Studies on the Heat Conductin in Wood

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The present study is a discussion on the results of investigations made hitherto by author on heat conduction in wood.

Introduction

The heat conduction in wood makes one of the most important problems in utilizing woods, and it is needed to study from that theory the efficient heat treatment of wood, for example, steaming and cooking as pretreatment in manufacturing veneer, bending wood, fireproof wood, and fungiproof wood, steaming as pre- and intermittent-treatment in drying of wood, and hotpressing in plywood manufacturing.

It was HUNT[15] (1915) who first observed that the effect of steaming of wood which had been a heat treatment most practisized from long ago has close relation with the internal temperature and moisture of wood. WIRKA[58] (1924) reported on the steaming of southern pine and MACLEAN[25] (1927-34) applied the theory of heat conduction to this problem for the first time to open the theoretical studies of steaming of wood. In our country, some works[36),43),44] were published on the steaming of wood.

There have been hardly published on the fundamental studies of hot-pressed plywood manufacturing except those of BITTNER[1], PERRY[46], and DELMONTE[5] who published some reports on how to select the time and temperature schedule in hot-pressing, and also hardly reported on the temperature distribution in wood drying[27] and on cooking of wood.

It is therefore easily imaginable that the insufficiency of the theoretical investigations on the heat conduction in wood which relates to widely and highly employed utilization of wood would prevent further development of reasonable utilization of wood considerably.

The auther treated theoretically and experimentally this problem and made investigations concerning to the equation of heat conduction of anisotropic wood and summarized solutions for important cases expected in heating of wood. For important application it hat been intended to simplify of calculation by showing practical method and by presenting nomograph, for special cases of application, also, the temperature calculation method has been shown.

I. The differential equation for heat conduction through wood

1. Equation by rectangular coordinates

Wood is a kind of heterogeneous, anisotropic, and porous material made by organic unions of many kind of cells which are composed chiefly of cellulose, lignin, pentosan, and others, and which contain air, water, and many other complicated chemical ingredients. The flow of heat in wood, therefore, takes place theoretically in close correlation with radiation, convection, and conduction. Since the dimensions of cells in wood vary with species, age, position of tree stem, location, and other factors, the following are some examples of maximum dimensions ; of vessel the diameter is 200-500 μ in *Castanea* (KURI) and the length is 800-1500 μ in *Clethra* (RYOBU) ; of wood fiber the diameter 25-30 μ in *Paulownia* (KIRI) and *Aesculus* (TOTI) and the length 1000-2000 μ in *Tilia* (SINA) ; of tracheid the diameter is 50-65 μ in *Ginkgo* (ICHO) and the length 3200-6000 μ in *Abies* (MOTI). In such dimensions of the cells, it has been shown hitherto[2),3)] that the influences of radiation and convection are very small so that the flow of heat through wood is only made by conduction along the cell membrane and the cell cavity.

For such large pieces of wood as used in industrial matrials the histological anisotropy is largely neglected and at least for one direction it can be assumed as a homogeneous body. Now suppose a micro-cube of wood as shown in Fig. 1 and let the radial-, tan-

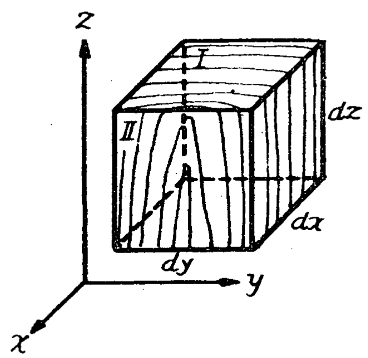


Fig. 1

gential-, and fiber-direction be parallel to x-, y-, and z-axis, and the thermal conductivity along the x-axis, i. e. the direction of the radius be λ_x, then the amount of heat flowing in this cube through the plate I in time $d\theta$ is $-\lambda_x \frac{\partial t}{\partial x} dy\, dz\, d\theta$, and that flowing out this cube through the plate II is $-\lambda_x \frac{\partial}{\partial x}\left(t+\frac{\partial t}{\partial x}dx\right) dy\ dz\ d\theta$. Consequently when λ_x is constant in this range of temperature change, the amount of heat remaining in the micro-cube is $\lambda_x \frac{\partial^2 t}{\partial x^2} dx\ dy\ dz\ d\theta$. The same relation is also obtained with y- and z-axis, and so the total amount of heat dQ remaing in this micro-cube is expressed as follows.

$$dQ=\left(\lambda_x\frac{\partial^2 t}{\partial x^2}+\lambda_y\frac{\partial^2 t}{\partial y^2}+\lambda_z\frac{d^2 t}{dz^2}\right)dx\ dy\ dz\ d\theta$$

when the rate of temperature raising of wood is $\frac{\partial t}{\partial \theta}$, density R and specific heat c, then $dQ=cR\ dx\ dy\ dz\frac{\partial t}{\partial \theta}d\theta$.

From the both equations,

$$\left.\begin{aligned}\frac{\partial t}{\partial \theta}&=\frac{\lambda_x}{cR}\frac{\partial^2 t}{\partial x^2}+\frac{\lambda_y}{cR}\frac{\partial^2 t}{\partial y^2}+\frac{\lambda_z}{cR}\frac{\partial^2 t}{\partial z^2}\\ \text{or}\qquad \frac{\partial t}{\partial \theta}&=a_x\frac{\partial^2 t}{\partial x^2}+a_y\frac{\partial^2 t}{\partial y^2}+a_z\frac{\partial^2 t}{\partial z^2}\end{aligned}\right\}\quad\cdots\cdots\cdots(1)$$

where a_x, a_y, a_z are the thermal diffusivity along the radial-, tangential-, and fiber-direction in wood. Equation (1) is the basic form for the heat conduction in wood.

2. Equation by cylindrical coordinates

As shown later, it can be written approximately $a_x=a_y$. For expressing formula (1) by cylindrical coordinates, let $a_x=a_y=a_\perp$, and $a_z=a_{//}$ and then we write $x=r\cos\varphi$, $y=r\sin\varphi$

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}=\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2}$$

and (1) is transformed

$$\frac{\partial t}{\partial \theta}=a_\perp\left(\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2}\right)+a_{//}\frac{\partial^2 t}{\partial z^2}\quad\cdots\cdots\cdots(2)$$

This can be regarded as an approximate heat conduction equation in round wood.

II. The factors influencing on the thermal diffusivity

In making solution of (1) and (2), generally it is requisite that the thermal diffusivity $a(=\lambda/cR)$ is constant. However, the factors λ, c, and R, which constitute a are influenced by temperature, density, moisture, the direction of fiber and others, and so before discussing the application of the above basic equation some consideration will be made about the influences of these factors on α, by referring literatures hitherto.

1. Relation of the density (i. e. wood species) and the thermal diffusivity

Long ago DEBYE gave a formula for the relation between the thermel conductivity and the density of insulating materials as follows:

$$\lambda = K\, cR^{\frac{1}{3}}\, \rho^{-\frac{1}{3}}$$

where K is a constant, c specific heat, R density, and ρ compressive rate, and MATANO[35] made a theoretical discussion on the same problem by the idea of combining the substancial and the vacuole parts in parallel and series but it is not available to applicate to wood. It is obvious that λ has theoretically the value near to the mean of that of cell membrane and of cell cavity, increasing with the increase of density between the two limits of the thermal conductivities of air and the true density of cell membrane. It has been regarded very difficult to deduce the relation of the density and the thermal conductivity physically by analyzing various factors which influence on wood structure, and hitherto chiefly experimental formulas have been presented.

KOLLMANN presented the following formula by the use of many estimated values[22] reported previously on any species of wood at 20°C and 10%.

$$\left.\begin{aligned} \lambda_{\perp} &= 0.15\left(\frac{R}{1000}\right)^{1.5} + 0.04 \\ \lambda_{//} &= 0.45\left(\frac{R}{1000}\right)^{1.5} + 0.04 \end{aligned}\right\} \quad \cdots\cdots\cdots(3)$$

where $\lambda_{\perp}$ and $\lambda_{//}$ are the thermal conductivity to the directions rectangular and parallel to the fiber respectively (kcal/m h°), R is density of wood (kg/m^3). He corrected these formulas later[24] that both $\lambda_{\perp}$ and $\lambda_{//}$ increase linearly with the increse of density. WAKASUGI[55] gave the following formula with dry *Ochroma* (Balsa), *Paulownia* (KIRI), *Cryptomeria* (SUGI), *Pinus* (MATU), and *Guajacum* (Ligunum-Vitae).

$$\left.\begin{aligned} \lambda_{\perp} &= 0.022 + 0.168\, r_u \quad &(\text{KOLLMANN. } 27^\circ\text{C } 12\%) \\ \lambda_{\perp} &= 0.035 + 0.00015\, R \quad &(\text{WAKASUGI, } 20^\circ\text{C}) \end{aligned}\right\} \quad \cdots\cdots\cdots(4)$$

The linear relation between $\lambda_{\perp}$ and R has also been found by ROWLEY[47], THNELL, NARAYANAMURTI[40], and others. Fig. 2 shows the results of these estimations, where good agreements can be seen.

As to the relation between the density and the specific heat, DUNLAP[6] reported that both have no relation in the range of $R=230-1100$. Therefore, the relation between the density and the thermal diffusivity can be determined by the ratio λ/R. As obvious from Fig. 2, in the range of $R=400-800$, it can be written $\lambda/R \fallingdotseq$ constant and so it may be

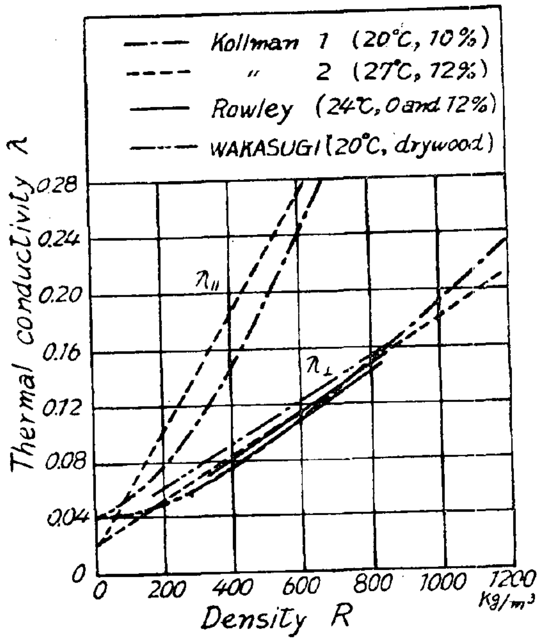


Fig. 2 Relation between the density and the thermal conductivity of wood

said that α is independent upon the density, accordingly upon the species of wood under the given temperature and the moisture content ; but for lighter ($R<400$) and heavier ($R>800$) woods, the value likely has inclination to increase slightly.

2. The relation of the direction of fiber and the thermal diffusivity

According to GRIFFITHS & KAYE[10] the value of $\lambda_{\perp}$ in the radial direction is larger than that in tangential direction by comparatively small difference, and for a practical purpose there can be written $\lambda_{\perp}=(\lambda_x+\lambda_y)/2$ or $\lambda_x=\lambda_y=\lambda_{\perp}$. The basic equation (2) by cylindrical coordinates is derived from on this condition.

The relation between that value parallel and rectangular to the fiber direction is given by formula (3), the former being 2.0—2.5 times as large as the latter. Table 1 compiles the values of the thermal conductivity of wood reported hitherto, which clearly indicates that the ratio $\lambda_{//}/\lambda_{\perp}$ can be regarded as 2.0—2.5 as in formula (3). R and c give no influence on direction so that the thermal diffusivity is determined solely by the thermal conductivity, and the value of $\lambda_{//}$ is regarded as 2.0—2.5 times as large as that of $\lambda_{\perp}$.

Table 1 Thermal conductivity of wood

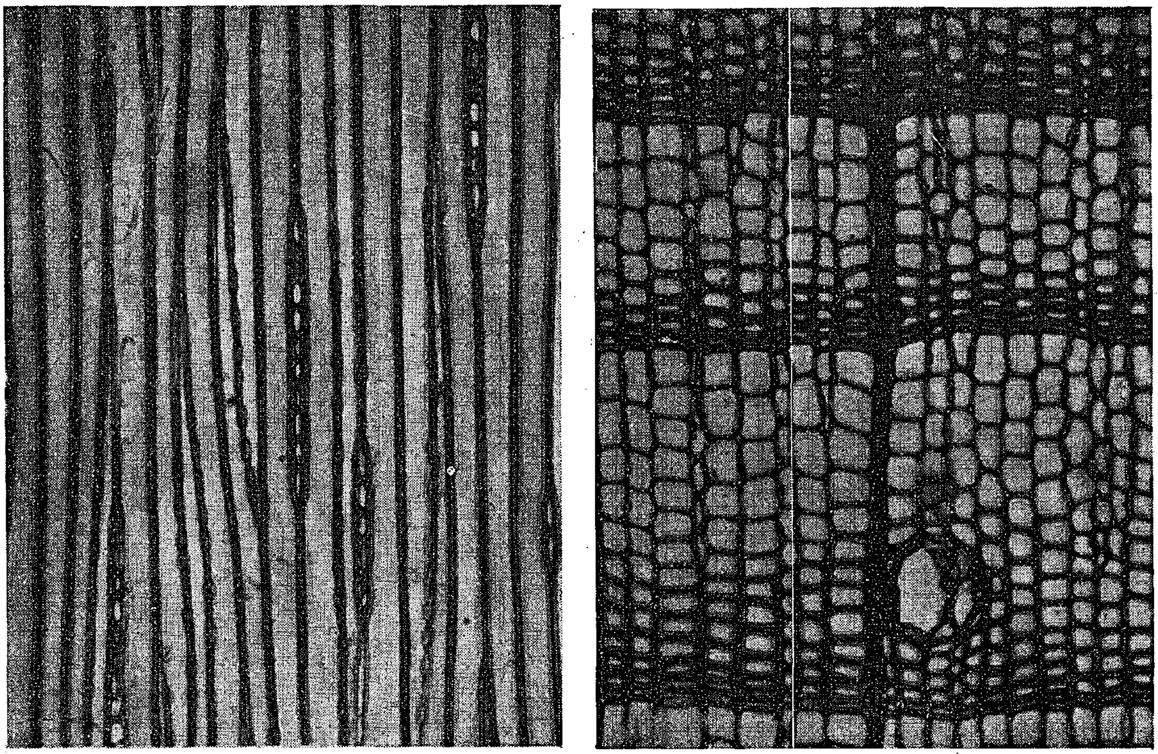
Species	Spec. Gravity r_u	Moisture content u %	Temperature °C	$\lambda_{//}$ Kcal/mh°	$\lambda_{\perp}$ Kcal/mh° λ_x	λ_y	$\lambda_{//}/\lambda_{\perp}$	Reporter
Fraxinus (Ash)	0.74	15	20	0.263	0.151	0.140	1.814	Griffiths and Kaye
Picea (Spruce)	0.41	16	〃	0.191	0.104	0.090	1.969	
Swietenia (Mahogany)	0.70	15	〃	0.266	0.144	0.133	1.927	
Juglans (Walnut)	0.65	12	〃	0.284	0.126	0.119	2.328	
Quercus (KASHIWA)	0.9～0.8	0～10	20～30	0.320	0.15～0.18		2.13～1.78	SHIBA (Physical Tables)
Aceracea (KAEDE)	0.72	〃	〃	0.360	0.150		2.40	
Fraxinus (TONERIKO)	0.74	15	〃	0.260	0.11		1.73	
Fagus (BUNA)	0.70	30	〃	0.300				
Swietenia (MAHOGANI)	0.70	13	〃	0.270	0.130		2.08	
Abies (MOMI)	0.41～0.42	10	〃	0.170	0.110		1.55	
〃	〃	20	〃	0.220	0.120		1.83	
〃	〃	30	〃	0.260	0.135		1.92	
Pinus (Kiefer)	0.49(r_0)	15	〃	0.300	0.120		2.5	Kollmann (Technologie des Holzes)
Abies (Tanne)	0.41〃	12	〃	0.220	0.092～0.112		2.39～1.96	
Acer (Ahorn)	0.59〃	15	〃	0.370	0.137～0.156		2.7～2.37	
Buxus (Buchsbaum)	0.92〃	10～15	〃	0.335	0.129～0.147		2.6～2.28	
Quercus (Stieleiche)	0.65〃	〃	〃	0.21～0.30	0.110～0.170		1.82	
Juglans (Walnup)	0.64〃	10	〃	0.238	0.091		2.62	
Tectona (Teak)	0.63〃	10～15	〃	0.330	0.140～0.170		2.36～1.94	
Fraxinus (Ash)	0.74〃	—	20	0.266	0.148		1.796	Smithsonian Physical Tables
Abies (Fir)	0.54〃	—	〃	0.292	0.119		2.45	
Swietenia (Mahogany)	0.70〃	—	〃	0.266	0.137		1.94	
Quercus (Oak)	0.82〃	—	15	0.310	0.180		1.72	
Pinus (White pine)	0.45〃	—	60	0.223	0.084		2.65	
Tectona (Teak)	0.64	—	15	0.328	0.151		2.17	

r_0, r_u : Spec. gravity at moisture content 0, u.

$\lambda_{//}$, $\lambda_{\perp}$: Thermal conductivity in fiber- and transverse direction.

3. Consideration on the relation of the specific gravity and the thermal conductivity

For the purpose of simplification, the constitutional factors of wood which infuluence on the thermal conductivity are classified into two categories ; anisotropic substance of cell membrane and isotropic substance in cell cavity. The histological classification is so complicated according to the species, individuality, spring wood, summer wood, and so on, conifers is comparatively less complicated of the constituents and is considered generally to be composed of following complex system (ref. Fig. 3).



Tangential section　　　　Cross section

Fig. 3 *Picea jesoensis* (EZOMATU)

a. The case parellel to fiber

—Cell membrane parallel to fiber—cell membrane rectangular to fiber
—Cell membrane slanting to fiber (adjacent wall of upper and lower cell)—cell cavity

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— 〃

b. The case rectangular to fiber

(i) Tangential direction

—Cell membarne rectangular to fiber (includes wood ray)
—Cell membrane rectangular to fiber—cell cavity

— 〃
— 〃

(ii) Radial direction

—Cell membrane rectangular to fiber—cell membrane parallel to fiber
—Cell membrane rectangular to fiber—cell cavity

— 〃
— 〃

Of the above systems the rate of wood ray in wood differs with species, individuality, the position of tree stem, and so on; conifers, *Chamaecyparis* (Eastern White Ceder) contains 3.9% and *Larix* (Western Larch) 10% (usually 5—8%); broad-leaved trees, *Tilia* (Basswood) contains 6.1% and *Morus* (Mulberry) 44.7% (usually 10-20%)[9,14,17,59]. These data indicate that if wood ray influences on the thermal conductivity there should be considerable differences between the conductivities along tangential and radial directions among species of wood. It is imaginable that the wood of large rate of wood ray would give larger λ_x than λ_y,, and that the wood of large rate of summer wood would show tendency to reverse the above relation. For the difference between λ_x and λ_y obtained experimentally, GRIFFITHS reported that λ_x is larger while ROWLEY reported that in the wood showing large difference between spring- and summer-wood λ_y is larger but in the others there is no difference between λ_x and λ_y, and WANGAAD[56] described that the broad-leaved trees give larger λ_x and that conifers give not so differnt λ_x and λ_y. Consequently, the difference is so small that it can be neglected in practice. This controversy of theory and experimental results may be explained by smaller volume-rate of wood ray and by its comparatively isolated presence in wood, but the details of the reason of this controversy has not yet clarified up to the present time. If it is allowed to neglect the influence of wood ray for practical purpose from the above descriptions, and to neglect, for the system a, the influence of "cell membrane slanting to fiber" by the reason that the ratio of the diameter of tracheid to the length is very small[17], the systems above mentioned are simplified as follows.

a. The case parallel to fiber

- —Cell membrane parallel to fiber
- —Cell cavity

- — 〃
- — 〃

b. The case transverse to fiber

- - Cell membrane rectangular to fiber
- —Cell membrane rectangular to fiber—cell cavity

- — 〃
- — 〃

a. The case parallel to fiber

This system is shown schematicalley in Fig. 4. When the fractional void volume and cell membrane volume is p and $1-p$, the mean thermal conductivity $\lambda_{//}$ in this system is

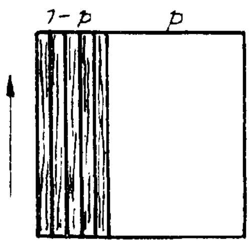

Fig. 4

$$\lambda_{//}=\lambda_{//}'+(\lambda_a-\lambda_{//}')p \quad \cdots\cdots (5)$$

where λ_a is the thermal conductivity of cell cavity and $\lambda'_{//}$ that of cell membrane in the direction of fiber.

