

Chiral-Symmetry Realization for Even- and Odd-Parity Baryon Resonances

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Baryon resonances with even and odd parity are collectively investigated from the viewpoint of chiral symmetry (ChS). We propose a quartet scheme where Δ 's and N^* 's with even and odd parity form a chiral multiplet. This scheme gives parameter-free constraints on the baryon masses in the quartet, which are consistent with observed masses with spin $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. The scheme also gives selection rules in the one-pion decay: The absence of the parity nonchanging decay $N(1720) \rightarrow \pi\Delta(1232)$ is a typical example which should be confirmed experimentally to unravel the role of ChS in baryon resonances.

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Chiral symmetry (ChS) and its dynamical breaking in quantum chromodynamics (QCD) are the key ingredients in low energy hadron dynamics. For instance, all hadrons can be classified in principle into some representation of the chiral group $SU(N_f)_L \times SU(N_f)_R$, and the interactions among hadrons are strongly constrained by this symmetry.

There are two ways to realize ChS in effective low-energy Lagrangians: nonlinear and linear representations. In the former, pions as Nambu-Goldstone (NG) bosons play a crucial role, which has been extensively studied and is summarized as the celebrated chiral perturbation theory [1]. In the latter, scalar mesons are introduced to form a linear chiral multiplet with the NG bosons. Although such heavy mesons do not allow systematic low-energy expansion, they are essential near the critical point of chiral phase transition where both the scalars and NG bosons behave as soft modes [2].

As for the baryons in the linear representation, the Gell-Mann Lévy sigma model [3] is a first example where the nucleon transforms linearly under both the vector and the axial-vector transformations. DeTar and Kunihiro (DK) [4] generalized the model so that N_+ (the nucleon) and its odd-parity partner N_- form a multiplet of the chiral group [5]. A unique aspect of the DK model is that the even- and the odd-parity nucleons can have nonvanishing mass even in the Wigner phase without violating ChS.

In the DK construction, N_{\pm} are represented as a superposition of N_1 and N_2 which are assigned to have opposite axial charges with each other. Subsequently, this was called the "mirror assignment" and distinguished from the "naive assignment" where N_1 and N_2 have the same axial change [6]: The two assignments are shown to have phenomenologically distinguishable predictions [7].

The purpose of this Letter is to develop the idea of the mirror assignment in baryon resonances with different parity ($P = \pm$) and different isospin ($I = \frac{1}{2}, \frac{3}{2}$), and to explore how ChS is realized in the excited baryons. Achieving this purpose is tantamount to constructing a linear sigma model in which both Δ_{\pm} 's and N_{\pm}^* 's are incor-

porated for a given spin sector. [Here we call N^* (Δ) as a resonance with $I = \frac{1}{2}$ ($\frac{3}{2}$), and the subscripts \pm denote their parity.] Thus we shall arrive at proposing a *quartet scheme* in which N_+^*, N_-^*, Δ_+ , and Δ_- form a chiral multiplet. It will be shown that this quartet scheme is consistent with the observed baryon spectra without fine tuning of the model parameters. We will also show some evidence of this scheme in the decay pattern of the resonances. Throughout the present Letter, we focus on $N_f = 2$, and neglect the explicit breaking of ChS due to quark masses.

To make the argument explicit, let us start with $\Delta(1232)$ ($J^P = \frac{3}{2}^+$) and its chiral partners. First of all, we need to choose the representation of Δ under $SU(2)_L \times SU(2)_R$. The quark fields $q = q_l + q_r$ belong to $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$, where the first and second numbers in the parentheses refer to $SU(2)_L$ and $SU(2)_R$ representations, respectively. Therefore, $(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$ and $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ are the two candidates for Δ ; both of them contain isospin $I = \frac{3}{2}$ and are constructed from three quarks $[(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})]^3$ [8]. Here, we choose $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ for Δ , because Δ is known to be a strong resonance in the N - π system, and $N \times \pi = [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})] \times [(\frac{1}{2}, \frac{1}{2})]$ does not contain $(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$. In the quark basis, this representation may be schematically written as $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1) = (q_L q_L)_{I=1} q_R \oplus q_L (q_R q_R)_{I=1}$ where Lorentz and color indices are suppressed [9]. Note that $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ contains both $I = \frac{3}{2}$ and $I = \frac{1}{2}$ baryons, thus we utilize the latter to incorporate N^* . From now on, we do not consider the quark structure of Δ and N^* , and simply introduce elementary Rarita-Schwinger (RS) fields for constructing an effective Lagrangian.

