of $\dot{\phi}^2/\dot{X}^2=O(10^{-1})$–$O(10^{-2})$. The amplitude of the curvature perturbation is rewritten as

$$\left.\frac{H^2}{2\pi|\dot{X}|}\right|_{t_k} = 2\frac{\delta_{GS}\langle M+\bar{M}\rangle_{inf}}{M_p^2}\sqrt{\frac{N}{6}}, \tag{78}$$

and we obtain the constraint on the model parameters,

$$\frac{\delta_{GS}\langle M+\bar{M}\rangle_{inf}}{M_p^2} \simeq 10^{-5}, \tag{79}$$

for the present horizon scale. For the GS coefficient δ_{GS}, a suppressed value is required like the heterotic case. Explicit models through type-IIB orientifold construction seem to derive $\delta_{GS}/M_s = O(10^{-1})$–$O(10^{-3})$ [20]. Anyway, the GS coefficient δ_{GS} is model dependent. If we find the model in which δ_{GS} is small enough like $\delta_{GS}/M_s \sim 10^{-3}$, the string scale can be almost comparable with M_p or slightly smaller. Otherwise, if the original FI terms are not small enough, we would need the mechanism to reduce the effective FI term as mentioned in Sec. II or a lower string scale.

On the other hand, the spectrum of gravitational wave perturbation produced by the inflation is given by $\mathcal{P}_g = |H/(2\pi M_p)|^2$ and its observational results show $\mathcal{P}_g \leqslant 10^{-10}$. This constraint is expressed as $V \leqslant 10^{-8} M_p^4$ in terms of the potential. Therefore we obtain

$$V = \frac{\delta_{GS}^2\langle M+\bar{M}\rangle_{inf}}{\tilde{\sigma}} \leqslant 10^{-8} M_p^4. \tag{80}$$

By combining Eqs. (79) and (80), we obtain $\tilde{\sigma}\langle M+\bar{M}\rangle_{inf} \geqslant 10^{-2}$. That can be satisfied by a stabilized value (41) of $\langle M+\bar{M}\rangle_{inf}$—e.g., $\langle M+\bar{M}\rangle_{inf} \sim 0.1\times M_s$ for $\sigma \sim 1$.

Finally we discuss the evolution of the universe after inflation. The relevant potential after inflation is given by

$$V = (\text{gaugino condensation potential}) + \lambda^2|X|^2|\phi_-|^2 + \frac{[\delta_{GS}K'(M,\bar{M}) - |\phi_-|^2]^2}{S+\bar{S}+\tilde{\sigma}(M+\bar{M})}. \tag{81}$$

After inflation, the scalar fields X and ϕ_- oscillate around their corresponding minimum $X=0$ and $\langle \phi_-\rangle = \sqrt{\xi}$, respectively. Here we should notice that ξ is a function of the twisted moduli M and still a time variant in the reheating stage, because the twisted moduli do not yet reach at the true vacuum at this moment. This is a feature which the simplest model of D-term inflation does not possess.

Now, we consider the reheating process according to Ref. [28]. Since there are fields which are charged under both anomalous U(1) and each of the subgroups of the standard model, the D term receives a contribution from these field. As a result, the D term takes the form of

$$D = g\left(|\phi_+|^2 - |\phi_-|^2 + \sum_i q_i|Q_i|^2 + \xi\right), \tag{82}$$

$$\xi = \delta_{GS}K'(M,\bar{M}), \tag{83}$$

where Q_i are charged fields for both anomalous U(1) and gauge groups of the standard model and q_i are their positive charge. In the case that fields φ_i of the minimal supersymmetry standard model belong to the fields Q_i, the ϕ_- field couples with them by the interaction

$$\mathcal{L}_{int} = g^2\sqrt{\xi}\sum_i q_i|\varphi_i|^2(\delta\phi_- + \delta\phi_-^\dagger), \tag{84}$$

where $\delta\phi_- \equiv \phi_- - \langle\phi_-\rangle$. Hence the ϕ_- field would immediately decay through this gauge interaction. On the other hand, the decay of X would be delayed. The mass of ϕ_- is $m_{\phi_-} = (g/\lambda)m_X$. Since $g>1$ in our model at the time of the end of inflation, the decay of X to ϕ_- would be forbidden kinematically. As a result, X decays to light matter through higher-dimensional interactions and the reheating can be completed with a high enough reheating temperature.