As for the specific gravity of cell membrane i. e. the true specific gravity of wood, the wood containing large amount of cellulose give higher value than the one which contains little amount of cellulose since cellulose is heavier than lignin. And different methods of estimation also give different values and hitherto 1.46–1.62 have been reported[7),12),13),39),48),49)]. At the present time for all species of wood the average value 1.56 is generally accepted, and taking this value the relation between the specific gravity γ_0 of oven dried wood

and the fractional void volume is written as $p=1-\gamma_0/1.56=1-0.64\gamma_0$.
The above formula, then, is

$$\lambda_{//}=\lambda_a+(\lambda_{//}'-\lambda_a)\cdot 0.641\ \gamma_0 \quad\cdots\cdots(6)$$

thus $\lambda_{//}=\lambda_a$, $\lambda_{//}'$ for $\gamma_0=0$ and 1.56 respectively.

The substance in cell cavity includes living-material, resin, tylose, and others, but in dried wood it can be regarded as air[16]. From the above formula it is obvious that $\lambda_{//}$ increases linearly with the increase of specific gravity between two limits of the thermal conductivities of air and the cell membrane in the fiber direction. Previous reports have never given the value measured for $\lambda'_{//}$ in the formula. Taking the value of the thermal conductivity of closed air $\lambda_a=0.022$, and converting the value of $\lambda_{//}$ given by GRIFFITHS and KOLLMANN to the conditions of 20°C and 0 % moisture content (ref. Table 2), the following formula is obtained.

$$\lambda_{//}=0.022+0.346\ \gamma_0 \quad\cdots\cdots(6)'$$

Table 2 Thermal conductivity of wood in fiber direction

Species	Spec. gravity γ_0	Therm. conductivity $\lambda_{//}$ *	Reporter
Fraxinus (Eshe)	0.70	0.216	Griffiths & Kaye
Picea (Fichte)	0.37	0.154	
Swietenia (Mahagoni)	0.66	0.218	
Juglans (Walnuss)	0.62	0.243	
Pinus (Kiefer)	0.49	0.246	Kollmann (Technologie des Holzes)
Abies (Tanne)	0.41	0.188	
Acer (Ahorn)	0.59	0.303	
Buxus (Buchsbaum)	0.92	0.287	
Quercus (Stieleiche)	0.62	0.218	
Juglans (Walnuss)	0.64	0.195	
Tectonia (Teak)	0.63	0.283	

* converted values to moisture content 0 from Table 1

From this, the value of $\lambda_{//}'$ is obtained as 0.562.

In this formula, $\gamma_0=0$ can not exist in reality and it is easily expected that as the specific gravity approaches to zero the influence of convection of air becomes larger so that the thermal conductivity surpasses the value of the closed air $\lambda_a=0.022$. This phenomenon has been confirmed experimentally with cork by WATZINGER, and WAKASUGI has obtained $\lambda_{\perp}=0.044$ for *Ochroma* (Balsa) of specific gravity 0.06 and 0.046 for that of specific gravity 0.08 and KOLLMANN also has employed the value 0.04. From these it seems reasonable to take the value $\lambda_a=0.04$ for general use. Of common woods the one having the least specific gravity is *Paulownia* (KIRI) showing average 0.3 and *Chamaecyparis* (Northern White Cedar) and *Populus* (Balsam) showing 0.35. It may be concluded that as shown in Figure 8 the value of $\lambda_{//}$ is 0.04 at $\gamma_0=0$ and move to meet the formula (6)' at the specific gravity about 0.3. For the practical range it can be obtained by formula (6)'.

b. The case transverse to fiber

When in this complex system the fractional volume of cell membrane in series part is q, then as obvious from Figure 5 the mean thermal conductivity $\lambda_{\perp}$ of the system is

$$\lambda_{\perp}=\lambda_{\perp}'+\left(\frac{p+q}{\frac{p}{\lambda_a}+\frac{q}{\lambda_{\perp}}}-\lambda_{\perp}'\right)(p+q) \cdots\cdots\cdots\cdots (7)$$

where $\lambda_{\perp}'$ is the thermal conductivity of the cell membrane rectangular to fiber.

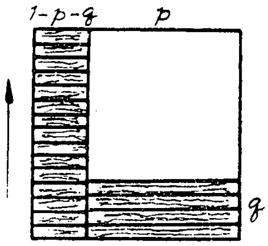

Fig. 5

The rate of the part represented by q in cell membrane is far from being obtained exactly because of so much complicated wood constituent. Now let us limit the object of discussion to conifers which is constituted by over 90% by tracheid and its cell-work is comparatively regularly arranged, as the cross section given in Figure 3. As well known the cross section of cell shows various types and among them the cells of approximately rectangular section occupy the greatest proportion. The cells in summer wood show thicker wall than in spring wood and in the cross section the latter give longer radial side while the former longer tangential side so that in a given annual ring q varies so greatly. However, in average the cross section of cells can be assumed to be square and q be constant in a given species of wood, and then the ratio of q to total cell membrane is determined by the ratio of the cell membrane thickness to the diameter of the cell (see Figure 6).

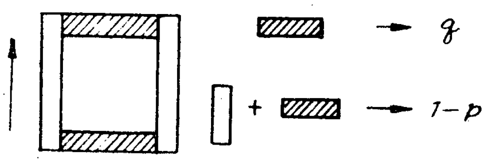

Fig. 6

To extend this relation from conifers to broad-leaved trees which shows more complicated structure is somewhat doubtful but from the facts that the broad-leaved trees contain generally over 50% wood fiber so that the fractional volume of cell membrane of

Table 3 $\frac{q}{1-p}$ of conifers (Tracheid)

Species		Spec. Gravity r_0	Diameter of Tracheid: Spring wood, radial (μ)	Diameter of Tracheid: Spring wood, tangential (μ)	Diameter of Tracheid: Summer wood, radial (μ)	Diameter of Tracheid: Summer wood, tangential (μ)	Thickness of cell wall (μ)	Thickness / Diameter	$\frac{q}{1-p}$
Ginkgo	ICHO	0.60	25—70	50—65			3—5	0.08	0.456
Taxus	ICHII	0.57	30—40	30—38	15—30	30—35	3—5	0.13	0.425
Torreya	KAYA	0.52	30—50	30—48	16—30	30—40	3—5	0.12	0.431
Picea	TOHI	0.44	40—56	38—52	12—40	30—45	2—5	0.09	0.450
〃	EZOMATU	0.45	40—55	40—50	10—40	30—40	2—5	0.09	0.450
Tsuga	TSUGA	0.66	40—55	40—50	15—45	35—42	2—6	0.10	0.445
Abies	MOMI	0.44	48—55	45—55	16—40	30—45	2—6	0.10	0.445
〃	SHIRABE	0.33	35—55	25—45	10—25	25—40	2—5	0.11	0.438
Pseudotsuga	TOGASAWARA	0.46	40—60	50—65	18—40	48—60	2—7	0.09	0.450
Larix	KARAMATU	0.57	40—65	40—60	20—40	40—50	2—8	0.11	0.438
Pinus	HIMEKOMATU	0.54	40—50	40—50	10—32	28—40	2—5	0.10	0.445
〃	CHOSENMATU	0.46	40—55	30—50	15—35	20—40	1.5—5	0.09	0.450
〃	AKAMATU	0.48	50—60	48—56	15—43	40—50	2.5—8	0.11	0.438
〃	KUROMATU	0.62	40—60	40—55	15—40	35—50	2—8	0.12	0.431
Cryptomeria	SUGI	0.40	30—50	30—48	15—30	25—40	2—7	0.13	0.425
Sciadopitys	KOYAMAKI	0.41	30—50	30—50	15—35	30—40	3—6	0.13	0.425
Chamaecyparis	HINOKI	0.50	35—50	30—50	10—25	30—35	2—4	0.09	0.450
〃	SAWARA	0.35	30—45	30—40	10—25	30—35	1—4	0.08	0.456
Thuja	NEZUKO	0.51	28—40	30—45	15—30	25—32	1—5	0.10	0.445
Thujopsis	HIBA	0.50	25—40	30—40	12—15	30—40	2—4	0.10	0.445

wood fiber surpasses half and that the both groups of wood of the same specific gravity give no significant difference in $\lambda_{\perp}$, this extention may be permissible.

On the basis of this way of thinking, Table 3 and 4 show the calculated ratio of (mean cell membrane thickness)/(mean diameter) and of $q/(1-p)$ using the previous data[17] for tracheid (conifers) and wood fiber (broad-leaved trees), and the relation with the specific gravity[50].

Table 4 $\frac{q}{1-p}$ of broad-leaved trees (Wood fiber)

Species		Spec. gravity r_0	Diameter of Wood fiber	Thickness of cell wall	$\frac{\text{Thickness}}{\text{Diameter}}$	$\frac{q}{1-p}$
Populus	DOROYANAGI	0.33	22—28	2—3	0.10	0.445
Juglans	ONIGURUMI	0.48	22—30	3—4	0.13	0.425
Pterocarya	SAWAGURUMI	0.36	20—30	1.5—2.5	0.08	0.456
Betula	SHIRAKABA	0.57	20—25	3—4	0.16	0.405
〃	MIZUME	0.74	24—28	3—4	0.13	0.425
Carpinus	INUSHIDE	0.71	20—25	2—3	0.11	0.438
Castanea	KURI	0.57	16—18	2—3	0.15	0.411
Fagus	BUNA	0.68	12—16	3—4	0.25	0.333
Quercus	MIZUNARA	0.76	14—16	4—5	0.30	0.286
〃	AKAGASHI	0.88	14—16	3—4	0.23	0.350
Zelkowa	KEYAKI	0.73	12—16	3—5	0.29	0.296
Cercidiphyllum	KATURA	0.47	18—22	3—4	0.18	0.390
Magnolia	HONOKI	0.46	24—28	2—4	0.12	0.431
Cinnamomum	KUSU	0.38	20—25	3—4	0.16	0.405
Prunus	KANZAKURA	0.63	20—24	3	0.14	0.419
Buxus	TUGE	0.89	12—16	4—5	0.32	0.265
Acer	KAEDE	0.64	18—23	2—3	0.10	0.445
Aesculus	TOCHI	0.52	25—30	2	0.07	0.462
Fraxinus	SIOJI	0.65	14—20	2—4	0.18	0.390
Paulownia	KIRI	0.27	25—30	2	0.07	0.462
Eriobotrya	BIWA	0.97	12—15	3.5—4	0.28	0.310
Betula	ONOOREKANBA	0.96	14—16	4.5	0.30	0.290
〃	URAJIRO	0.96	18—20	2—3	0.13	0.425
Myrica	YAMAMOMO	0.92	15—20	3—5	0.23	0.350
Osmanthus	HIIRAGI	0.89	12—14	2—3	0.19	0.380

Figure 7 shows this results diagrammatically and indicates clearly that there exists a close relation between $q/(1-p)$ and the specific gravity. And in this relation $q/(1-p)$ should be 0.5 and 0 for γ_0 is 0 and 1.56, respectively, and therefore

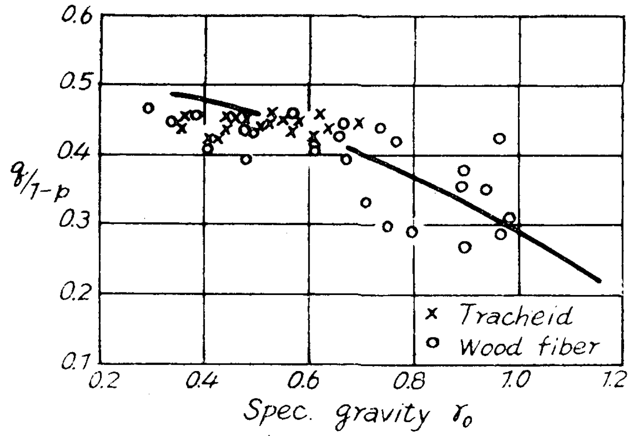

Fig. 7 Relation between $q/1-p$ and the spec. gravity

$$\frac{q}{1-p}=0.5-0.0321\gamma_0-0.185\gamma_0^2$$

thus

$$\left.\begin{aligned} q&=0.32\gamma_0-0.0206\gamma_0^2-0.118\gamma_0^3 \\ p+q&=1-0.321\gamma_0-0.0206\gamma_0^2-0.118\gamma_0^3 \end{aligned}\right\} \cdots\cdots(8)$$

Introducing the above equations into equation (7), man can obtain the relation between $\lambda_\perp$ and γ_0. There have been previously no reports that give the estimated value for $\lambda_\perp'$ as in the case a, and so the values of $\lambda_\perp$ were experimentally obtained for determining $\lambda_\perp'$. With the quarter sawn, dry pieces of ***Chamaecyparis*** (HINOKI), ***Betula*** (KABA), and ***Fagus*** (BUNA) compressed to various degrees by the pressure 35–1000 kg/cm² to obtain varied specific gravity in a given species of wood, the relation between specific gravity and thermal conductivity was obtained. The results are given in Table 5. Using the mean specific gravity and thermal conductivity at various stages of compression and the value $\lambda_a=0.022$, $\lambda_\perp'$ can be calculated from formula (7) and (8), and the mean value of $\lambda_\perp'$ is obtained to be 0.362 Kcal/m h°.

On this value the relation of γ_0 and $\lambda_\perp$ from formula (7) and (8) is reprensented by the thick line in Figure 8. Approximately it is shown

$$\lambda_\perp=0.02+0.0724\gamma_0+0.0931\gamma_0^2 \cdots\cdots(7)'$$

In the range where γ_0 is extremely small, the thermal conductivity of vacant space in

Table 5 Thermal conductivity in various spec. gravity of wood

Species	Applying pressure kg/cm²	Spec. gravity of dry wood	Therm. conductivity Kcal/mh°
Chamaecyparis (HINOKI)	35	0.628	0.103
	〃	0.586	0.092
	〃	0.555	0.099
	50	0.628	0.124
	〃	0.622	0.112
	〃	0.632	0.124
	100	0.614	0.122
	〃	0.649	0.127
	〃	0.627	0.114
	500	0.806	0.179
	〃	0.850	0.169
	〃	0.812	0.153
	1000	0.998	0.213
	〃	0.994	0.206
	〃	0.958	0.176
Betula (KABA)	50	0.695	0.112
	〃	0.674	0.099
	〃	0.683	0.098
	100	0.678	0.121
	〃	0.673	0.122
	〃	0.660	0.118
	500	0.837	0.162
	〃	0.843	0.168
	〃	0.827	0.138
	1000	1.001	0.182
	〃	1.100	0.195
	〃	1.162	0.191
Fagus (BUNA)	50	0.645	0.094
	〃	0.625	0.091
	〃	0.669	0.115
	100	0.678	0.109
	〃	0.658	0.121
	〃	0.676	0.115
	500	0.836	0.131
	〃	0.866	0.161
	〃	0.864	0.125
	1000	1.037	0.172
	〃	1.015	0.155
	〃	1.008	0.181

wood is influenced by radiation and convection, and so the curve swerves from the vicinity of $\gamma_0=0.3$ to reach $\lambda_L=0.04$ at $\gamma_0=0$. However, in the practical use this can be neglected as in the case of a. In the figure the curve above mentioned and the experimental formula reported previously have been given in reference. From these it is indicated that the values obtained by formula (6)', (7), and (8) are in good agreement with the experimental results hitherto obtained.

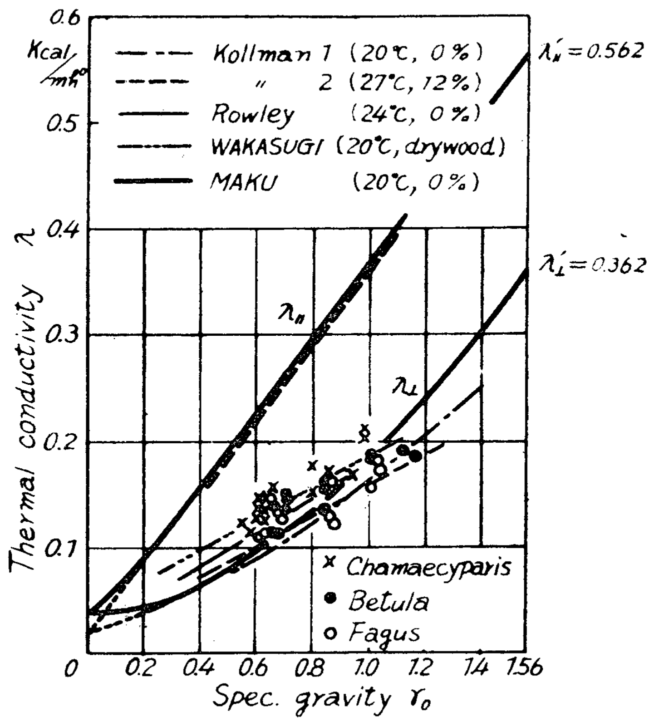

Fig. 8 Relation between the spec. gravity and the thermal conductivity

4. The relation of the temperature and the thermal diffusivity

For many materials it has been found from long ago by NUSSELT [41] that λ is proportional to absolute temperature and KOLLMANN pointed out that this relation was applicable for wood, thus

$$\lambda_1 = \lambda_2 \frac{T_1}{T_2} = \lambda_2 \frac{273+t_1}{273+t_2} \quad \cdots\cdots (9)$$

where λ_1 and λ_2 are the thermal conductivity at temperature t_1 and t_2, respectively. WAKASUGI gave the following formula for the direction perpendicular to fiber

$$\lambda_t = \lambda_0 + 0.000075t \quad \cdots\cdots (10)$$

where λ_0 and λ_t are the thermal conductivity at temperature 0 and t°C. For the relation between specific heat and temperature DUNLAP [6] has given for oven dried wood of any species

$$c = 0.266 + 0.00116t \quad \cdots\cdots (11)$$

Since the influence of density on temperature can be neglected practically the change of α by temperature is determined from formula (9), (10), and (11), and as clear from these formulas λ and c increase with the rise of temperature so that α is slightly influenced by temperature but it can be regarded as a constant and the mean value is used in practice.

5. The relation of the moisture content and the thermal diffusivity

The thermal conductivity of water at 20°C is about 0.507 Kcal/m h° and so with the increase of moisture content thermal conductivity of wood increases, but after the free water begins to appear it decreases in order. When a large amount of water is contained in wood, the relation between moisture content and the thermal conductivity is often considerably complicated since the distribution of free water which is controlled by capillary force can not be free from heterogeneity in the range from the fiber saturation point to water saturation point and the moisture gradient is often influenced by the change of the temperature gradient. When these influences are comparatively small there is a considerably definite relation between the both, and according to GRIFFITHS & KAYE[10] and ROWLEY[47], particularly to the latter, there can be obtained the following experimental formula for any species of wood in the range of moisture content $u=5\sim30\%$

$$\lambda_2=\lambda_1\ \{1-0.012(u_1-u_2)\} \quad \cdots\cdots(12)$$

where λ_1 and λ_2 are the thermal conductivities at the moisture content u_1 and u_2 (%). MACLEAN[53] has presented for the range below fiber saturation point.

$$\lambda=G(1.39+0.028u)+0.165 \quad \cdots\cdots(13)$$

where λ is B. T. U./h ft^2 (deg.F/inch) and G specific gravity (dry weight/wet volume). And for the range above it

$$\lambda=G(1.39+0.038u)+0.165 \quad \cdots\cdots(14)$$

These indicate under a given temperature and a specific gravity there are tendencies similar to each other in both cases.

No report has been published on the relation between moisture content and the thermal conductivity in fiber direction but according to MACLEAN[53] it is at least 2.5 times as large as that in the direction perpendicular to fiber. It seems reasonable that the relation given in article 1–3 is satisfied for any moisture content.

In the next, the following formula is obtained for the relation between moisture content and specific heat, when the latter are c_0 and c_x at oven dried and x % moisture content in wood and c_w in water,

$$c_x = x \cdot c_w + (1-x)c_0$$

Taking relation $x = u/(1+u)$, where x, u are the moisture content based on wet and dry weight and taking $c_w = 1$,

$$c_u = \frac{u+c_0}{u+1} \quad \cdots\cdots(15)$$

As for the relation between moisture content and density KOLLMANN has given the following formula in the range of $u = 0 \sim 25\%$ on the data by MÖRATH[37] and by U. S. Forest Products Laboratory[21]

$$\gamma_u = \gamma_0 \frac{1+u}{1+0.84\, u\gamma_0} \quad \cdots\cdots(16)$$

and under the consideration that maximum swelling by moisture absorption occurs at a little higher moisture content than the fiber saturation point, he has presented a graph shown in Fig. 9, which has been confirmed to be applicable in good agreement for many species of wood. Therefore the change of a by the change of moisture content is determined by the use of formula (12)—(16) and Fig. 9.