To accommodate the parity partners of the baryon resonances, let us define ψ_1 and ψ_2 as two independent $J = \frac{3}{2}$ RS fields with even and odd parity, respectively. The Lorentz index $\mu = 0, \dots, 3$ for the RS fields is suppressed for brevity. We then define the chiral decomposition: $\psi_i = \psi_{il} + \psi_{ir}$ with $\gamma_5 \psi_{il,ir} = \mp \psi_{il,ir}$ ($i = 1, 2$). In the $J = \frac{3}{2}$ chiral quartet, ψ_1 and ψ_2 are mixed to

form four resonances: $\Delta_+(P_{33})$, $\Delta_-(D_{33})$, $N_+(P_{13})$, and $N_-(D_{13})$.

In the mirror assignment, ψ_{1l} and ψ_{2r} belong to $(1, \frac{1}{2})$, while ψ_{1r} and ψ_{2l} belong to $(\frac{1}{2}, 1)$, so that ψ_1 and ψ_2 have opposite axial charge. Thus, these fields have three indices, $(\psi_{1,2})_{\alpha\beta}^\gamma$, with α, β , and γ take 1 or 2. Here $(\alpha\beta)$ is the index for $I = 1$ triplet and γ for $I = \frac{1}{2}$ doublet. Since ψ is traceless for the triplet index $(\alpha\beta)$, it is convenient to introduce a component field $(\psi_i)^{A,\gamma}$ ($A = 1, 2, 3$ for triplet and $\gamma = 1, 2$ for doublet) as

$$(\psi_{1,2})_{\alpha\beta}^\gamma = \sum_{A=1,2,3} (\tau^A)_{\alpha\beta} (\psi_{1,2})^{A,\gamma}, \quad (1)$$

where τ^A ($A = 1, 2, 3$) is the 2×2 Pauli matrix.

The transformation rules of ψ_i under $SU(2)_L \times SU(2)_R$ are then represented by

$$(\tau^A)_{\alpha\beta} (\psi_{1l,2r})^{A,\gamma} \rightarrow (L\tau^A L^\dagger)_{\alpha\beta} (R\psi_{1l,2r})^{A,\gamma}, \quad (2)$$

$$(\tau^A)_{\alpha\beta} (\psi_{2l,1r})^{A,\gamma} \rightarrow (R\tau^A R^\dagger)_{\alpha\beta} (L\psi_{2l,1r})^{A,\gamma}, \quad (3)$$

where L (R) corresponds to the $SU(2)_L$ ($SU(2)_R$) rotation. The meson field $M \equiv \sigma + i\vec{\pi} \cdot \vec{\tau}$ belongs to $(\frac{1}{2}, \frac{1}{2})$ multiplet, and obeys the standard transformation rule, $M \rightarrow LMR^\dagger$.

Now let us construct the mass term and the Yukawa coupling of ψ_i with M . Here we consider only the simplest interaction which has only single M without derivatives as in the case of the Gell-Mann-Lévy and DeTar-Kunihiko models. It can be shown that the chiral invariance under Eqs. (2) and (3) together with parity and time-reversal invariance allow only three terms:

$$\begin{aligned} \mathcal{L}_{\text{int}} = & m_0 (\bar{\psi}_2^A \gamma_5 \psi_1^A - \bar{\psi}_1^A \gamma_5 \psi_2^A) \\ & + a \bar{\psi}_1^A \tau^B (\sigma - i\vec{\pi} \cdot \vec{\tau} \gamma_5) \tau^A \psi_1^B \\ & + b \bar{\psi}_2^A \tau^B (\sigma + i\vec{\pi} \cdot \vec{\tau} \gamma_5) \tau^A \psi_2^B, \end{aligned} \quad (4)$$

where m_0 , a , and b are free parameters not constrained by ChS. These three parameters give strong constraints on the masses and couplings of Δ 's and N^* 's. The interaction Eq. (4) for the $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ chiral quartet is a natural generalization of that for the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ chiral doublet in [4]. Instead of working with the linear basis $(\sigma, \vec{\pi})$, one can also adopt the nonlinear basis [$M = \rho \exp(i\gamma_5 \vec{\tau} \cdot \vec{\phi})$] together with a suitable redefinition of the baryon fields along the same line with the second reference in [7]. This could be also a good starting point of the pion-baryon phenomenology in the quartet scheme.