In this scenario, a huge entropy after the reheating by the inflaton is produced by the decay of the dilaton and moduli fields in late time. Therefore, this D-term inflation model should be incorporated with the Affleck-Dine baryogenesis [29]. The Affleck-Dine baryogenesis with the dilution by the dilaton and moduli decay have been investigated [30,31]. However, an actual estimation of the baryon to entropy ratio is beyond the scope of this paper, because their full potential strongly depends on the form of the Kähler potential of twisted moduli and the condensation gauge groups [15].

Furthermore, it is pointed out that cosmic strings generated after D-term inflation can modify the CMB anisotropy significantly [32]. Huge entropy production at a late time is desirable from this point of view, because dilution by the entropy production is one possible way to remove the effect of cosmic strings. Moreover, there is one thing to note. The FI term is given by Eq. (25) and its magnitude is parametrized by the expectation value of the twisted moduli. Therefore the tension of cosmic strings in late time is not necessarily the same as the scale of the FI term during inflation—namely, $\langle M+\bar{M}\rangle_{inf} \neq \langle M+\bar{M}\rangle$, where $\langle\cdots\rangle$ denotes the VEV. This feature might change the situation of the effect of cosmic strings.

V. CONCLUSIONS AND DISCUSSION

We have studied D-term inflation within the framework of type-I string-inspired models. In this case, the twisted moduli field plays a significant role. For stabilization of the twisted moduli, the polynomial form of the Kähler potential is important. Stabilization of the twisted moduli during inflation is achieved with the help of the gaugino condensation potential and this stabilization mechanism does not spoil the situation that the potential is dominated by the D term. Furthermore, it is a favorable property that the expectation value of the twisted moduli during inflation is smaller enough than unity where the Kähler potential $K=(M+\bar{M})^2/2$ is valid, while this result leads to one shortcoming—that the inflaton X must take a somewhat large value $X>M_p$ during inflation.

At first, we found that the magnitude of the FI term would be reducible to a desirable value within this framework even if we do not assume a hierarchically large gap between the string scale M_s and the Planck scale M_p.

Concerning the second problem—or dilaton runaway problem—we found the condition to avoid this difficulty for the initial value of the dilaton. In the potential with a fixed twisted moduli, we could find the field region where the dilaton does not run away, $S+\overline{S} \ll \tilde{\sigma}\langle M+\overline{M}\rangle_{inf}$. If the dilaton takes such an initial value at the preinflation stage, it is possible that the universe undergoes a quasi de Sitter expansion, because the dilaton can evolve slowly. Of course, at the moment, we cannot answer the question why the dilaton takes such an initial value. The point we would like to emphasize in the present paper is that D-term inflation is possible in type-I string-inspired models under certain conditions. On the other hand, the small expectation value of the dilaton during inflation might be a good point from another point of view. In studies of the evolution of the dilaton in string cosmology, the initial condition—that the dilaton take a smaller value than the VEV at the beginning, $(S+\overline{S})|_{t_i} < \langle S+\overline{S}\rangle$—has been adopted [10,31,33]. Therefore, the initial value of the dilaton set by this inflation is consistent with the initial condition in these studies. Although these are based on a heterotic model, the situation would not change essentially. That might lead to dilaton stabilization at the correct vacuum.

Moreover, the ingenious point in this model is that the condition which prohibits the dilaton from rolling down the potential also suppresses the growth of the unnecessary isocurvature fluctuation. Thus, we can obtain an almost adiabatic density perturbation.

What we have to investigate further is the late time evolution of the universe. In the present paper, we have not estimated baryogenesis and the effect of cosmic strings and so on, because they would involve us in other factors such as a concrete potential for the supersymmetry breaking than the potential of inflation. These are important issues for future works.

ACKNOWLEDGMENTS

T.K. was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan (No. 14540256). O.S. was supported by the National Science Council of Taiwan under Grant Nos. NSC 92-2811-M-009-006 and NSC 92-2811-M-009-018.

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