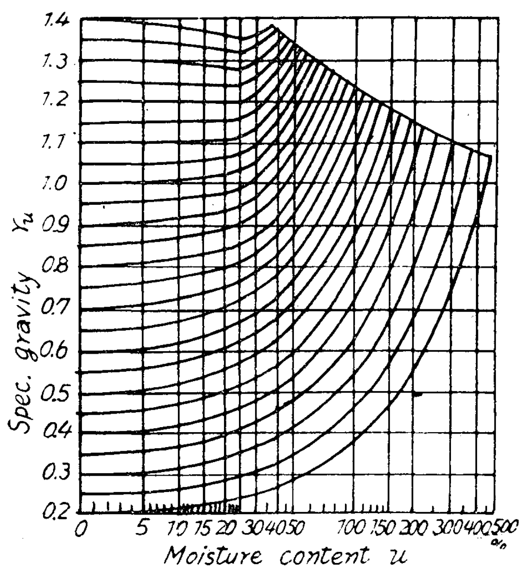

Fig. 9 Relation between the moisture content and the spec. gravity (KOLLMANN)

Now in article 1-5 the results of hitherto studies on the thermal diffusivity of wood have been summarized and explained. Some examples of calculation are given in Fig. 10—12. The broken lines in these figures are based on an assumption that above mois-

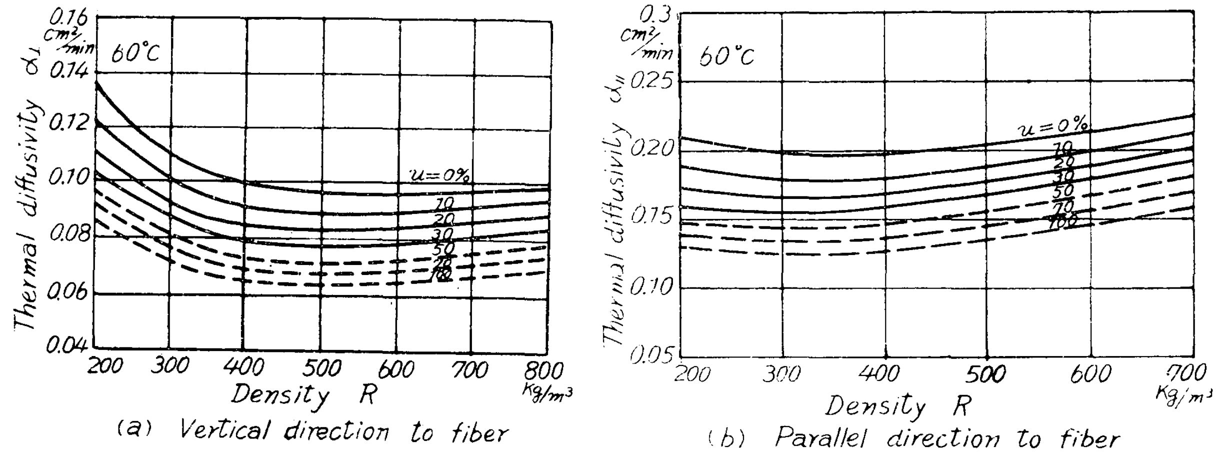

Fig. 10 Relation between the density and the thermal deffusivity

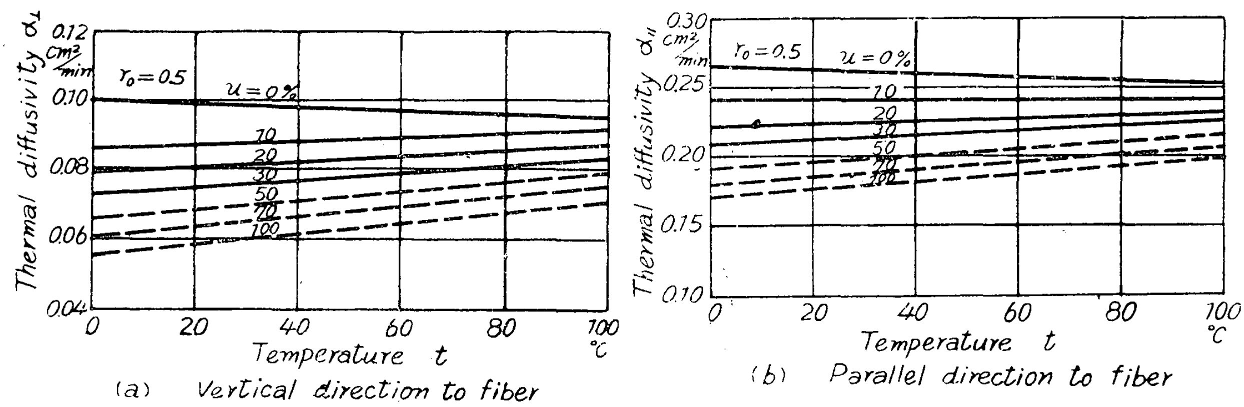

Fig. 11 Relation between the temperature and the thermal diffusivity

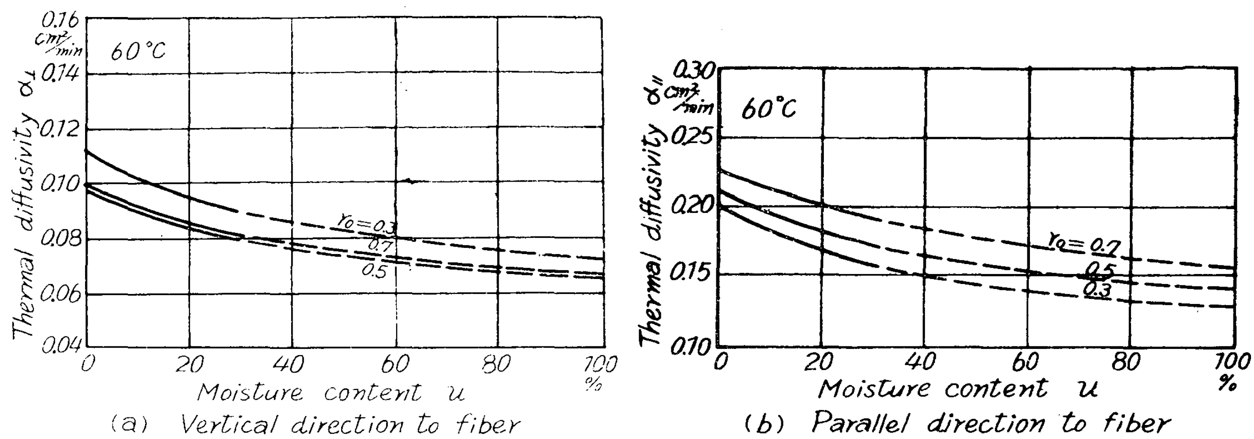

Fig. 12 Relation between the moisture content and the thermal diffusivity

ture content 35% formula (12) is applicable. From these figures it is seen that, in the relation of density and thermal diffusivity, $\alpha_\perp$ is practically constant in the range of

$R=350$—800 and that for $a_{//}$ the changes in $R>500$ can be neglected since the degree of the changes may be within the range of individual error of wood, in the relation of temperature and thermal diffusivity both $a_{\perp}$ and $a_{//}$ show different tendencies with the change of moisture content, decreasing in lower moisture range and increasing in higher range, as in the case of some kinds of insulating material. However, for general usage, the mean value is quite free from the errors. Finally in the relation of moisture content and thermal diffusivity $a_{\perp}$ and $a_{//}$ decrease with the increase of moisture content particularly remarkably up to the vicinity of the fiber saturation point.

In summarizing these the thermal diffusivity of wood keeps to be practically constant in the change of temperature and over-all species of wood and so the theoretical solution is easily obtained, but in the case where moisture content varies during heating the value of a varies with these change, for this case appropriate consideration is needed. However, as shown later, some degrees of change in a produce not so large error due to temperature and for practical use the changes by 10—30% at high moisture content and by a few % below the fiber saturation point can be neglected and the mean value of a is used without committing great error.

III. Experiments on the factors influencing on the thermal diffusivity

In chapter II the value of a obtained indirectly from the results reported hitherto on λ, c and R has been discussed. The value of a in this meaning will be denoted as calculated value hereafter. Very few reports have been published on the availability of this value into the pratical use, and attempts have been made by the present auther on this problem on wood-plate using the theoretical solution of the simplest case and by measuring the interior distribution of temperature the experimental value a has been given and the influences of various factors have been investigated. The value of a thus obtained will be denoted as experimental value hereafter.

1. The heat conduction in wood plate under the ordinary boundary condition

When a very wide plate is heated from both surfaces by hot water or air of a constant temperature, the heat flows solely in the direction perpendicular to the surface and through fluid film at the boundary of plate. Taking the thickness of plate $2a$, the initial temperature distribution $f(x)$, and the temperature of heating medium t_1 °C, there exist the following equations (cf. Fig. 13).

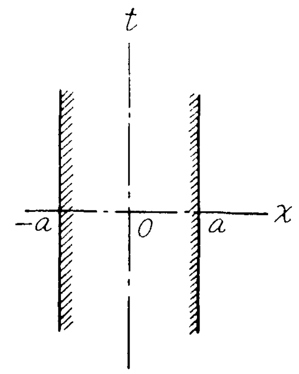

Fig. 13

$$\begin{cases} \dfrac{\partial t}{\partial \theta} = a_{\perp} \dfrac{\partial^2 t}{\partial x^2} & \cdots\cdots(17)' \\ \theta = 0 : t = f(x) & \cdots\cdots(18) \\ x = \mp a : \dfrac{\partial t}{\partial x} \mp h(t - t_1) = 0 & \cdots\cdots(19) \\ x = 0 : \dfrac{\partial t}{\partial x} = 0 & \cdots\cdots(20) \end{cases}$$

where $h = h'/\lambda_{\perp}$, (h' is film coefficient of heat transfer)

Let $T = t - t_1$

$$\begin{cases} \dfrac{\partial T}{\partial \theta} = a_{\perp} \dfrac{\partial^2 T}{\partial x^2} & \cdots\cdots(17)' \\ \theta = 0 : T = f(x) - t_1 = F(x) & \cdots\cdots(18)' \\ x = \mp a : \dfrac{\partial T}{\partial x} \mp h\ T = 0 & \cdots\cdots(19)' \\ x = 0 : \dfrac{\partial T}{\partial x} = 0 & \cdots\cdots(20)' \end{cases}$$

the particular solution is

$$T = \frac{1}{a} \sum_{n=1}^{\infty} \frac{u_n}{u_n + \sin u_n \cos u_n} e^{-\alpha_{\perp} \left(\frac{u_n}{a}\right)^2 \theta} \cos \frac{u_n}{a} x \int_{-a}^{a} F(\lambda) \cos \frac{u_n}{a} \lambda \, d\lambda$$

where u_n is the nth real root of equation $\cot u = u/ha$

thus the solution is

$$t - t_1 = \frac{1}{a} \sum_{n=1}^{\infty} e^{-\alpha_{\perp} \left(\frac{u_n}{a}\right)^2 \theta} \cos \frac{u_n}{a} x \frac{u_n}{u_n + \sin u_n \cos u_n} \int_{-a}^{a} \left\{ f(\lambda) - t_1 \right\} \cos \frac{u_n}{a} \lambda d\lambda \quad (21)$$

When the initial temperature distribution of the plate is uniform, that is t_0, let $f(\lambda)=t_0$
then

$$\frac{t_1-t}{t_1-t_0}=2\sum_{n=1}^{\infty}e^{-\alpha_\perp\left(\frac{u_n}{a}\right)^2\theta}\cos\frac{u_n}{a}x\frac{\sin u_n}{u_n+\sin u_n\cos u_n} \quad\cdots\cdots\cdots(22)$$

For temperature t_m at the mid-plane of the plate, let $x=0$

$$\frac{t_1-t_m}{t_1-t_0}=2\sum_{n=1}^{\infty}e^{-\alpha_\perp\left(\frac{u_n}{a}\right)^2\theta}\frac{\sin u_n}{u_n+\sin u_n\cos u_n} \quad\cdots\cdots\cdots(23)$$

Formula (21)—(23) are well known regarding to the heat conduction of homogeneous body, and the values of u_n at various ha are given in Table 6. Therefore with known h' and a the interior temperature t of the plate is obtained from these solutions. However, the exact calculation is very complicated so that GURNEY and LURIE,[11] considering solution (22) as a dimensionless equation regarding to $\frac{t_1-t}{t_1-t_0}$, $\frac{x}{a}$, and $\alpha\frac{\theta}{a^2}$, have given Fig. 14 for a simplified calculation.

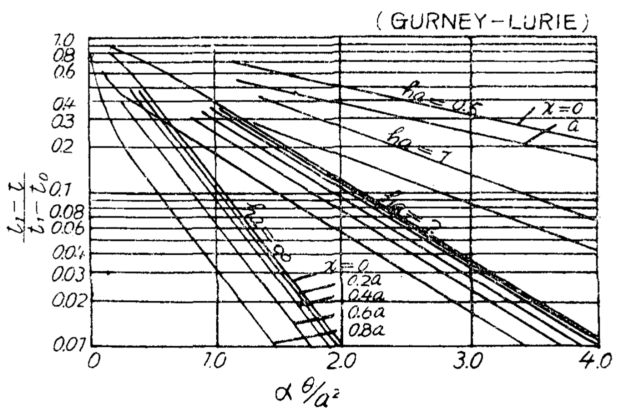

Fig. 14 Diagram of eq. (22)

The film coefficient of heat transfer h' is varied depending on temperature, velocity, the kind of heating medium, the surface conditions of the heated body and other factors, and the value of h' becomes very large when condensing vapor is used as heating medium, being reported 7000--12000 Kcal/m²h°. There are hardly published on the estimated values of h' on woods. According to the present auther under the conditions of hot air of temperarture 80°C, relative humidity 40%, and velocity nearly 0, h' was nearly 15 Kcal/m² h° irrespective of the surface conditions and wood species.

In this case the time θ_{eq} required to reach equilibrium between the temperature of plate and the heating medium is obtained when the factor $e^{-\alpha\left(\frac{u_n}{a}\right)^2\theta}$ which is influenced by time is extremely small, and for practical use it is sufficient that $e^{-\alpha\left(\frac{u_n}{a}\right)^2\theta}=0.01$

then

$$\theta_{eq}=\frac{4.6a^2}{\alpha_L u_n^2} \quad \cdots\cdots (24)$$

and this can be obtained with the use of the minimum value u_1.

Table 6 Roots of eq. cot $u=u/ha$

ha	u_1	u_2	u_3	u_4
0	0	π	2π	3π
0.001	0.032	3.142	6.283	9.425
0.002	0.044	3.142	6.284	9.425
0.005	0.071	3.143	6.284	9.425
0.01	0.100	3.145	6.285	9.426
0.02	0.141	3.148	6.286	9.427
0.05	0.222	3.157	6.291	9.430
0.1	0.311	3.173	6.299	9.435
0.2	0.433	3.204	6.315	9.446
0.5	0.653	3.292	6.362	9.477
1	0.861	3.426	6.437	9.529
2	1.079	3.644	6.578	9.630
5	1.300	3.936	6.814	9.811
10	1.428	4.305	7.229	10.200
20	1.498	4.491	7.495	10.513
50	1.536	4.619	7.703	10.783
∞	$1/2\cdot\pi$	$3/2\cdot\pi$	$5/2\cdot\pi$	$7/2\cdot\pi$

Now, when condensing steam is used as heating medium, h' is extremely large so that putting $h'\rightarrow\infty$ i. e. $h\rightarrow\infty$,

$$u_n=\frac{(2n-1)\pi}{2}$$

$$\sin u_n=(-1)^{n-1}$$

$$\cos u_n=0$$

thus, equation (21)—(23) become

$$t-t_1=\frac{1}{a}\sum_{n=1}^{\infty} e^{-\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\int_{-a}^{a}\left\{f(\lambda)-t_1\right\}\cos\frac{\overline{2n-1}\pi}{2a}\lambda d\lambda \quad (25)$$

$$\frac{t_1-t}{t_1-t_0}=\frac{4}{\pi}\sum_{n=1}^{\infty} e^{-\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\frac{(-1)^{n-1}}{2n-1} \quad \cdots\cdots(26)$$

$$\frac{t_1-t_m}{t_1-t_0}=\frac{4}{\pi}\sum_{n=1}^{\infty} e^{-\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\theta}\frac{(-1)^{n-1}}{2n-1} \quad \cdots\cdots(27)$$

using $u_1=\frac{\pi}{2}$, solution (24) becomes

$$\theta_{eq}=\frac{18.4a^2}{\alpha_\perp\pi^2} \quad \cdots\cdots(28)$$

Under this condition, the boundary condition (19) becomes

$$x=\mp a \quad : \quad t=t_1 \quad \cdots\cdots(19)''$$

This indicates that when heated with steaming, the wood surface reaches approximately the temperature of steam at once and the boundary condition come to an accordance with that of the case where the plate is hotpressed from both surfaces. It is concluded henceforth that the raise of the interior temperature of plate is most rapid when heated with steaming or by hotpress.

2. Experiments on the relation between density and the thermal diffusivity

a. Experiments by hotpressing

When a plate is heated from both surfaces by hotpress, for satisfying condition (19)″, the problem comes to the perfectness of the contact between hot plate and the wood plate, or technically the control of pressure of hotpress. The present author made experiments with oven dried *Fagus* (BUNA) of measuring the temperature of mid-plane of the plate under hotpressing at the pressure below 0.05, 10, 20, and 30 kg/cm^2 and the results are shown in Fig. 15. The curve therein is the one calculated theoretically putting $\alpha=0.1$. In this case, from solution (28) let $a=0.72$,

$$\theta_{eq}=\frac{18.4\cdot 0.72^2}{0.1\cdot 3.14^2}\fallingdotseq 9.7\text{ min}$$

This indicates that at 10 minutes after being heated the temperature of mid-plane reaches about 130°C and gives good agreement with experimental result of hotpressing above

10 kg/cm². From this agreement it may be regarded that the boundary condition (19)'' is satisfied at the pressure over 10 kg/cm².

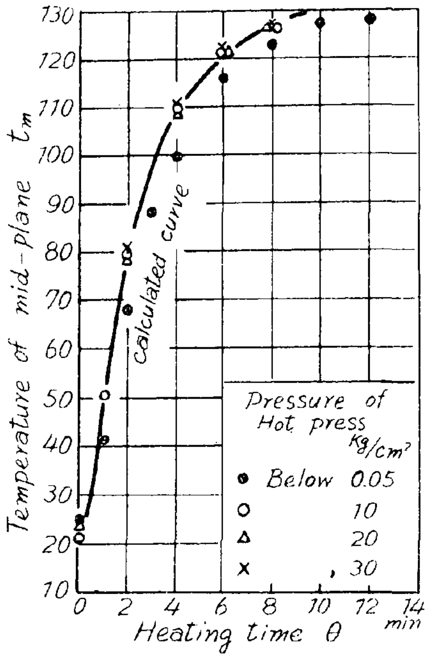

Fig. 15 Change of temperature in mid-plane (Hotpressing). Oven dry Beech, spec. grav. 0,65–0,67, thickness 1.43–1,45 cm, heating temp. 130°C

With the pressure of 10 kg/cm², oven dried, plain sawn pieces of various species were hotpressed at $t_1=130$ and $t_0=20$, and measuring the temperatures of its mid-plane, the value of $a_\perp$ were calculated from solution (27). The results are shown in Table 7. From this table it is indicated that (a) $a_\perp$ is essentially a constant, (b) $a_\perp$ is almost the same in the range of common wood species irrespective of the specific gravity and (c) $a_\perp$ gives

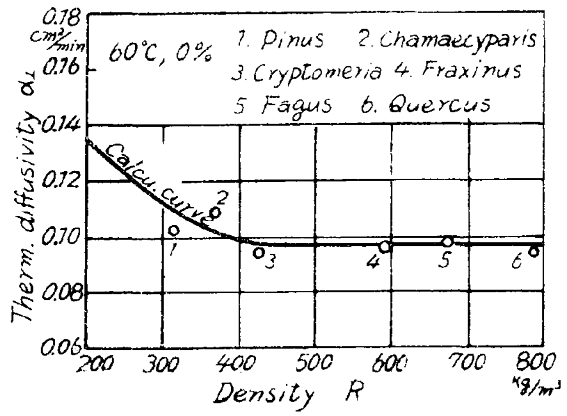

Fig. 16 Relation between the density and therm. diffusivity (Hotpressing)

a good agreement with the value calculated indirectly in II (cf. Fig. 16).