A shortcut to derive Eq. (4) is to use LMR^\dagger together with the rotated fields in the right hand side of Eqs. (2) and (3) and to look for combinations in which L and R do not appear in the final expression. Since L and R are independent transformations, the indices related to the left (right) rotation must be always contracted with the left (right) rotation. One of the chiral invariant mass terms, for example, comes from the combination $\text{Tr}[(R\tau^A R^\dagger) \times$

$(R\tau^B R^\dagger)][(\bar{\psi}_{1r}^A L^\dagger)(L\psi_{2l}^B)]$. Also, one of the Yukawa terms is obtained from $[(\bar{\psi}_{1r}^A L^\dagger)(R\tau^B R^\dagger)(RM^\dagger L^\dagger) \times (L\tau^A L^\dagger)(L\psi_{1r}^B)]$.

As already mentioned, $\psi_i^{A,\gamma}$ contains both $I = \frac{3}{2}$ field $\Delta_{i,M}$ ($M = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$) and $I = \frac{1}{2}$ field $N_{i,m}^*$ ($m = \frac{1}{2}, -\frac{1}{2}$) which are obtained by the following isospin decomposition: $\psi_i^{A,\gamma} = \sum_M (T_{3/2}^A)_{\gamma M} \Delta_{i,M} + \sum_m (T_{1/2}^A)_{\gamma m} N_{i,m}^*$ where the isospin projection matrices $T_{3/2}^A$ and $T_{1/2}^A$ are defined through the Clebsch-Gordan coefficients, $(T_{3/2}^A)_{\gamma M} = \sum_{r,\gamma'} (1r \frac{1}{2} \gamma' | \frac{3}{2} M) \epsilon_r^A \chi_{\gamma'}^\gamma$ and $(T_{1/2}^A)_{\gamma m} = \sum_{r,\gamma'} (1r \frac{1}{2} \gamma' | \frac{1}{2} m) \epsilon_r^A \chi_{\gamma'}^\gamma$. $\vec{\epsilon}_r$ are vectors relating the $A = (1, 2, 3)$ basis to $r = (+1, 0, -1)$ basis, and $\vec{\chi}_{\gamma'}$ relates the $\gamma = (1, 2)$ basis to $\gamma' = (\frac{1}{2}, -\frac{1}{2})$ basis [10]. Their explicit forms are $\epsilon_1 = -1/\sqrt{2}(1, i, 0)$, $\epsilon_0 = (0, 0, 1)$, $\epsilon_{-1} = 1/\sqrt{2}(1, -i, 0)$, $\chi_{1/2} = (1, 0)$, $\chi_{-1/2} = (0, 1)$.

With the invariant Lagrangian (4), we shall next show its phenomenological consequences on the masses of Δ 's and N^* 's. After the spontaneously symmetry breaking $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ due to the finite σ condensate $\langle \sigma \rangle \equiv \sigma_0 > 0$, the mass term in Eq. (4) becomes

$$\begin{aligned} \mathcal{L}_m = & -(\bar{\Delta}_1, \bar{\Delta}_2) \begin{pmatrix} -2a\sigma_0 & \gamma_5 m_0 \\ -\gamma_5 m_0 & -2b\sigma_0 \end{pmatrix} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \\ & -(\bar{N}_1^*, \bar{N}_2^*) \begin{pmatrix} a\sigma_0 & \gamma_5 m_0 \\ -\gamma_5 m_0 & b\sigma_0 \end{pmatrix} \begin{pmatrix} N_1^* \\ N_2^* \end{pmatrix}. \end{aligned} \quad (5)$$

The physical bases Δ_\pm and N_\pm^* diagonalizing the mass matrices are given by

$$\begin{pmatrix} \Delta_+ \\ \Delta_- \end{pmatrix} = \frac{1}{\sqrt{2 \cosh \xi}} \begin{pmatrix} e^{\xi/2} & \gamma_5 e^{-\xi/2} \\ \gamma_5 e^{-\xi/2} & -e^{\xi/2} \end{pmatrix} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix},$$

together with a similar formula for N_\pm^* with the replacement $\xi \rightarrow \eta$. The mixing angles ξ , η are given by $\sinh \xi = -(a+b)\sigma_0/m_0$ and $\sinh \eta = (a+b)\sigma_0/(2m_0)$. These bases are chosen so that the masses of Δ 's and N^* 's are all reduced to the chiral-invariant mass $m_0 > 0$ when ChS is unbroken ($\sigma_0 = 0$).