Table 7 Thermal diffusivity of oven dry wood $\alpha_\perp$ (Hotpressing)

	Quercus (NARA) $2a=1.0$ cm $r_0=0.78$						*Fagus* (BUNA) $2a=1.43$ cm $r_0=0.67$					
	Initial temperature of wood $t_0=20$°C						〃 20°C					
θ/a^2	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$
4	1′	75.5	54.5	0.497	0.380	0.095	2′02″	77.3	52.7	0.479	0.396	0.099
8	2′	106.2	23.8	0.217	0.720	0.090	4′04″	111.1	19.9	0.182	0.792	0.099
12	3′	121.1	8.9	0.0810	1.120	0.093	6′08″	122.6	7.4	0.0670	1.200	0.100
16	4′	126.7	3.3	0.0301	1.520	0.095	8′10″	126.9	3.1	0.0280	1.550	0.097
						$\alpha_{av}=0.093$						$\alpha_{av}=0.099$

	Fraxinus (TAMO) $2a=1.17$ $r_0=0.59$						*Chamaecyparis* (TAIWANHINOKI) $2a=1.50$ $r_0=0.36$					
	Initial temperature of wood $t_0=20$						〃 22.3					
θ/a^2	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$
4	1′22″	75.8	54.2	0.493	0.384	0.096	2′15″	82.3	47.7	0.443	0.428	0.107
8	2′44″	108.0	22.0	0.200	0.752	0.094	4′30″	112.9	17.1	0.159	0.848	0.106
12	4′06″	120.7	9.3	0.0850	1.10	0.092	6′45″	125.2	4.8	0.0450	1.360	0.113
16	5′28″	127.1	2.9	0.0266	1.57	0.098	9′00″	128.2	1.8	0.0167	1.760	0.110
						$\alpha_{av}=0.095$						$\alpha_{av}=0.109$

	Cryptomeria (SUGI) $2a=1.39$ $r_0=0.42$						*Pinus* (MATU) $2a=1.49$ $r_0=0.31$					
	Initial temperature of wood $t_0=20$						〃 20					
θ/a^2	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_\perp\theta/a^2$	$\alpha_\perp$
4	1′56″	75.0	55.0	0.500	0.376	0.094	2′13″	80.5	49.5	0.450	0.420	0.105
8	3′52″	107.5	22.5	0.205	0.744	0.093	4′26″	111.5	18.5	0.168	0.824	0.103
12	5′48″	120.7	9.3	0.0850	1.100	0.092	6′39″	123.3	6.7	0.0605	1.240	0.103
16	7′44″	126.5	3.5	0.0321	1.470	0.092	8′52″	127.7	2.3	0.0213	1.660	0.104
20	9′47″	128.3	1.7	0.0152	1.800	0.095	11′05″	128.7	1.3	0.0117	1.920	0.096
						$\alpha_{av}=0.093$						$\alpha_{av}=0.102$

In Table 8 a part of results made on $\alpha_{//}$, by similar method, is given. Fig. 17 summarizes these results. In this case $\alpha_{//}$ was essentially constant and was independent on the specific gravity as same as $\alpha_\perp$ but the measured values differed from the ones obtained

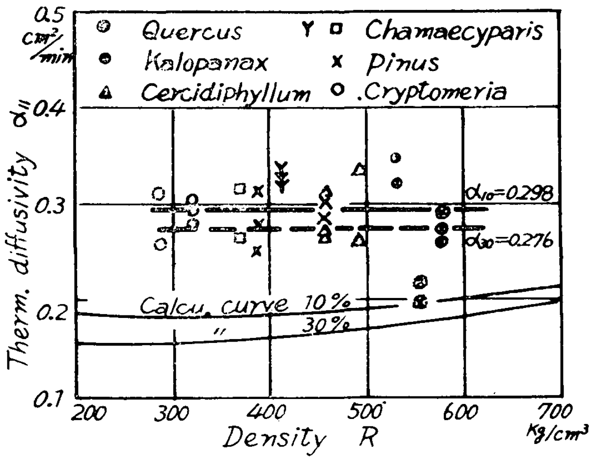

Fig. 17 Relation between the density and the thermal diffusivity (Hotpressing)

Table 8 Thermal diffusivity parallel to the grain $\alpha_{//}$ (Hotpressing)

Species	θ/a^2	θ	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$\alpha_{//}\theta/a^2$	$\alpha_{//}$	Remarks
	0.5	4′23	32.4	66.4	0.879	0.13	0.26	$t_i=99$
	0.75	6′33	43.2	55.8	0.739	0.219	0.342	$t_0=23.5$
	1.0	8′45	55.6	43.4	0.574	0.319	0.319	$r_0=0.45$
Pinus	1.5	13′07	71.2	27.8	0.378	0.492	0.304	$2a=5.92$
(MATU)	2.0	17′30	79.7	19.3	0.256	0.650	0.325	$u=24.9-22.9$ (23.9)
	2.5	21′11	83.5	15.5	0.206	0.740	0.338	
	3.0	26′37	87.5	11.5	0.152	0.860	0.286	
							$\alpha_{av}=0.311$	$^*\alpha_{30}=0.306$
	0.5	4′29	27.6	71.4	0.950	0.085	0.17	$t_i=99$
	0.75	6′42	32.7	66.3	0.881	0.125	0.167	$t_0=23.8$
	1.0	8′57	40.3	58.7	0.781	0.190	0.190	$r_0=0.55$
	1.5	13′25	54.2	44.8	0.595	0.305	0.202	$2a=5.98$
Quercus	2.0	17′54	64.8	34.2	0.455	0.417	0.208	$u=34.0\sim32.2$ (33.1)
(NARA)	2.5	22′24	73.2	25.8	0.343	0.528	0.211	
	3.0	26′48	79.2	19.8	0.254	0.650	0.217	
	3.5	31′24	83.6	15.4	0.205	0.74	0.211	
	4.0	35′48	86.4	12.6	0.168	0.823	0.206	
							$\alpha_{av}=0.199$	$^*\alpha_{30}=0.280$
	0.5	3′18	32.5	66.5	0.899	0.112	0.224	$t_i=99$
	0.75	4′32	39.6	59.4	0.803	0.170	0.227	$t_0=25.0$
	1.0	6′03	48.5	50.5	0.683	0.248	0.248	$r_0=0.49$
	1.25	7′34	57.4	41.6	0.564	0.323	0.258	$2a=4.92$
Kalopanax	1.5	9′05	63.7	35.3	0.487	0.382	0.254	$u=34.2\sim32.6$ (33.4)
(SEN)	1.75	10′36	69.0	30.0	0.405	0.460	0.263	
	2.0	12′06	74.0	25.0	0.338	0.540	0.270	
	2.5	15′07	81.6	17.4	0.235	0.680	0.272	
	3.0	18′09	86.6	12.4	0.168	0.823	0.274	
							$\alpha_{av}=0.254$	$^*\alpha_{30}=0.256$

Cryptomeria (SUGI)	0.5	3'22	33.9	65.1	0.903	0.112	0.224	$t_1=99$
	0.75	5'01	47.1	51.9	0.719	0.232	0.309	$t_0=26.9$
	1.0	6'42	58.5	40.5	0.561	0.325	0.325	$r_0=0.286$
	1.25	8'23	66.1	32.9	0.456	0.415	0.332	$2a=5.17$
	1.5	10'05	72.0	27.0	0.374	0.495	0.330	$u=32.2\sim30.2$ (31.2)
	2.0	13'24	80.7	18.3	0.254	0.651	0.326	
	2.5	16'48	86.4	12.6	0.175	0.805	0.322	
							$\alpha_{av}=0.310$	$^*\alpha_{30}=0.310$

* converted values to moisture content 30%

by calculation in II. This difference appears only in α'' as shown later and also in the case of steaming of wood of low moisture content. The reason has not yet clarified.

b. Experiments by steaming

Air seasoned wood plates of 3 cm thick and of various specific gravities were heated with steaming at 100°C and the measured values of the temperature t_m at mid-plane were shown in Fig. 18. From these results $a_\perp$ was calculated by Gurney-Lurie diagram of solution (27) and converted to the values at 20% moisture content. The relation with the density is given in Fig. 19 and further the changes of $a_\perp$ during heating are shown in

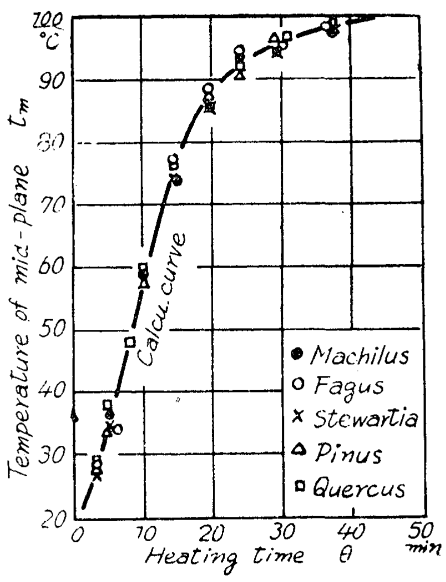

Fig. 18 Change of temperature in mid-plane (Steaming)

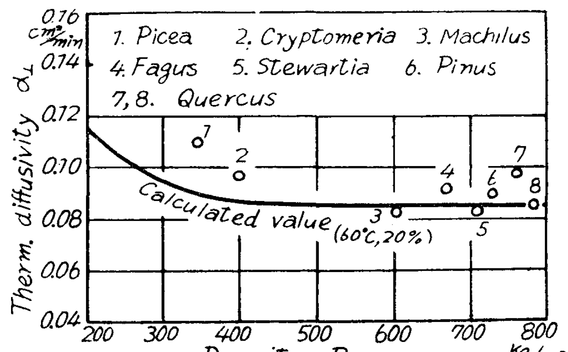

Fig. 19 Relation between the density and the thermal diffusivity (Steaming)

Table 9. This indicates that, as previously shown, the heat conduction in woods containing moisture is complicated by moving of interior moisture at the early stage of heating, condensing on surface, adsorption, evaporation, and so on, so that it does not give so exact value as in the case where oven dried wood is hotpressed. However, from the

above results it can be regarded that $\alpha_\perp$ is nearly constant. The curve in Fig. 18 represents a theoretical one obtained with the use of an average value $\alpha_\perp = 0.0904$ of many species except *Picea* (EZOMATU) which is extremely light. In this case, from solution (28)

$$\theta_{eq} = \frac{1.84 \cdot 1.5^2}{0.0904 \cdot 3.142^2} \doteqdot 46.5 \text{ min}$$

Table 9 Thermal diffusivity of *Cryptomeria* (Steaming)

Species	θ/a^2	θ	t_m	$t_1 - t_m$	$\frac{t_1 - t_m}{t_1 - t_0}$	$\alpha_\perp \theta/a^2$	$\alpha_\perp$	Remarks
	2.22	5	29.5	67.5	0.813	0.175	0.0788	$t_1 = 97$
	4.44	10	65.0	32.0	0.386	0.485	0.109	$t_0 = 14$
	6.66	15	79.5	17.5	0.210	0.735	0.110	$r_0 = 0.4$
	8.88	20	86.0	11.0	0.132	0.920	0.104	$2a = 3.0$
Cryptomeria (SUGI)	11.1	25	91.0	6.0	0.0724	1.165	0.104	$u = 16.4 \sim 23.3$
	13.3	30	93.0	4.0	0.0481	1.33	0.10	(19.9)
	17.8	40	95.0	2.0	0.0241	1.61	0.0905	
	22.2	50	96.0	1.0	0.0120	1.89	0.0851	
							$\alpha_{av} = 0.0977$	$\alpha_{20} = 0.0976$

This makes a good agreement with experimental data and the same conclusion as in the case of a can be drawn.

Fig. 20 gives the results of the similar experiments on the direction parallel to fiber. As in the case of hotpressing, there is a considerable difference between experimental and

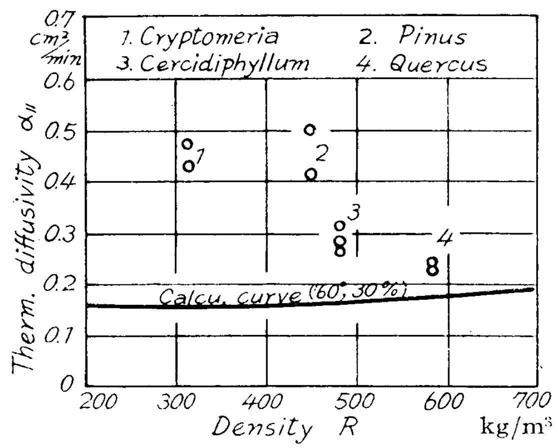

Fig. 20 Relation between the density and the thermal diffusivity (Steaming)

calculated values and further between conifers and the broad-leaved trees in $\alpha_{/\!/}$. The cause of these differences in $\alpha_{/\!/}$ has not yet been clarified and it is suggested that the moisture condensation, absorption, and evaporation at the wood surface which inevitably occur in steaming or movement of interior moisture during heating may play some role in that problem. Some discussions on this matter will appear later again. In summarizing, in the direction perpendicular to fiber $\alpha_{\perp}$ is constant independent on the density of wood and the experimental values agree well with the calculated one shown in II, and in the direction parallel to fiber $\alpha_{/\!/}$ give considerable differences between in conifers and broad-leaved trees and also between the experimental and calculated values.

3. Experiments on the relation between temperature and thermal diffusivity

Table 10 Thermal diffusivity of *Quercus* (Steaming)

Species	θ/a^2	θ	t_m	t_1-t_m	$\frac{t_1-t_m}{t_1-t_0}$	$\alpha_{\perp}\theta/a^2$	$\alpha_{\perp}$	Remarks
	2.22	5	24	73.0	0.87	0.14	0.063	$t_1=97$
	4.44	10	50	47.0	0.56	0.33	0.0742	$t_0=13$
	6.66	15	70	27.0	0.321	0.565	0.085	$r_0=0.61$
	8.88	20	81	16.0	0.190	0.775	0.0873	$2a=3.0$
Quercus	11.1	25	87.5	9.5	0.113	0.99	0.089	$u=103\sim96.1$
(NARA)	13.3	30	91	6.0	0.0715	1.17	0.088	(99.5)
	15.6	35	93.2	3.8	0.0452	1.36	0.087	
	17.8	40	94.5	2.5	0.0298	1.525	0.0865	
	22.2	50	96.0	1.0	0.0119	1.89	0.085	
							$\alpha_{av}=0.0827$	

Table 11 Thermal diffusivity of *Cryptomeria* (Steaming)

Species	θ/a^2	θ	t_m	t_1-t_m	$\frac{t_1-t_m}{t_1-t_0}$	$\alpha_{/\!/}\theta/a^2$	$\alpha_{/\!/}$	Remarks
	0.5	3′20″	24.8	65.2	0.975	0.08	0.160	$t_1=90$
	1.0	6′40″	35.0	55.0	0.824	0.163	0.163	$t_0=23.1$
	2	13′19″	45.9	44.0	0.660	0.261	0.130	$r_0=0.32$
	3	20′	59.5	30.5	0.456	0.418	0.139	$2a=5.17$
Cryptomeria (SUGI)	4	26′38″	69.1	20.9	0.313	0.569	0.142	$u=146\sim130$
	5	33′19″	76.4	13.6	0.204	0.740	0.148	(138)
	6	40′	81.2	8.8	0.132	0.919	0.153	
	7	46′38″	84.2	5.4	0.0808	1.23	0.161	
							$\alpha_{av}=0.149$	

The cases where the moisture change is comparatively small during heating are discussed. For $a_{\perp}$ Table 7 and 10, for $a_{//}$ Table 8 and 11 may be referred. Hotpressing of wet pieces or steaming may produce delicate influences on heat conduction by moisture shift or surface action caused by temperature difference between wood surface and interior, even if the apparent change in moisture content is not observed, so that both show irregular change as compared to the case where oven dried wood is hotpressed. These

Table 12 Thermal diffusivity in various moisture content (Steaming)

Species	Moisture content before heating %	Moisture content after heating %	Average moisture content %	Thermal diffusivity $\alpha_{\perp}$ cm²/min	Heating temperature °C	Thickness cm	Spec. gravity	Remarks
Fagus (BUNA)	4.6	9.8	7.2	0.119	97	2	0.63	diagonal direction
	15.2	18.7	17.0	0.0955	〃	〃		
	26.9	31.2	29.1	0.0874	〃	〃		
	43.6	43.8	43.7	0.0850	〃	〃		
	117.9	93.5	105.7	0.0842	〃	〃		
Cryptomeria (SUGI)	1.5	9.7	5.6	0.110	〃	3	0.40	〃
	16.4	23.3	19.9	0.0977	〃	〃		
Quercus (NARA)	3.1	9.1	6.1	0.121	〃	3	0.61	〃
	22.0	24.8	23.4	0.0768	〃	2		
	57.9	57.9	57.9	0.0814	〃	〃		
	100.3	96.1	99.5	0.0835	〃	3		
Cercidiphyllum (KATURA)	20.6	23.6	22.1	0.0915	〃	2	0.48	〃
	28.5	32.0	30.3	0.0831	〃	〃		
Machilus (TABU)	14.8	19.8	17.3	0.0833	100	3	0.61	radial direction
Fagus (INUBUNA)	15.1	20.1	17.6	0.0908	〃	〃	0.67	〃
Stewartia (HIME-SHARA)	13.9	18.9	16.4	0.0856	〃	〃	0.71	〃
Pinus (AKA-MATU)	14.8	19.8	17.3	0.0891	〃	〃	0.72	〃
Quercus (URAZIRO-KASHI)	13.4	22.4	17.9	0.0862	〃	〃	0.78	〃
〃 (NARA)	13.6	18.6	16.1	0.0856	〃	〃	0.76	〃
Picea (EZOMATU)	13.9	19.1	16.5	0.113	〃	〃	0.35	〃

irregularities, however, are practically neglected.

4. Experiments on the relation between moisture content and the thermal diffusivity.

a. $a_{\perp}$

Wood plate, 2-3 cm thick, of various moisture contents are heated with steaming at $t_1=97$ and the temperatures t_m of the mid-plane are measured. The results are shown in Table 12 together with the mean value of $a_{\perp}$ obtained from solution (27) and with the previous experimental results. It indicates that the moisture change between before and after heating is a few percents in low moisture content and about 20 % in high moisture content. As stated previously these deviations are practically negligible so that a is nearly constant during heating.

As there have hardly been published on the studies of cooking of wood, the present author has studied thereon and measured the temperature t_m of mid-plane of wood plate, 2 cm thick, prepared from *Fagus* (BUNA) and *Cryptomeria* (SUGI) of various moisture contents during heating in water at 65°C. The values of film coefficient of heat transfer h' and ha have never been determined hitherto, which should satisfy the boundary condition (19) and so solution (23) can not be directly used here. The present author has determined ha and $a_{\perp}$ by the following manner. By the use of t_m, $a_{\perp}\theta/a^2$ and $a_{\perp}$ are obtained corresponding to each ha in Table 13, from the diagram of solution (23), and the average a_{av} is calculated, and ha for which the difference of $a_{\perp}\theta/a^2$ and $a_{av}\theta/a^2$ is minimum is designated to be the first order. The order of ha thus obtained for various moisture contents is given in Table 14, and from this the most probable values of ha are

for *Cryptomeria* (SUGI) : 100

for *Fagus* (BUNA) : ∞

This result indicates that the effect of cooking can be regarded practically as similar as that of steaming. Table 15 summarizes the values of $a_{\perp}$ thus obtained.

Using these results together with those given in Table 16 (Fig. 16), the relation between average moisture content and the thermal diffusivity is graphically given in Fig. 21. The thermal diffusivity decreases abruptly for early stage with the increase of moisture content, slowly gradually from the fiber saturation point and was nearly constant above moisture content 50%. The broken line in this figure represents the values calculated in II for the sake of comparison, which indicates that below fiber saturation point they show good agreement with those obtained in the experiment here described. For the phenomena in the range of higher moisture content than the fiber saturation point, MACLEAN[25] wrote that the rate of temperature rise is constant independently on the moisture content,

Table 13 Thermal diffusivity of *Fagus* (BUNA) (Cooking, $t_1=65$, $t_0=3.3$°C)

θ min	t_m	t_1-t_m	$\frac{t_1-t_m}{t_1-t_0}$	$ha=\infty$				100				50				20				10			
				$\alpha_\perp\frac{\theta}{a^2}$	$\alpha_\perp$	$\alpha_{av}\frac{\theta}{a^2}$	deviation	$\alpha_\perp\frac{\theta}{a^2}$	$\alpha_\perp$	$\alpha_{av}\frac{\theta}{a^2}$	deviation	$\alpha_\perp\frac{\theta}{a^2}$	$\alpha_\perp$	$\alpha_{av}\frac{\theta}{a^2}$	deviation	$\alpha_\perp\frac{\theta}{a^2}$	$\alpha_\perp$	$\alpha_{av}\frac{\theta}{a^2}$	deviation	$\alpha_\perp\frac{\theta}{a^2}$	$\alpha_\perp$	$\alpha_{av}\frac{\theta}{a^2}$	deviation
0	3.3																						
2	13.7	51.3	0.831	0.165	0.0825	0.143	-0.022	0.167	0.0835	0.144	-0.023	0.175	0.0875	0.149	-0.026	0.185	0.0925	0.158	-0.027	0.20	0.100	0.171	-0.029
4	25.7	39.3	0.637	0.280	0.0700	0.285	0.005	0.284	0.0710	0.288	0.004	0.295	0.0738	0.297	0.002	0.315	0.0788	0.315	0	0.335	0.0838	0.342	0.007
6	36.2	28.8	0.467	0.405	0.0675	0.428	0.023	0.413	0.0688	0.432	0.019	0.425	0.0708	0.446	0.021	0.452	0.0753	0.473	0.021	0.490	0.0817	0.513	0.023
8	44.8	20.2	0.327	0.548	0.0685	0.570	0.022	0.557	0.0696	0.576	0.019	0.575	0.0719	0.594	0.019	0.610	0.0763	0.631	0.021	0.657	0.0821	0.684	0.027
10	51.3	13.7	0.222	0.706	0.0706	0.710	0.007	0.716	0.0716	0.721	0.005	0.735	0.0735	0.743	-0.008	0.778	0.0778	0.788	0.010	0.847	0.0847	0.855	0.008
12	55.5	9.5	0.154	0.856	0.0713	0.855	-0.001	0.865	0.0712	0.865	0	0.888	0.0740	0.892	0.004	0.942	0.0785	0.946	0.004	1.027	0.0856	1.026	-0.001
14	58.2	6.8	0.110	0.995	0.0711	0.998	0.003	1.005	0.0718	1.009	0.004	1.030	0.0736	1.040	0.010	1.096	0.0780	1.104	0.008	1.193	0.0852	1.194	0.001
16	60.6	4.4	0.0713	1.174	0.0734	1.140	-0.034	1.183	0.0739	1.153	-0.03	1.210	0.0756	1.189	-0.021	1.285	0.0803	1.201	-0.024	1.40	0.0875	1.364	-0.036
18	61.4	3.6	0.0583	1.255	0.0697	1.280	0.028	1.267	0.0704	1.297	0.03	1.297	0.0721	1.338	0.041	1.375	0.0764	1.419	0.044	1.502	0.0834	1.540	0.038
20	62.2	2.8	0.0454	1.360	0.0680	1.425	0.065	1.373	0.0687	1.441	0.068	1.406	0.0703	1.486	0.080	1.487	0.0744	1.577	0.090	1.625	0.0813	1.711	0.086
				$\alpha_{av}=0.0713$			+0.096	$\alpha_{av}=0.0720$			+0.096	$\alpha_{av}=0.0743$			+0.130	$\alpha_{av}=0.0788$			+0.147	$\alpha_{av}=0.0855$			+0.124

Table 14 Order of *ha*

Fagus (BUNA)

Moisture content %	$ha=\infty$	100	50	20	10
4.1	1	2	3	4	5
24.6	1	2	3	4	5
35.2	3	4	1	2	5
54.7	1	1	4	5	3
86.1	1	2	3	5	4

Cryptomeria (SUGI)

Moisture content %	$ha=\infty$	100	50	20	10
3.9	3	5	1	2	4
20.7	1	2	4	5	3
31.3	2	1	3	4	5
55.2	5	4	3	1	2
71.2	4	1	2	5	2

Table 15 Thermal diffusivity in various moisture content (Cooking)

Species	Moisture content before heating %	Moisture content after heating %	Average moisture content %	Thermal diffusivity $\alpha_{\perp}$ cm²/min	Heating temperature °C	Thickness cm	Spec. gravity	Remarks
Fagus (BUNA)	1.6	6.6	4.1	0.093	65	2	0.63	diagonal direction
	18.4	30.7	24.6	0.0863	〃	〃		
	32.4	38.0	35.2	0.0689	〃	〃		
	54.0	55.3	54.7	0.0712	〃	〃		
	86.7	85.5	86.1	0.0706	〃	〃		
Cryptomeria (SUGI)	0.7	7.1	3.9	0.116	〃	〃	0.43	〃
	17.5	23.9	20.7	0.0867	〃	〃		
	29.7	32.9	31.3	0.0834	〃	〃		
	53.8	56.6	55.2	0.0774	〃	〃		
	69.1	73.3	71.2	0.0702	〃	〃		

Table 16 Thermal diffusivity in various moisture content (Hotpressing)

Species	Moisture content before heating %	Moisture content after heating %	Average moisture content %	Thermal diffusivity $\alpha_{\perp}$ cm²/min	Heating temperature °C	Thickness cm	Spec. gravity
Quercus (NARA)	0	0	0	0.093	130	1.0	0.78
Fagus (BUNA)	〃	〃	〃	0.099	〃	1.43	0.67
Fraxinus (TAMO)	〃	〃	〃	0.095	〃	1.17	0.59
Cryptomeria (SUGI)	〃	〃	〃	0.093	〃	1.39	0.42
Pinus (MATU)	〃	〃	〃	0.102	〃	1.49	0.35
Chamaecyparis (TAIWANHINOKI)	〃	〃	〃	0.109	〃	1.50	0.31
Cryptomeria (SUGI)	24.5	24.4	24.4	0.077	80	1.48	0.41

while MATUMOTO[36] wrote that the rate decreases even above fiber saturation point with the increase of moisture content. These conclusions lack sufficient experimental evidences. The results of the present author indicate that the rate of temperature rise is constant above moisture content 50%.

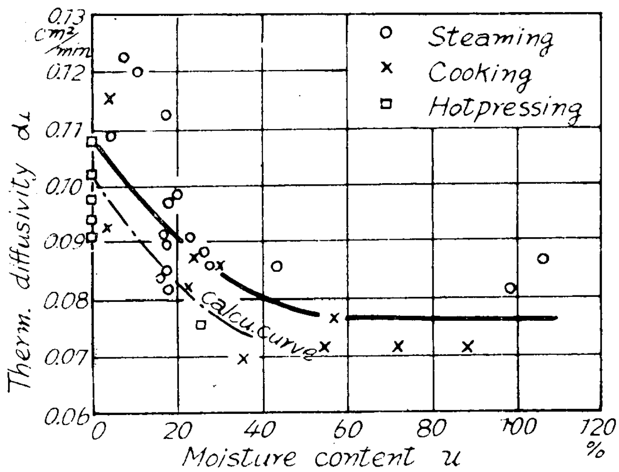

Fig. 21 Relation between the miosture content and the thermal diffusivity vertical to the grain

Fig. 22 gives the temperature behaviors of *Fagus* (BUNA) contained 17.0, 29.1, and 43.7 % moisture content during steaming and $t_m - \theta$ curves calculated by $\alpha = 0.092$, 0.083, 0.078, respectively, derived from Figure 21. It indicates that changes of α by some degree give not so large influence on temperature. From solution (28) the values of θ_{eq} are obtained as nearly 20, 22.5, and 24 min. respectively. In Fig. 23 and 24 the temperature gradients calculated and experimented are given on *Cryptomeria* (SUGI) which was cooked and on *Fagus* (BUNA) which was steamed.

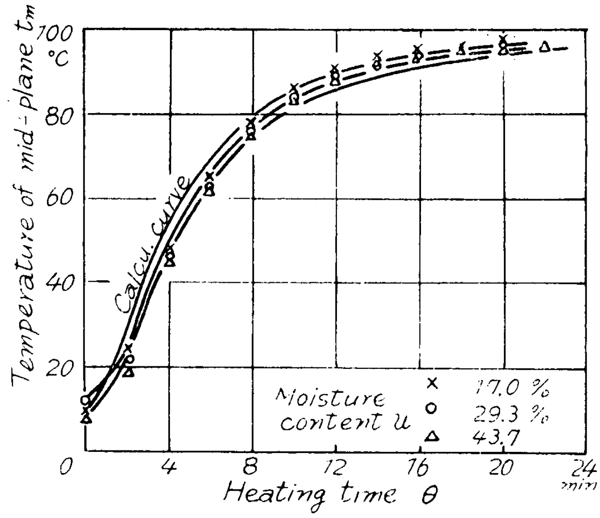

Fig. 22 Change of temperature in mid-plane (Steaming), *Fagus* 2 cm thickness

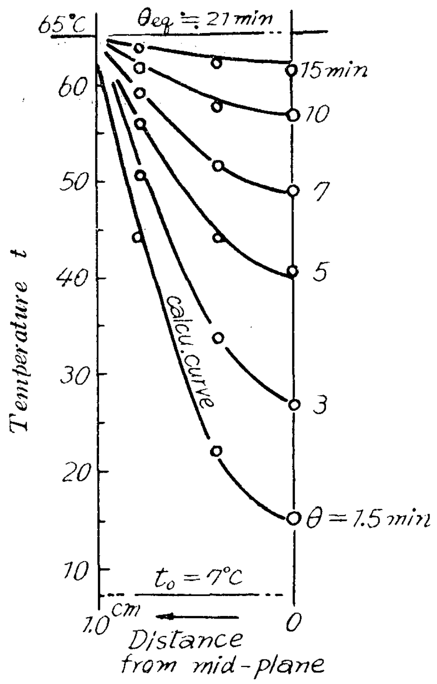

Fig. 23 Temperature distribution (Cooking), *Cryptomeria* 2 cm thickness

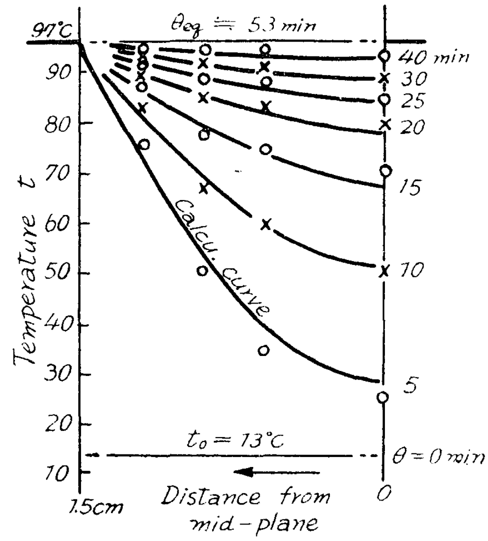

Fig. 24 Temperature distribution (Steaming), *Fagus* 3cm thickness

Table 17 Thermal diffusivity in various moisture content of wood (Steaming)

Species	Moisture content before heating %	Moisture content after heating %	Average Moisture content %	Thermal diffusivity $\alpha_{//}$ cm²/min	Heating temperature °C	Thickness cm	Spec. gravity	Remarks
Cercidiphyllum (KATURA)	7.9	12.1	10.0	0.439	90	4.04	0.48	
	8.0	12.0	10.0	0.431	〃	〃		
	8.1	12.8	10.4	0.447	〃	〃		
	13.1	19.1	16.1	0.388	86	〃		
	14.0	19.9	17.0	0.336	90	〃		
	16.1	23.0	19.6	0.403	〃	〃		
	23.9	30.0	26.9	0.326	〃	〃		
	25.0	30.9	27.9	0.265	〃	〃		
	26.0	31.9	28.9	0.302	〃	〃		$*\alpha_{30}$
	28.9	35.0	32.0	0.270	〃	〃		0.276
	29.0	35.1	32.1	0.313	〃	〃		0.315
	29.7	35.4	32.3	0.278	〃	〃		0.284
	54.0	58.1	56.1	0.208	〃	〃		
	74.5	76.5	75.5	0.196	〃	〃		

	7.1	11.6	9.4	0.491	〃	4.06		
	8.0	12.9	10.4	0.398	〃	〃		
	8.8	13.9	11.4	0.345	〃	〃		
	14.9	20.0	17.5	0.289	〃	〃		
	15.0	21.9	18.5	0.279	〃	〃		
Quercus	15.4	21.6	18.5	0.269	〃	〃		
(NARA)	26.1	31.1	28.6	0.238	〃	〃	0.58	0.237
	26.7	33.1	29.9	0.204	〃	〃		0.237
	28.0	35.0	31.5	0.236	〃	〃		
	39.8	46.8	43.3	0.202	〃	〃		
	40.9	47.0	43.9	0.204	〃	〃		
	67.9	68.2	68.0	0.121	〃	〃		
	96.9	91.0	94.0	0.108	〃	〃		
	108.0	103.5	105.8	0.0930	〃	〃		
	7.9	12.5	10.2	0.479	90	4.08		
	8.8	13.1	11.0	0.636	〃	〃	0.45	
	14.2	22.1	18.2	0.610	〃	〃		
	14.3	21.1	17.7	0.584	〃	〃		
	18.0	24.5	21.2	0.531	〃	4.06		
Pinus	18.6	24.0	21.3	0.544	〃	〃	0.37	
(MATU)	19.0	25.0	22.0	0.491	〃	〃		0.502
	31.0	39.0	35.0	0.497	〃	4.08		0.410
	32.0	39.0	35.5	0.407	〃	〃		
	68.9	77.9	72.4	0.244	〃	〃	0.45	
	78.0	83.5	80.7	0.240	〃	〃		
	120	118	119	0.185	〃	〃		
	164	156	160	0.126	〃	〃		
	13.1	19.9	16.5	0.597	〃	5.17	0.32	
	15.9	22.8	19.4	0.477	〃	4.00		
	16.7	22.1	19.4	0.343	〃	〃	0.29	
	16.9	23.1	20.0	0.476	〃	〃		
	23.8	27.1	25.5	0.561	〃	5.17		
Cryptomeria	25.0	32.5	28.8	0.374	〃	〃		
(SUGI)	25.3	31.8	28.6	0.483	〃	〃		
	25.8	33.6	29.7	0.442	〃	〃		0.478

	26.9	18.1	22.5	0.612	〃	〃	
	27.0	34.8	30.9	0.306	〃	〃	
	84.0	82.5	83.3	0.159	〃	〃	0.32
	99.0	92.7	95.9	0.194	〃	〃	
	112.5	104	108.3	0.177	〃	〃	
	140	128	134	0.163	〃	〃	
	146	130	138	0.149	〃	〃	
	161.5	146	153.8	0.144	〃	〃	
	10.0	15.9	13.0	0.556	〃	3.98	
	11.0	17.0	14.0	0.377	〃	〃	
Chamaecyparis	11.4	17.8	14.6	0.413	〃	〃	0.45
(HINOKI)	17.0	22.7	19.8	0.420	〃	〃	
	17.0	23.1	20.1	0.414	〃	〃	
	18.0	23.9	21.0	0.462	〃	〃	

* converted values to moisture content 30 %

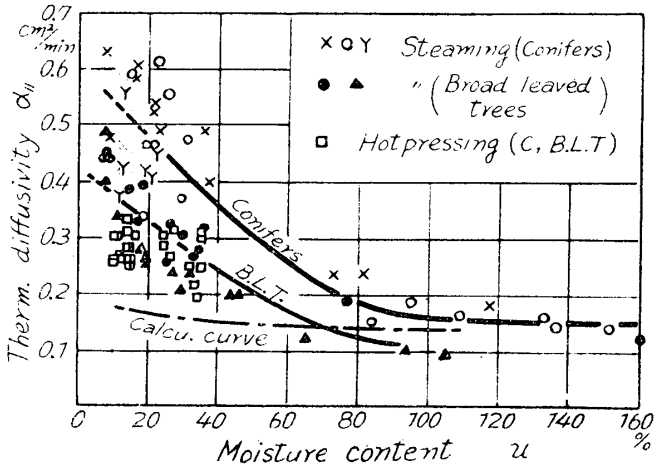

Fig. 25 Relation between the moisture content and the thermal diffusivity parallel to the grain

b. $\alpha_{//}$

Table 17 shows $\alpha_{//}$ of *Pinus* (MATU), *Cryptomeria* (SUGI), *Chamaecyparis* (HINOKI), *Cercidiphyllum* (KATURA), and *Quercus* (NARA) with various moisture contents obtained with steaming, and Fig. 25 shows the relation between average moisture content and $\alpha_{//}$, including the results by hotpressing in Fig. 17 (ref. Table 18) and for the sake of comparison the calculated values in II is shown by chain line.

Table 18 Thermal diffusivity in various moisture content of wood (Hotpressing)

Species	Moisture content before heating %	Moisture content after heating %	Average moisture content %	Thermal diffusivity $\alpha_{,,}$ cm²/min	Heating tempera-ture °C	Thickness cm	Spec. gravity	Remarks
Quercus (NARA)	13.7 13.8 14.8	12.9 13.0 14.0	13.3 13.4 14.4	0.282 0.262 0.252	90 81 90	2.96 〃 〃	0.57	*α_{10} 0.290 0.269 0.261
Cryptomeria (SUGI)	14.1 15.6 15.8	12.9 13.7 13.5	13.5 14.7 14.7	0.280 0.284 0.266	〃 〃 〃	3.0 〃 〃	0.32	0.293 0.301 0.282
Cercidiphyllum (KATURA)	12.5 12.9 13.0	11.8 12.0 11.9	12.2 12.4 12.4	0.274 0.266 0.306	〃 〃 〃	2.97 〃 〃	0.45	0.277 0.270 0.310
Chamaecyparis (HINOKI)	14.3 14.5 15.0	13.9 13.1 14.7	14.1 13.8 14.8	0.322 0.313 0.306	〃 〃 〃	2.99 〃 〃	0.41	0.335 0.326 0.322
Kalopanax (SEN)	13.9 14.0	12.8 13.0	13.4 13.5	0.341 0.316	〃 〃	〃 〃	0.53	0.348 0.322
Pinus (MATU)	12.0 12.1 12.1	10.9 10.5 11.0	11.4 11.3 11.6	0.259 0.277 0.308	〃 〃 〃	4.01 〃 〃	0.38	0.262 0.280 0.313
〃 〃	24.9 24.9	22.1 22.9	23.5 23.9	0.295 0.311	〃 99	5.92 〃	0.45	*α_{30} 0.291 0.306
Quercus (NARA)	33.0 34.0	31.6 32.2	32.3 33.1	0.218 0.199	〃 〃	5.98 〃	0.55	0.218 0.200
Kalopanax (SEN)	34.2 43.8	32.6 41.8	33.4 42.8	0.254 0.286	〃 〃	4.92 〃	0.49	0.256 0.339
Cryptomeria (SUGI)	32.0 32.2	30.0 30.2	31.0 31.2	0.257 0.310	〃 〃	5.17 〃	0.28	
Chamaecyparis (SAWARA)	27.1 27.8	25.9 25.2	26.5 26.5	0.321 0.269	〃 〃	4.95 〃	0.36	0.315 0.264

* α_{10}, α_{30} : converted into moisture content 10, 30 %

These indicate that by steaming $\alpha_{,,}$ decreases, as in the case of a, initially with the increase of moisture content and then gradually to reach a nearly constant value in high moisture range, and that there is a distinct difference in conifers and in broadleaved trees, the former reaching the constant at 100% and the latter at 80%. In both kinds of trees the experimental values differ considerably from calculated ones in moisture absorbing range (hygroscopic range) and the difference become lesser as the moisture content increases so that above the limiting point stated above they give considerable agreement. On the contrary, in the case of hotpressing there are no difference by species and density, all agreeing with the curves for broad-leaved trees.

The causes of higher values of the thermal diffusivity experimented in the fiber direction in moisture absorption range than the calculated ones and of their distinct difference between conifers and broad-leaved trees have not been clarified but the followings are to be considered as giving considerable influences on them.