Thus we finally reach the mass formula,

$$m_{\Delta_\pm} = \sqrt{(a+b)^2 \sigma_0^2 + m_0^2} \mp \sigma_0(a-b), \quad (6)$$

$$m_{N_\pm^*} = \sqrt{\left(\frac{a+b}{2}\right)^2 \sigma_0^2 + m_0^2} \pm \frac{\sigma_0}{2}(a-b). \quad (7)$$

Equations (6) and (7) show that the spontaneous breaking of ChS lifts the degeneracy between parity partners (Δ_+ vs Δ_- , and N_+^* vs N_-^*) and the degeneracy between isospin states (Δ vs N^*) simultaneously [11].

A remarkable consequence of our quartet scheme is the following mass relations which hold irrespectively of the choice of the parameters (m_0, a, b):

(1) The ordering in parity doublet of N^* is always opposite to that of Δ :

$$\text{sgn}[m_{\Delta_+} - m_{\Delta_-}] = -\text{sgn}[m_{N_+^*} - m_{N_-^*}]. \quad (8)$$

(2) The mass difference between the two parity doublets

space is large enough, is insignificant, or has not been shown to exist in the recent analysis of πN scattering amplitudes [13]. (The existence has been suggested in an old analysis of $\pi N \rightarrow \pi\pi N$ though [14].) On the other hand, $N_-(1520) \rightarrow \pi\Delta_+(1232)$ and $\Delta_-(1700) \rightarrow \pi\Delta_+(1232)$ in the S -wave channel, which are not suppressed in Eq. (11), have been seen with the partial decay rates 5%–12% and 25%–50%, respectively [12].

There exist some works investigating the spectrum and decay of the excited baryons simultaneously: the constituent quark model [15] and the collective string model [16], in which Δ 's are in a different spin-flavor multiplet from that of N^* 's. Although the symmetry and dynamics are totally different from ours, the parity nonchanging decay $N_+(1720) \rightarrow \pi\Delta_+(1232)$ in these models is suppressed by a factor 25–50 compared to the parity changing decay $\Delta_-(1700) \rightarrow \pi\Delta_+(1232)$. The physical reason behind this suppression and its relation to our approach is not clear at the moment.

The suppression of $\pi\Delta_+\Delta_\pm$ and $\pi N_\pm^* N_\pm^*$ cannot be checked in the decays, but empirical studies of the $\pi N \rightarrow \pi\pi N$ process [17] seem to suggest that the $\pi\Delta_+(1232)\Delta_+(1232)$ coupling is less than half of the quark model prediction given by $g_{\pi\Delta\Delta} = (4/5)g_{\pi NN}$ [10].

For $J = \frac{1}{2}, \frac{5}{2}$ sectors, similar analysis is not possible at present, because of large uncertainties and/or the absence of experimental data for relevant decays. More experimental data on the decays among the quartet shown in Fig. 1 would be quite helpful for future theoretical studies.

We note here that the selection rule discussed above may in principle be modified by chiral invariant terms not considered here, such as the terms containing derivatives as well as multi- M fields. This is the situation similar to that for g_A of the nucleon in the linear sigma model, where the simplest Yukawa coupling in the tree level gives $g_A = 1$, while the higher dimensional derivative coupling as well as quantum corrections could shift it to 1.25 [18]. Therefore, detailed studies with those terms should be also done in the future.

In summary, we have investigated baryon resonances with both parities from the viewpoint of chiral symmetry. We have constructed a linear sigma model in which Δ_\pm 's and N_\pm^* 's with a given spin are assigned to be a representation $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ of the chiral $SU(2)_L \times SU(2)_R$ group. Adopting the mirror assignment for the axial charge of baryons, we have arrived at the *quartet scheme* where N_+^*, N_-^*, Δ_+ , and Δ_- form a chiral multiplet. We have shown that the quartet scheme gives constraints not only on the baryon masses but also their couplings; it turns out that the constraints are consistent with the observed baryon spectra. We have shown that experimental confirmation of the absence of parity nonchanging decay in the $J = \frac{3}{2}$ sector such as $N_+(1720) \rightarrow \pi\Delta_+(1232)$ together with the

measurement of the decay patterns in $J = \frac{1}{2}, \frac{5}{2}$ sectors is important to test the quartet scheme and to explore the role of ChS in excited baryons.

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