When wet wood is heated and if the initial moisture distribution is uniform and the surface evaporation does not take place, the moisture of high temperature at the surface partially evaporates to diffuse into the interior of the wood by the temperature gradient which produced in the interior of wood immediately after heating, and condenses so that a kind of flow of heat other than the pure conduction locally takes place and in the same time the interior thermal diffusivity decreases by the increase of moisture content while in outer parts it increases. Meanwhile with the lessening of the temperature gradient, this diffusion becomes less active and on the contrary the interior moisture begins to move towards outer so that the internal thermal diffusivity is liable to increase while the outer one to decrease. The thermal diffusivity thus has an inclination to vary by time and by position. Therefore, whether actually a varies or not or whether abnormal temperature rise takes place or not depend chiefly on the correlation of heat conduction and the moisture diffusion and it can be assumed that in the direction perpendicular to fiber this correlation is held at nearly an equilibrium state on an average. In the case of steaming, other than the above phenomena, the condensation of moisture on the wood surface, the rate of moisture absorption, the capillary condensation effected by the number and the dimension of meniscus, and the others complicate the heat conduction. However, as will be stated later, as there is observed no distinct difference between the longitudinal and cross-section of wood on the change of moisture content by steaming, the phenomena can be included in the assumption $h \to \infty$ in II irrespectively to the direction of fiber. The anomaly of $a_{//}$ in moisture absorption range, therefore, seems to be caused mainly by the correlation of heat conduction and the internal moisture diffusion. Now discussion will be made in more detail on this point. On the moisture diffusion constant $k_{//}$ in fiber direction and $k_{\perp}$ in radial direction there are few papers particularly on the former. According to EGNER[8], in the hygroscopic moisture range of *Picea* (Fichte), there are differences depending on temperature and moisture, for example, at 60°C and moisture content 5, 20, and 27%, the ratio $k_{//}/k_r$ are 65, 7, and 10, at 80°C and moisture content 5 and 20%, $k_{//}/k_r$ 100 and 10. The present author[28] gave the ratio $k_{//}/k_{\perp} \fallingdotseq 4 \sim 8$ independently of the moisture content on a few species of wood which were dried under various conditions and STAMM[7] has given the ratios of diffusion coefficient 12.6 and 23.1 at specific gravity (wet volume basis) 0.365 and 0.60 respectively when dried to 10% from the fiber saturation point at 40°C.

Theory of STAMM indicates that as the specific gravity is smaller so the ratio of the both is smaller, while KOLLMANN[22] has written that, with the understanding that it can not be applied to all cases, the ratio of moisture passing the unit square in the unit time in the directions parallel and perpendicular to fiber decreases with the increase of specific graivity, that's, at the specific gravity 0.4 and 0.8 the ratios are 7 and 4 respectively. On the other hand, the calculated value of $\lambda_{//}/\lambda_{\perp}$ or $a_{//}/a_{\perp}$ is 2.0 - 2.5, as stated previously, the ratio of the moisture diffusion constant in the direction perpendicular to fiber to that in the direction parallel to fiber reaches to several fold or much more so that the equilibrium which is possible in the direction perpendicular to fiber breaks in fiber direction and it is supposed that the influence by moisture diffusion appears positively. Thus the apparent values of $a_{//}$ are constant and considerably higher than the calculated ones, but in extreme cases, below 20% moisture content in conifers or below 10% in broad-leaved trees, both in steaming, $a_{//}$ gradually changes with progressing of heating as exemplified in Table 19, and employment of mean value produces unnegligible error. The dotted line in Figure 25 indicates the range where the change of a during heating is so large that it can not be regarded as a constant. It is readily understood that with the increase of moisture content the above phenomena gradually disappear by the facts that with the increase of cell cavity which is occupied by free water diffusion of vapor due to pressure difference is inhibited and that the thermal diffusivity does not so much increase as moisture does. In summarizing, it can be said that the heat conduction in the direction parallel to fiber is influenced by movement of internal moisture and other factors in comparatively lower moisture content so that the values of thermal diffusivity obtained in the steady state can not be

Table 19 Thermal diffusivity of *Cryptomeria* (Steaming)

Species	θ/a^2	θ	t_m	t_1-t_m	$\frac{t_1-t_m}{t_1-t_0}$	$\alpha_{//}\ \theta/a^2$	$\alpha_{//}$	Remarks
	0.5	2′	34.5	55.5	0.879	0.123	0.246	$t_1=90$
	0.75	3	52.8	37.2	0.589	0.309	0.412	$t_0=26.8$
	1.00	4	64.8	25.2	0.398	0.475	0.475	$r_0=0.29$
Cryptomeria	1.25	5	72.6	17.4	0.276	0.611	0.489	$2a=4.0$
(SUGI)	1.5	6	77.7	12.3	0.195	0.760	0.506	$u=15.9\sim22.8$ (19.4)
	2.0	8	83.5	6.5	0.103	1.02	0.510	
	3.0	12	88.0	2.0	0.0316	1.50	0.500	
						$\alpha_{av}=0.477$		

directly employed and the apparent values of the thermal diffusivity are fairly higher than those calculated in II.

c. The relation of $a_{//}$ and $a_{\perp}$

Fig. 26 shows the ratio $k=a_{//}/a_{\perp}$ in conifers and broadleaved trees at every moisture content. Thus previous determining of k favors practical calculation in dealing theoretically with the heat conduction of short balk and round timber in which the heat conduction from the both end cross section is not negligible.

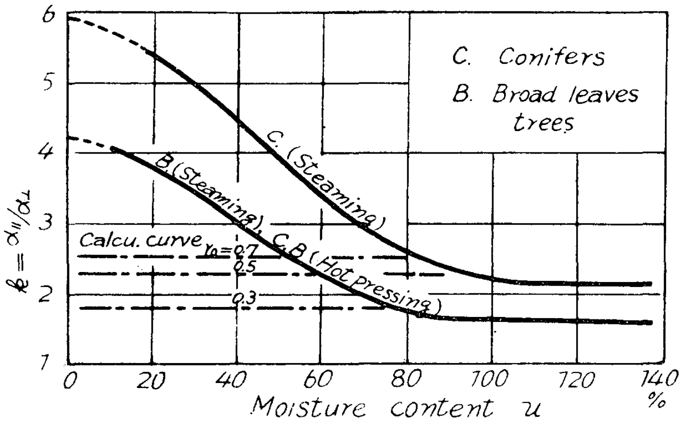

Fig. 26 Relation $k=\alpha_{//}/\alpha_{\perp}$ and the moisture content

5. The change of moisture content in wood by steaming

The change of the interior moisture content of wood by steaming is influenced by absorption in lower moisture content and by evaporation in higher moisture content so that it theoretically directs to the common critical point (the fiber saturation point) after infinitive period, but practically it is complicated in rather shorter definite period by various accompanying phenomena to direct a limiting point which is considerably higher than the fiber saturation point[15),23),43)]. Few papers have been published considering of the difference of this change by the direction to fiber and any only fragmentarily. For example, OGURA[43)] heated with steaming the piece of *Ulmus* (AKADAMO), 0.8•3.5•3.5 inches, of various moisture contents and observed the limiting moisture content where no apparent change in moisture content takes place to be about 58%, and KOLLMANN[23)] heated small piece of air-seasoned *Fraxinus* (Esche) and *Fagus* (Buche) with steaming under 0, 1, 2, 3 atmospheric pressure and observed that (1) when condensed water is absent there is a tendency to direct a limiting moisture content 25%, independently on the wood species and vapor-pressure, and practically where some capillary condensation is unavoidable it does never go over 30%, (2) when condensed water is present water absorption becomes active after some degree of moisture absorption to appear capillary water and the limiting moisture becomes 40 - 50%, and (3) the capillary water is more readily formed in diffuse-porous hardwoods than in ring-porous hardwoods. He has made no reference to the relation with the direction of fiber and the present auther has studied thereupon with edge-and end-grained pieces (perpendicular and parallel to fiber respectively) of *Fagus* (BUNA) and *Cryptomeria* (SUGI) of various initial moisture contents with steaming, the results being

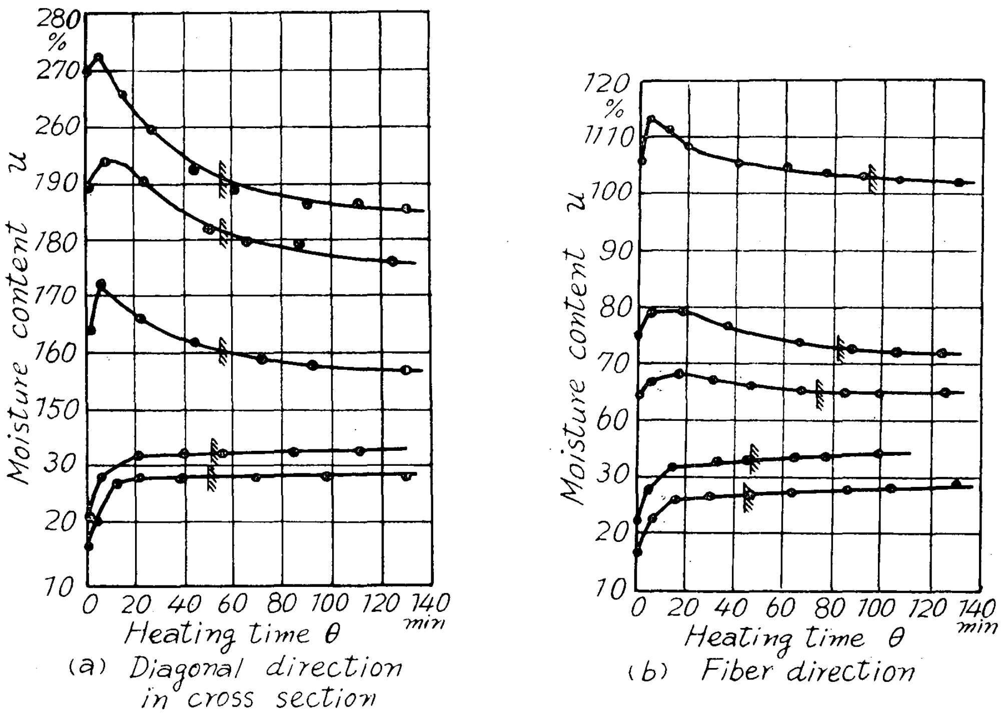

Fig. 27 Change of moisture content in *Cryptomeria* (Steaming)

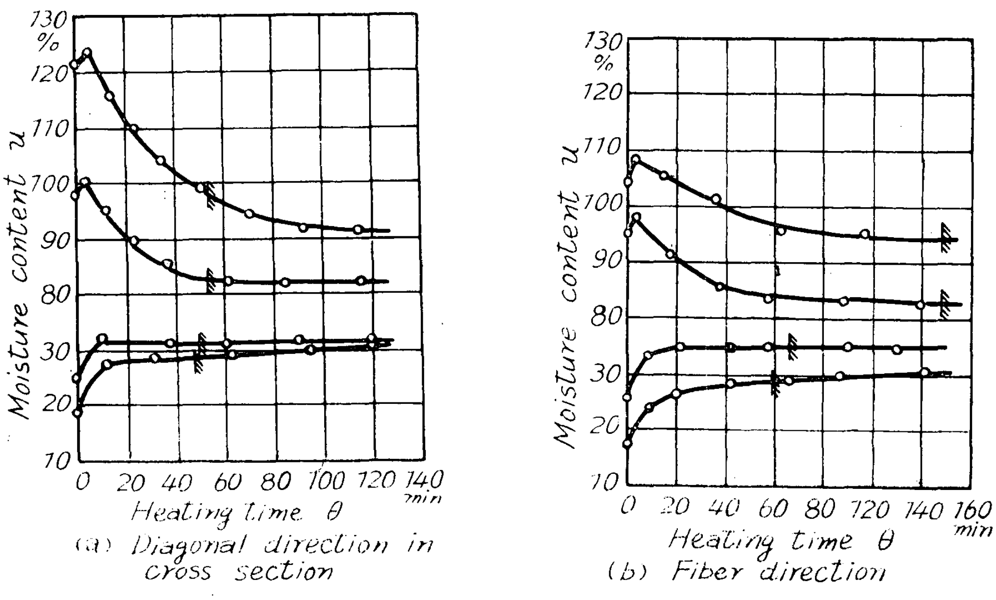

Fig. 28 Change of moisture content in *Fagus* (Steaming)

shown in Fig. 27, 28. The hatched point represents θ_{eq} obtained by nomograph, given later (Fig. 31), and by the value of α from Fig. 21 and 25. It is clear from these results that the change of the apparent average moisture content by time and the degree of change are essentially independent on the direction of fiber in both species. In steaming, even if there are some differences in surface phenomena, such as condensation on wood surface, moisture absorption, and capillary condensation, between two direction, their influences on the difference of each apparent thermal diffusivity are not so determinative, but the speed of moisture diffusion in the interior of wood gives rather larger influence.

Summary to I - III.

1. Wood is a heterogeneous and anisotropic substance, but as industrial material it can be regarded as ortho-anisotropy in reference to its heat conduction.

2. The thermal diffusivity in the direction perpendicular to fiber is nearly constant independently on the density of common wood species, but in particularly light wood (specific gravity below 0.3) and particularly heavy ones (specific gravity above 0.8) the diffusivity shows inclination to be rather larger. The experimental values give fairly good agreement with the calculated ones derived indirectly from previous formulas concerning to thermal conductivity, specific heat and density. On the contrary, the experimental thermal diffusivity in fiber direction shows difference between conifers and broadleaved trees, the former being higher and the latter lower. Both usually give higher estimated values than the calculated ones. The discrepancy of the calculated values with the experimental values can be explained by mainly, among others, insufficiency of consideration of the influence of the movement of the interior moisture caused by unsteady heat conduction.

3. The change of the thermal diffusivity by temperature is irregular in the range of low moisture content, but practically it can be almost neglected.

4. With the increase of moisture content, the thermal diffusivity lowers, particularly in the hygroscopic range, but it shows almost constant value in the range of moisture content above 50% in the direction perpendicular to fiber, and above 110% in conifers and above 90% in broad-leaved trees in the direction parallel to fiber. As stated above, the measured values give a good agreement with the calculated ones in the direction perpendiculer to fiber while in fiber direction there are considerable differences between both values in low moisture content so that employment of the calculated values may produce unneglegible error. Also in the steaming of conifers of moisture content below 20% or broad-leaved

trees below 10%, the values of the thermal diffusivity varies by time so that considerable errors may similarly produced (Fig. 21 and 25).

5. When a wood-plate is hotpressed, the change in moisture content between before and after heating in time θ_{eq} is fairly small but when steamed evaporation takes place in the range of high moisture content and absorption in low moisture content and generally considerable change of moisture content are produced between before and after heating. However, except the special case of the fiber direction as mentioned in 4, the thermal diffusivity for average moisture content can be used.

6. The difference of the thermal diffusivity in radial and tangential directions can be practically neglected. The ratio of the thermal diffusivity in the fiber direction to that in the direction perpendicular to fiber differs with the moisture content, conifers, and broad-leaved trees, in conifers and with low moisture content being large, but with the increase of the moisture content the difference tends to diminish to approach to the ratio obtained by calculation.

As obvious from the above statements, the thermal diffusivity can be regarded to keep nearly constant during heating, except special cases, in dealing with the theory of heat conduction in wood, and the value of the diffusivity in a given condition can be practically obtained from Fig. 21, 25, and 26. From these results the solutions of heat conduction frequently encountered in utilization of wood are readily obtained. The heating with steaming is, as stated in 1, a special case but in the following chapters this will be treated separately for the sake of practical convenience.

IV.´ The heat conduction in plate timber[30), 31)]

1. Heating by hot water or by hot air

a. Case where the temperature of heating medium is constant

As obvious from III - 1, the conditions for this case are

$$\begin{cases} \dfrac{\partial t}{\partial \theta} = a_L \dfrac{\partial^2 t}{\partial x^2} & \cdots\cdots(17) \\ \theta = 0 : t = f(x) & \cdots\cdots(18) \\ x = \mp a : t = t_1 & \cdots\cdots(19) \\ x = 0 : \dfrac{\partial t}{\partial x} = 0 & \cdots\cdots(20) \end{cases}$$

Thus particular solutions are

$$t - t_1 = \frac{1}{a} \sum_{n=1}^{\infty} e^{-\alpha_\perp \left(\frac{u_n}{a}\right)^2 \theta} \cos \frac{u_n}{a} x \frac{u_n}{u_n + \sin u_n \cos u_n} \int_{-a}^{a} \left\{ f(\lambda) - t_1 \right\} \cos \frac{u_n}{a} \lambda d\lambda \quad (21)$$

$$\frac{t_1 - t}{t_1 - t_0} = 2 \sum_{n=1}^{\infty} e^{-\alpha_\perp \left(\frac{u_n}{a}\right)^2 \theta} \cos \frac{u_n}{a} x \frac{\sin u_n}{u_n + \sin u_n \cos u_n} \quad \cdots\cdots (22)$$

$$\frac{t_1 - t_m}{t_1 - t_0} = 2 \sum_{n=1}^{\infty} e^{-\alpha_\perp \left(\frac{u_n}{a}\right)^2 \theta} \frac{\sin u_n}{u_n + \sin u_n \cos u_n} \quad \cdots\cdots (23)$$

And also

$$\theta_{cq} = \frac{4.6 a^2}{\alpha_\perp u_1^2} \quad \cdots\cdots (24)$$

b. Case where the temperature of heating medium varies with time

When the temperature of the heating medium changes by function of time $\varphi(\theta)$, we have

$$\begin{cases} \dfrac{\partial t}{\partial \theta} = \alpha_\perp \dfrac{\partial^2 t}{\partial x^2} & \cdots\cdots (17) \\ \theta = \ : t = f(x) & \cdots\cdots (18) \\ x = \pm a : \dfrac{\partial t}{\partial x} \pm h \left\{ t - \varphi(\theta) \right\} = 0 & \cdots\cdots (29) \end{cases}$$

By the Stoke's method assuming these solution as

$$\begin{cases} t(\theta, x) = \sum_{n=1}^{\infty} A_n(\theta) \cos \dfrac{u_n}{a} x \\ A_n(\theta) = \dfrac{1}{a} \dfrac{u_n}{u_n + \sin u_n \cos u_n} \displaystyle\int_{-a}^{a} t(\theta, \lambda) \cos \dfrac{u_n}{a} \lambda \, d\lambda \end{cases}$$

we determine A_n by using $\frac{\partial^2 t}{\partial x^2}$ instead of t

$$A_n = e^{-\alpha_\perp \left(\frac{u_n}{a}\right)^2 \theta} \left\{ K \, 2a \frac{\sin u_n}{u_n} \alpha_\perp \left(\frac{u_n}{a}\right)^2 \int_0^{\theta} e^{\alpha_\perp \left(\frac{u_n}{a}\right)^2 \xi} \varphi(\xi) \, d\xi + C_n \right\}$$

where

$$K = \frac{1}{a} \frac{u_n}{u_n + \sin u_n \cos u_n}$$

Thus finally

$$t(\theta,\ x)=\sum_{n=1}^{\infty} e^{-\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\theta} \cos\frac{u_n}{a}x\left\{K2a\frac{\sin u_n}{u_n}\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\int_0^{\theta} e^{\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\xi}\varphi(\xi)d\xi+C_n\right\}$$

We can determine C_n by using the intial condition

$$C_n=K\int_{-a}^{a} f(\lambda)\cos\frac{u_n}{a}\lambda\ d\lambda$$

therefore

$$t=\frac{1}{a}\sum_{n=1}^{\infty} e^{-\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\theta}\cos\frac{u_n}{a}x\frac{u_n}{u_n+\sin u_n\cos u_n}\left[\int_{-a}^{a} f(\lambda)\cos\frac{u_n}{a}\lambda\ d\lambda\right.$$

$$\left.+2a\frac{\sin u_n}{u_n}\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\int_0^{\theta} e^{\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\xi}\varphi(\xi)\ d\xi\right] \quad \cdots\cdots (30)$$

If the initial temperature distribution is t_0, putting $f(\lambda)=t_0$

$$t=2\sum_{n=1}^{\infty} e^{-\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\theta}\cos\frac{u_n}{a}x\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\left[t_0+\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\int_0^{\theta} e^{\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\xi}\varphi(\xi)d\xi\right] \quad \cdots\cdots (31)$$

For the temperature t_m of mid-plane, putting $x=0$

$$t_m=2\sum_{n=1}^{\infty} e^{-\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\theta}\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\left[t_0+\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\int_0^{\theta} e^{\alpha_{\perp}\left(\frac{u_n}{a}\right)^2\xi}\varphi(\xi)d\xi\right] \cdots\cdots (32)$$

Practically heating process is made so as $\varphi(\theta)$ increases, as shown in Figure 29, from t_1 to t_1' during θ' approximately linearly and then maintains t_1' till θ''. In this case $\varphi(\theta)$ is conveniently expressed by Fourier's Series in the range θ''

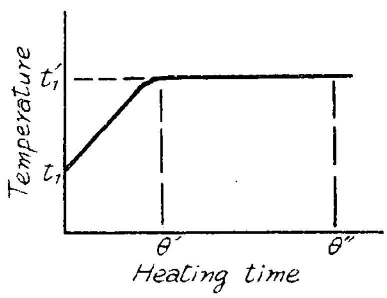

Fig. 29

$$\varphi(\theta)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{t_1'-t_1}{n}\left\{\frac{\theta''}{n\pi\theta'}\sin\frac{n\pi\theta'}{\theta''}+(-1)^{n-1}\right\}\sin\frac{n\pi\theta}{\theta''}+t_1 \cdots\cdots (33)$$

The solution is obtained by introducing this into the second term of equation (30)—(32).

Heating of wood by hot air is not usually treated except for drying. In this case the purpose is evaporation of moisture and so the above solution are not usually employed[33]. The heating by warm water, i. e. cooking, is often employed next to steaming in wood industry, but as stated in the foregoing chapters studies on this problem has hitherto scarcely made and according to the present author this process can be treated similarly as steaming and hotpressing. Therefore in the theory of the heat conduction in wood the process of steaming plays the most important role for practical application.

2. Heating by hotpressing or with steaming

a. Case where the temperature of hotpress or of steam is constant

From III—1, the following particular solutions are obtained.

$$t-t_1=\frac{1}{a}\sum_{n=1}^{\infty}e^{-\alpha\left(\frac{(2n-1)\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\int_{-a}^{a}\{f(\lambda)-t_1\}\cos\frac{(2n-1)\pi}{2a}\lambda\, d\lambda \cdots (25)$$

$$\frac{t_1-t}{t_1-t_0}=\frac{4}{\pi}\sum_{n=1}^{\infty}e^{-\alpha\left(\frac{(2n-1)\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\,\frac{(-1)^{n-1}}{2n-1}\cdots\cdots (26)$$

$$\frac{t_1-t_m}{t_1-t_0}=\frac{4}{\pi}\sum_{n=1}^{\infty}e^{-\alpha\left(\frac{(2n-1)\pi}{2a}\right)^2\theta}\frac{(-1)^{n-1}}{2n-1}\cdots\cdots (27)$$

$$\theta_{eq}=\frac{18.4\cdot a^2}{\alpha\pi^2}\cdots\cdots (28)$$

Solution (25)-(27) converge very fast and unless the temperature immediately after heating does matter the first term is sufficiently used. Figure 30 is a nomograph for practical calculation of the time required to reach the temperature of the mid-plane lower by 2.5, 5, 10, and 20°C than the external temperature with the use of the first term of solution (27), and Figure 31 is the nomograph for θ_{eq} by solution (28).

b. Case where the temperature of hotpress and steam varies with time

By introducing the condition for $h\to\infty$ into solution (30), (31), and (32)

$$t=\frac{1}{a}\sum_{n=1}^{\infty}e^{-\alpha\left(\frac{(2n-1)\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\left[\int_{-a}^{a}f(\lambda)\cos\frac{(2n-1)\pi}{2a}\lambda\, d\lambda\right.$$

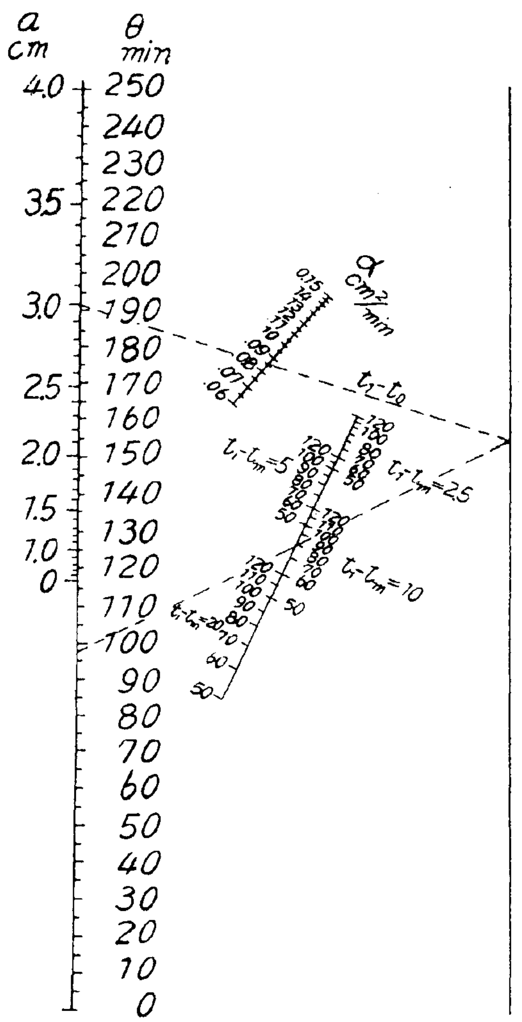

Fig. 30 Nomograph of $\frac{t_1-t_m}{t_1-t_0}=\frac{4}{\pi}e^{-\alpha\left(\frac{\pi}{2a}\right)^2\theta}$

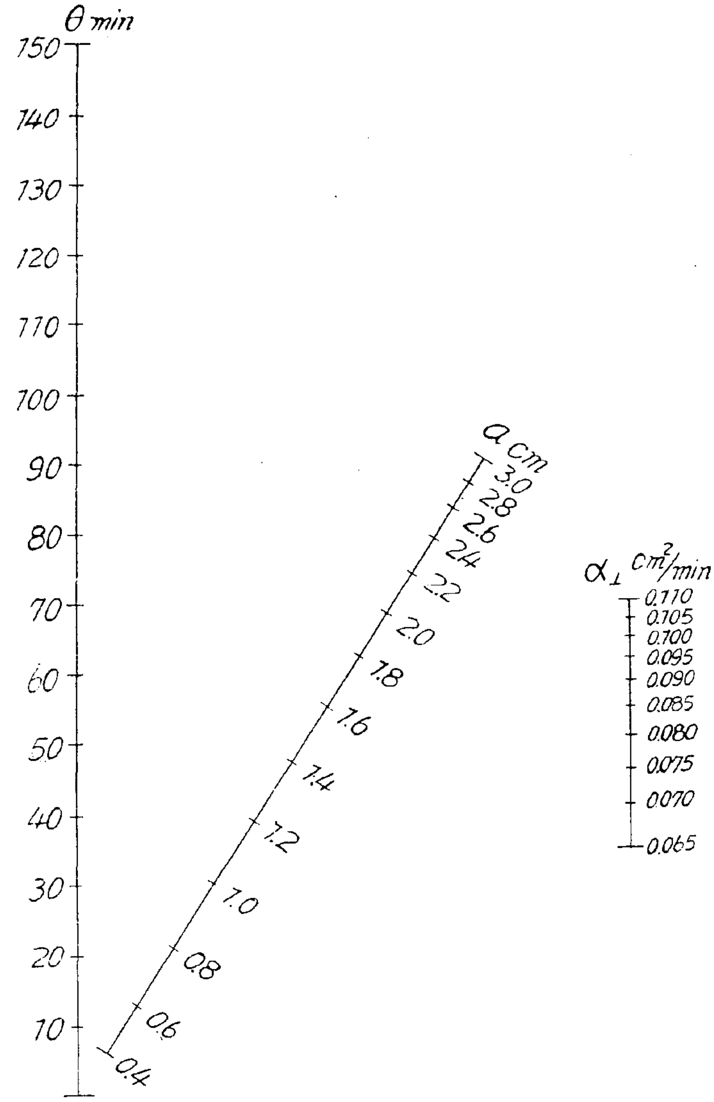

Fig. 31 Nomograph of $\theta_{eq}=\frac{18.4a^2}{\alpha\pi^2}$

$$+\left(\frac{4}{\pi}\right)a\frac{(-1)^{n-1}}{2n-1}\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\int^\theta e^{\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\xi}\varphi(\xi)\,d\xi\Big] \quad\cdots\cdots\cdots\cdots (34)$$

$$t=\frac{4}{\pi}\sum_{n=1}^{\infty}e^{-\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\theta}\cos\frac{(2n-1)\pi}{2a}x\frac{(-1)^{n-1}}{2n-1}$$

$$\times\left[t_0+\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\int_0^\theta e^{\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots\cdots\cdots (35)$$

$$t_m=\frac{4}{\pi}\sum_{n=1}^{\infty}e^{-\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\theta}\frac{(-1)^{n-1}}{2n-1}\left[t_0+\alpha_\perp\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\int_0^\theta e^{\alpha_T\left(\frac{\overline{2n-1}\pi}{2a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots(36)$$

V. The heat conduction in long balk timber[30,31]

1. Heating by hot water or by hot air

a. Case where the temperature of heating medium is constant

When a long balk timber with $2a \times 2b$ cross section is heated by a medium of temperature t_1, putting the initial temperature distribution as $f(x,y)$ (Figure 32)

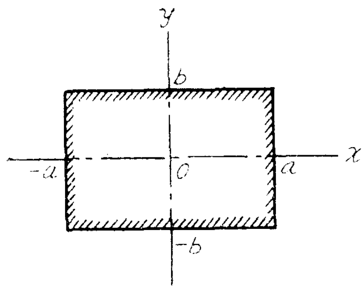

Fig. 32

$$\frac{\partial t}{\partial \theta} = \alpha_x \frac{\partial^2 t}{\partial x^2} + \alpha_y \frac{\partial^2 t}{\partial y^2} \quad \cdots\cdots (37)$$

$$\theta = 0 \quad : \quad t = f(x,y) \quad \cdots\cdots (38)$$

$$x = \pm a \ : \ \frac{\partial t}{\partial x} + h(t - t_1) = 0 \quad \cdots\cdots (39)$$

$$y = \pm b \ : \ \frac{\partial t}{\partial y} + h(t - t_1) = 0 \quad \cdots\cdots (40)$$

$$x = 0, \ y = 0 \ : \ \frac{\partial t}{\partial x} = 0, \ \frac{\partial t}{\partial y} = 0 \quad \cdots\cdots (41)$$

Transforming with $T = t - t_1$ so as to obtain the solutions which satisfy equation (37)–(41) in the similar way as in III–1

$$t - t_1 = \frac{1}{ab} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e^{-\left\{\alpha_x \left(\frac{u_n}{a}\right)^2 + \alpha_y \left(\frac{u_m}{b}\right)^2\right\}\theta} \cos\frac{u_n}{a}x \cos\frac{u_m}{b}y \frac{u_n}{u_n + \sin u_n \cos u_n} \frac{u_m}{u_m + \sin u_m \cos u_m}$$
$$\times \int_{-a}^{a} \int_{-b}^{b} \left\{f(\lambda,\mu) - t_1\right\} \cos\frac{u_n}{a}\lambda \cos\frac{u_m}{b}\mu \, d\lambda \, d\mu \quad \cdots\cdots (42)$$

where u_n and u_m are the nth and mth real roots of $\cot u = u/ha$ and $\cot u = u/hb$ respectively.

If the initial temperature distribution is t_0, putting $f(x,y) = t_0$

$$\frac{t_1 - t}{t_1 - t_0} = 4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e^{-\left\{\alpha_x \left(\frac{u_n}{a}\right)^2 + \alpha_y \left(\frac{u_m}{b}\right)^2\right\}\theta} \cos\frac{u_n}{a}x \cos\frac{u_m}{b}y \frac{\sin u_n}{u_n + \sin u_n \cos u_n} \frac{\sin u_m}{u_m + \sin u_m \cos u_m}$$
$$\cdots\cdots (43)$$

For temperature t_m of the central axis, let $x=0,\ y=0$

$$\frac{t_1-t_m}{t_1-t_0}=4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\left\{\alpha_x\left(\frac{u_n}{a}\right)^2+\alpha_y\left(\frac{u_m}{b}\right)^2\right\}\theta}\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\quad\cdots(44)$$

For square cross section, let $a=b$, from (43) and (44) respectively

$$\frac{t_1-t}{t_1-t_0}=4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_\perp\left(\frac{u_n^2+u_m^2}{a^2}\right)\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{a}y\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}$$
$$\cdots\cdots\cdots\cdots(45)$$

and

$$\frac{t_1-t_m}{t_1-t_0}=4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_\perp\left(\frac{u_n^2+u_m^2}{a^2}\right)\theta}\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\quad\cdots\cdots(46)$$

As obvious in these equations the actual calculations of formulas (43)—(46) are readily made as the product of $\frac{t_1-t}{t_1-t_0}$ obtained for x and y, respectively, from the Gurney-Lurie's diagram (Fig. 14). This is more simplified by the present author by employing a nomograph given in Fig. 33. For example, the temperature t at time θ on a point, 0.6a and 0.4b of the cross section is given from $\frac{t_1-t}{t_1-t_0}$ obtained by calculating $\alpha\frac{\theta}{a^2}$ and $\alpha\frac{\theta}{b^2}$, putting these values on the corresponding ha and hb on the nomograph, and moving to the reference line. The θ_{eq} of balk timber is obtained in the similar way as in plate, thus

$$\theta_{eq}=\frac{4.6}{\alpha_x\left(\frac{u_{1a}}{a}\right)^2+\alpha_y\left(\frac{u_{1b}}{b}\right)^2}\quad\cdots\cdots\cdots\cdots(47)$$

where u_{1a} and u_{1b} correspond to ha and hb.

For a square cross section, $a=b$, $ha=hb$, thus

$$\theta_{eq}=\frac{2.3a^2}{\alpha_\perp u_1^2}\quad\cdots\cdots\cdots\cdots(48)$$

b. Case where the temperature of heating medium varies with time

In this case

$$\frac{\partial t}{\partial\theta}=\alpha_\perp\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}\right)\quad\cdots\cdots\cdots\cdots(37)$$

$$\theta=0\ :\ t=f(x,y)\quad\cdots\cdots\cdots\cdots(38)$$

$$x=\mp a:\ \frac{\partial t}{\partial x}\mp h\left\{t-\varphi(\theta)\right\}=0\quad\cdots\cdots\cdots\cdots(49)$$

$$y=\mp b:\ \frac{\partial t}{\partial y}\mp h\left\{t-\varphi(\theta)\right\}=0\quad\cdots\cdots\cdots\cdots(50)$$

By Stoke's method, assuming the solution as

$$t(\theta,x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}(\theta)\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y$$

$$A_{nm}(\theta)=\frac{1}{ab}\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}\int_{-a}^{a}\int_{-b}^{b}t(\theta,\lambda,\mu)\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\,d\lambda\,d\mu$$

and replacing t with $\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}$ to obtain A_{nm}, as follows

$$A_{nm}=e^{-\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\theta}\left[K\,4ab\frac{\sin u_n}{u_n}\frac{\sin u_m}{u_m}\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2\right.\right.$$

$$\left.\left.+\left(\frac{u_m}{b}\right)^2\right\}\int_0^{\theta}e^{\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\xi}\varphi(\xi)d\xi+C_{nm}\right]$$

where

$$K=\frac{1}{ab}\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}$$

From the initial condition

$$C_{nm}=K\int_{-a}^{a}\int_{-b}^{b}f(\lambda,\mu)\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\,d\lambda\,d\mu$$

Using these

$$t=\frac{1}{ab}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}$$

$$\times\left[\int_{-a}^{a}\int_{-b}^{b}f(\lambda,\mu)\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\,d\lambda\,d\mu+4ab\frac{\sin u_n}{u_n}\frac{\sin u_m}{u_m}\right.$$

$$\left.\times\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\int_0^{\theta}e^{\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots\cdots\cdots(51)$$

When the initial temperature distribution is t_0

$$t=4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}$$

$$\times\left[t_0+\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\int_0^{\theta}e^{\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots\cdots\cdots(52)$$

The temperature t_m of central axis is

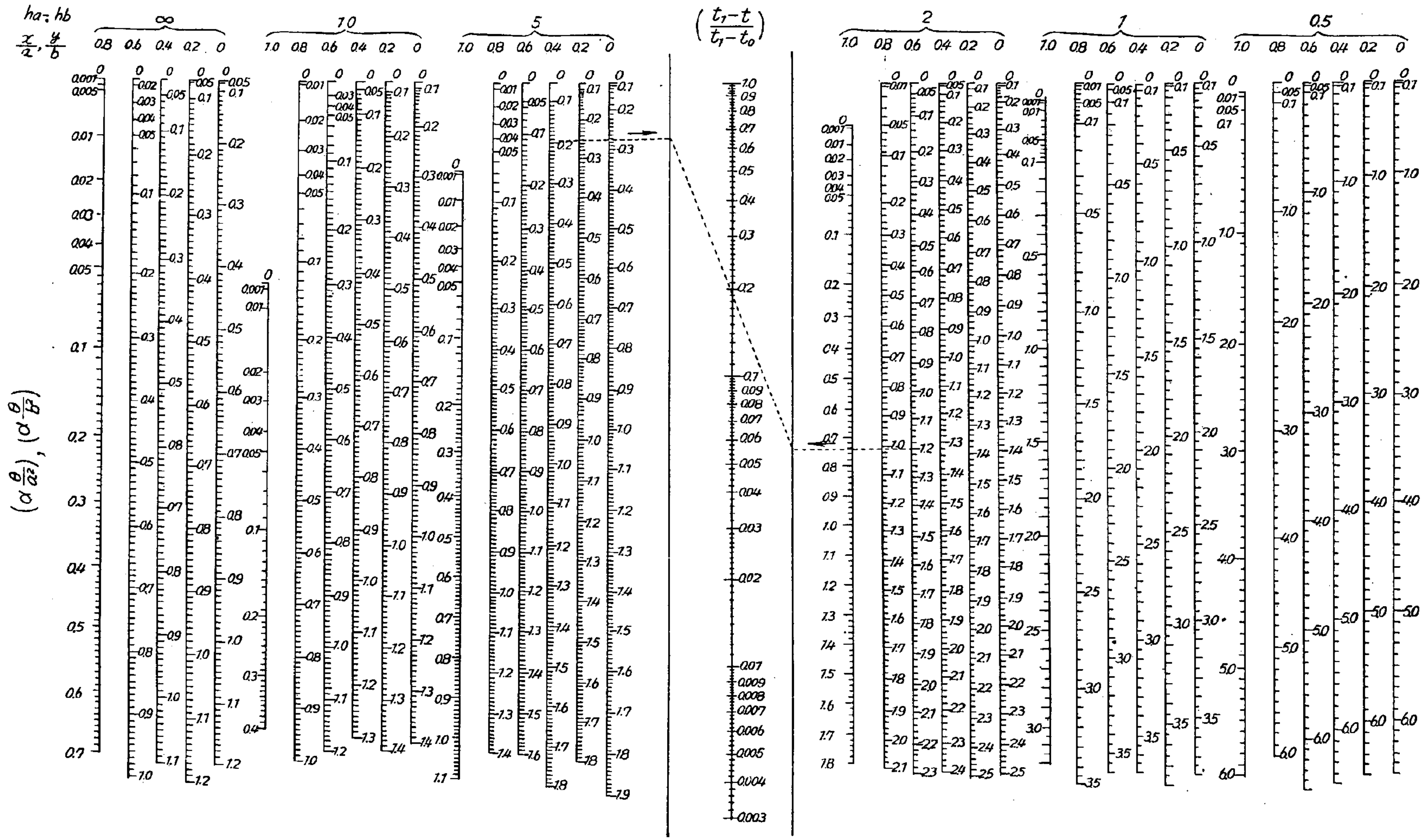

Fig. 33 Nomograph of eq. (43)

$$t_m = 4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e^{-\alpha_\perp \left\{ \left(\frac{u_n}{a} \right)^2 + \left(\frac{u_m}{b} \right)^2 \right\} \theta} \frac{\sin u_n}{u_n + \sin u_n \cos u_n} \frac{\sin u_m}{u_m + \sin u_m \cos u_m}$$

$$\times \left[t_0 + \alpha_\perp \left\{ \left(\frac{u_n}{a} \right)^2 + \left(\frac{u_m}{b} \right)^2 \right\} \int_0^\theta e^{\alpha_\perp \left\{ \left(\frac{u_n}{a} \right)^2 + \left(\frac{u_m}{b} \right)^2 \right\} \xi} \varphi(\xi)\, d\xi \right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots (53)$$

2. Heating with steaming

a. Case where the temperature of steaming is constant

By introducing into equation (42)—(44) conditions for $h \rightarrow \infty$, $u_n = (2n-1)\pi/2$ and $u_m = (2m-1)\pi/2$, the solutions are

$$t - t_1 = \frac{1}{ab} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e^{-\left\{ \alpha_x \left(\frac{\overline{2n-1}\pi}{2a} \right)^2 + \alpha_y \left(\frac{\overline{2m-1}\pi}{2b} \right)^2 \right\} \theta} \cos \frac{(2n-1)\pi}{2a} x \cos \frac{(2m-1)\pi}{2b} y$$

$$\times \int_{-a}^{a} \int_{-b}^{b} \left\{ f(\lambda, \mu) - t_1 \right\} \cos \frac{(2n-1)\pi}{2a} \lambda \cos \frac{(2m-1)\pi}{2b} \mu \, d\lambda \, d\mu \quad \cdots\cdots\cdots\cdots (54)$$

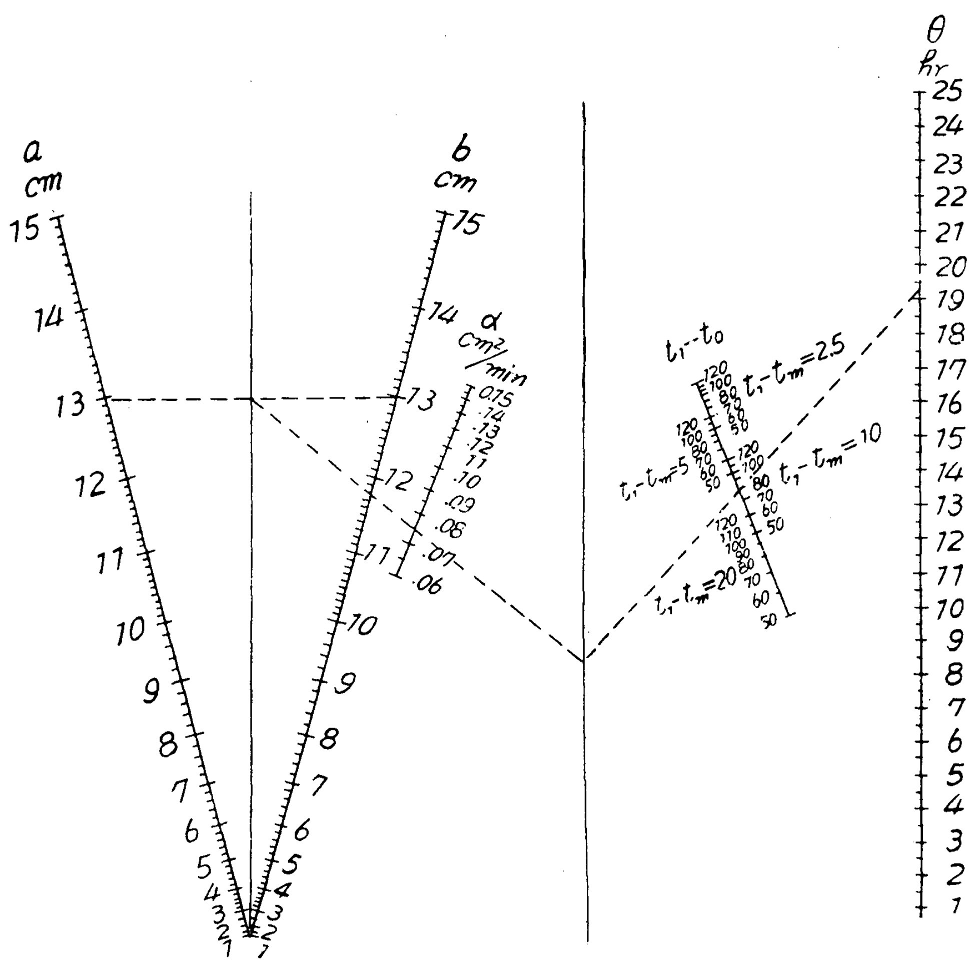

Fig. 34 Nomograph of $\frac{t_1 - t_m}{t_1 - t_0} = \left(\frac{4}{\pi} \right)^2 e^{-\alpha \left\{ \left(\frac{\pi}{2a} \right)^2 + \left(\frac{\pi}{2b} \right)^2 \right\} \theta}$

$$\frac{t_1-t}{t_1-t_0}=\left(\frac{4}{\pi}\right)^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\left\{\alpha_x\left(\frac{2n-1\pi}{2a}\right)^2+\alpha_y\left(\frac{2m-1\pi}{2b}\right)^2\right\}\theta}\cos\frac{(2n-1)\pi}{2a}x\cos\frac{(2m-1)\pi}{2b}y$$
$$\times\frac{(-1)^{n+m-2}}{(2n-1)(2m-1)}\cdots\cdots(55)$$

$$\frac{t_1-t_m}{t_1-t_0}=\left(\frac{4}{\pi}\right)^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-\left\{\alpha_x\left(\frac{2n-1\pi}{2a}\right)^2+\alpha_y\left(\frac{2m-1\pi}{2b}\right)^2\right\}\theta}\frac{(-1)^{n+m-2}}{(2n-1)(2m-1)}\cdots\cdots(56)$$

For actual calculation of equation (55), the scales for $ha=\infty$ at the left side of Fig. 33 is employed. θ_{eq} in this case is, by replacing u_1 in equation (47) by $\pi/2$

$$\theta_{eq}=\frac{1.86}{\alpha\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}\cdots\cdots(57)$$

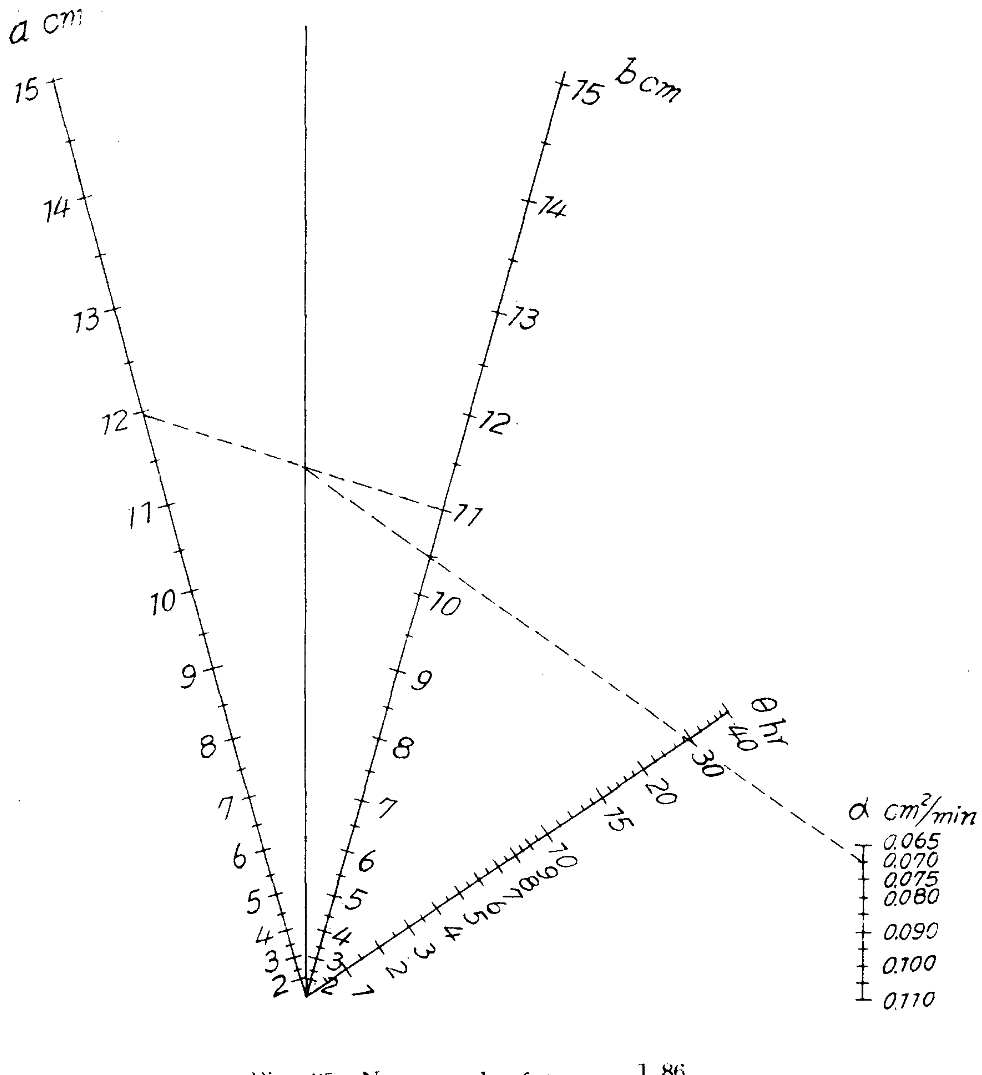

Fig. 35 Nomograph of $\theta_{eq}=\frac{1.86}{\alpha\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}$

Fig. 34 is the nomograph for the time required to reach $t_1 - t_m = 2.5$, 5, 10, and 20℃ at the central axis using the first term in equation (56), and Fig. 35 is that for equation (57).

b. Case where the temperature of steaming changes with time

Introducing the condition $h \to \infty$ in equation (51)—(53)

$$t=\frac{1}{ab}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\theta} \cos\frac{(2n-1)\pi}{2a}x \cos\frac{(2m-1)\pi}{2b}y$$
$$\times\left[\int_{-a}^{a}\int_{-b}^{b} f(\lambda,\mu)\cos\frac{(2n-1)\pi}{2a}\lambda \cos\frac{(2m-1)\pi}{2b}\mu \, d\lambda\, d\mu+\left(\frac{4}{\pi}\right)^2 ab\frac{(-1)^{n+m-2}}{(2n-1)(2m-1)}\right.$$
$$\left.\times\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\int_0^\theta e^{\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\xi}\varphi(\xi)\, d\xi\right] \cdots\cdots\cdots\cdots(58)$$

$$t=\left(\frac{4}{\pi}\right)^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\theta} \cos\frac{(2n-1)\pi}{2a}x \cos\frac{(2m-1)\pi}{2b}y\frac{(-1)^{n+m-2}}{(2n-1)(2m-1)}$$
$$\times\left[t_0+\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\int_0^\theta e^{\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\xi}\varphi(\xi)\, d\xi\right]\cdots\cdots\cdots(59)$$

$$t_m=\left(\frac{4}{\pi}\right)^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\theta}\frac{(-1)^{n+m-2}}{(2n-1)(2m-1)}$$
$$\times\left[t_0+\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\int_0^\theta e^{\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2+\left(\frac{2m-1\pi}{2b}\right)^2\right\}\xi}\varphi(\xi)\, d\xi\right] \cdots\cdots(60)$$

VI. The heat conduction in short balk timber[30),31)]

1. Heating by hot water or by hot air

a. Case where the temperature of the heating medium is constant

In this case the rapid heat conduction from the end surface can not be neglected. When a balk, $2a \cdot 2b \cdot 2c$, the initial temperature distribution being $f(x,y,z)$ is heated by a medium of temperature t_1, then

$$\begin{cases}\dfrac{\partial t}{\partial\theta}=a_x\dfrac{\partial^2 t}{\partial x^2}+a_y\dfrac{\partial^2 t}{\partial y^2}+a_z\dfrac{\partial^2 t}{\partial z^2} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)\\ \theta=0: \quad t=f(x,y,z) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(61)\\ x=\mp a: \quad \dfrac{\partial t}{\partial x}\mp h(t-t_1)=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(39)\\ y=\mp b: \quad \dfrac{\partial t}{\partial y}\mp h(t-t_1)=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(40)\\ z=\mp c: \quad \dfrac{\partial t}{\partial z}\mp h(t-t_1)=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(62)\\ x=0,\ y=0,\ z=0: \quad \dfrac{\partial t}{\partial x}=0,\ \dfrac{\partial t}{\partial y}=0,\ \dfrac{\partial t}{\partial z}=0 \cdots\cdots\cdots\cdots(63)\end{cases}$$

Thansforming by $T=t-t_1$ to obtain a particular solution which satisfy these equations in the similar way as in IV and V

$$t-t_1=\frac{1}{abc}\sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{u_n}{a}\right)^2+\alpha_y\left(\frac{u_m}{b}\right)^2+\alpha_z\left(\frac{u_p}{c}\right)^2\right\}\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\cos\frac{u_p}{c}z$$

$$\times\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}\frac{u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}\left\{f(\lambda,\mu,\nu)-t_1\right\}\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\cos\frac{u_p}{c}\nu\,d\lambda\,d\mu\,d\nu \quad\cdots\cdots(64)$$

where u_p is the pth real root of $\cot u=u/hc$, when the initial temperature distribution is t_0, let $f(\lambda,\mu,\nu)=t_0$,

$$\frac{t_1-t}{t_1-t_0}=8\sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{u_n}{a}\right)^2+\alpha_y\left(\frac{u_m}{b}\right)^2+\alpha_z\left(\frac{u_p}{c}\right)^2\right\}\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\cos\frac{u_p}{c}z$$

$$\times\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}\quad\cdots\cdots(65)$$

The temperature t_m of the center is, putting $x=0,\ y=0,\ z=0,$

$$\frac{t_1-t_m}{t_1-t_0}=8\sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{u_n}{a}\right)^2+\alpha_y\left(\frac{u_m}{b}\right)^2+\alpha_z\left(\frac{u_p}{c}\right)^2\right\}\theta}$$

$$\times\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}\quad\cdots\cdots(66)$$

The time θ_{eq} for the thermal equilibrium of the center is

$$\theta_{eq}=\frac{4.6}{\alpha_x\left(\frac{u_{1a}}{a}\right)^2+\alpha_y\left(\frac{u_{1b}}{b}\right)^2+\alpha_z\left(\frac{u_{1c}}{c}\right)^2}\quad\cdots\cdots(67)$$

where, u_{1a}, u_{1b}, u_{1c} correspond to ha, hb, hc respectively.

b. Case where the temperature of heating medium changes with time

We can write

$$\begin{cases}\dfrac{\partial t}{\partial\theta}=\alpha_\perp\left(\dfrac{\partial^2 t}{\partial x^2}+\dfrac{\partial^2 t}{\partial y^2}\right)+\alpha_{/\!/}\dfrac{\partial^2 t}{\partial z^2} & \cdots\cdots(1)\\ \theta=0:\quad t=f(x,y,z) & \cdots\cdots(61)\\ x=\mp a:\quad \dfrac{\partial t}{\partial x}\mp h\left\{t-\varphi(\theta)\right\}=0 & \cdots\cdots(49)\\ y=\mp b:\quad \dfrac{\partial t}{\partial y}\mp h\left\{t-\varphi(\theta)\right\}=0 & \cdots\cdots(50)\\ z=\mp c:\quad \dfrac{\partial t}{\partial z}\pm h\left\{t-\varphi(\theta)\right\}=0 & \cdots\cdots(68)\end{cases}$$

and by Stoke's method, assuming this solution as

$$\begin{cases} t(\theta,x,y,z)=\sum\limits_n\sum\limits_m\sum\limits_p A_{nmp}(\theta)\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\cos\frac{u_p}{c}z \\ A_{nmp}(\theta)=\frac{1}{abc}\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}\frac{u_p}{u_p+\sin u_p\cos u_p} \end{cases}$$

$$\times\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}t(\theta,\lambda,\mu,\nu)\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\cos\frac{u_p}{c}\nu\,d\lambda\,d\mu\,d\nu$$

replacing t with $\alpha_\perp\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}\right)+\alpha_{/\!/}\frac{\partial^2 t}{\partial z^2}$ as similar as in the foregoing chapter

$$t=\frac{1}{abc}\sum_n\sum_m\sum_p e^{-\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\cos\frac{u_p}{c}z$$

$$\times\frac{u_n}{u_n+\sin u_n\cos u_n}\frac{u_m}{u_m+\sin u_m\cos u_m}\frac{u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\left[\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}f(\lambda,\mu,\nu)\cos\frac{u_n}{a}\lambda\cos\frac{u_m}{b}\mu\cos\frac{u_p}{c}\nu\,d\lambda\,d\mu\,d\nu+8abc\frac{\sin u_n}{u_n}\frac{\sin u_m}{u_m}\frac{\sin u_p}{u_p}\right.$$

$$\left.\times\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\int_0^\theta e^{\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots\cdots(69)$$

When the initial temperature distribution is t_0

$$t=8\sum_n\sum_m\sum_p e^{-\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\theta}\cos\frac{u_n}{a}x\cos\frac{u_m}{b}y\cos\frac{u_p}{c}z$$

$$\times\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\left[t_0+\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\int_0^\theta e^{\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots(70)$$

The temperature t_m of the center is

$$t_m=8\sum_n\sum_m\sum_p e^{-\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\theta}$$

$$\times\frac{\sin u_n}{u_n+\sin u_n\cos u_n}\frac{\sin u_m}{u_m+\sin u_m\cos u_m}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\left[t_0+\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\int_0^\theta e^{\left(\alpha_\perp\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{/\!/}\left(\frac{u_p}{c}\right)^2\right)\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots(71)$$

2. Heating with steaming

a. Case where the temperature of steaming is constant.

We introduce the conditions for $h \to \infty$ i. e. $u_i = \frac{(2i-1)}{2}\pi$, $\sin u_i = (-1)^{i-1}$, and $\cos u_i = 0$ $(i = n, m, p)$ into equation (64) – (66), to obtain the solutions as follows

$$t - t_1 = \frac{1}{abc}\sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{2n-1\pi}{2a}\right)^2 + \alpha_y\left(\frac{2m-1\pi}{2b}\right)^2 + \alpha_z\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta}$$

$$\times \cos\frac{(2n-1)\pi}{2a}x \cos\frac{(2m-1)\pi}{2b}y \cos\frac{(2p-1)\pi}{2c}z$$

$$\times \int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}\left\{f(\lambda,\mu,\nu) - t_1\right\} \cos\frac{(2n-1)\pi}{2a}\lambda \cos\frac{(2m-1)\pi}{2b}\mu \cos\frac{(2p-1)\pi}{2c}\nu \, d\lambda \, d\mu \, d\nu \cdots (72)$$

$$\frac{t_1 - t}{t_1 - t_0} = \left(\frac{4}{\pi}\right)^3 \sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{2n-1\pi}{2a}\right)^2 + \alpha_y\left(\frac{2m-1\pi}{2b}\right)^2 + \alpha_z\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta}$$

$$\times \cos\frac{(2n-1)\pi}{2a}x \cos\frac{(2m-1)\pi}{2b}y \cos\frac{(2p-1)\pi}{2c}z \frac{(-1)^{n+m+p-3}}{(2n-1)(2m-1)(2p-1)} \cdots\cdots (73)$$

$$\frac{t_1 - t_m}{t_1 - t_0} = \left(\frac{4}{\pi}\right)^3 \sum_n\sum_m\sum_p e^{-\left\{\alpha_x\left(\frac{2n-1\pi}{2a}\right)^2 + \alpha_y\left(\frac{2m-1\pi}{2b}\right)^2 + \alpha_z\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta} \frac{(-1)^{n+m+p-3}}{(2n-1)(2m-1)(2p-1)} \cdots\cdots (74)$$

From equation (67)

$$\theta_{eq} = \frac{1.86}{\alpha_\perp\left(\frac{1}{a^2} + \frac{1}{b^2}\right) + \alpha_{//}\frac{1}{c^2}} \cdots\cdots (75)$$

b. Case where the temperature of steaming changes with time.

Similarly by using the conditions for $h \to \infty$, from equation (69)—(71)

$$t = \frac{1}{abc}\sum_n\sum_m\sum_p e^{-\left(\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2 + \left(\frac{2m-1\pi}{2b}\right)^2\right\} + \alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right)\theta}$$

$$\times \cos\frac{(2n-1)\pi}{2a}x \cos\frac{(2m-1)\pi}{2b}y \cos\frac{(2p-1)\pi}{2c}z$$

$$\times \Bigg[\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c} f(\lambda,\mu,\nu) \cos\frac{(2n-1)\pi}{2a}\lambda \cos\frac{(2m-1)\pi}{2b}\mu \cos\frac{(2p-1)\pi}{2c}\nu \, d\lambda \, d\mu \, d\nu$$

$$+ \left(\frac{4}{\pi}\right)^3 abc \frac{(-1)^{n+m+p-3}}{(2n-1)(2m-1)(2p-1)} \left(\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2 + \left(\frac{2m-1\pi}{2b}\right)^2\right\} + \alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right)$$

$$\times \int_0^\theta e^{\left(\alpha_\perp\left\{\left(\frac{2n-1\pi}{2a}\right)^2 + \left(\frac{2m-1\pi}{2b}\right)^2\right\} + \alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right)\xi} \varphi(\xi) \, d\xi\Bigg] \cdots\cdots (76)$$

$$t=\left(\frac{4}{\pi}\right)^3\sum_n\sum_m\sum_p e^{-\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\theta}$$

$$\times\cos\frac{\overline{2n-1}\pi}{2a}x\ \cos\frac{\overline{2m-1}\pi}{2b}y\ \cos\frac{\overline{2p-1}\pi}{2c}z\ \frac{(-1)^{n+m+p-3}}{(2n-1)(2m-1)(2p-1)}$$

$$\times\left[t_0+\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\right.$$

$$\left.\times\int_0^{\theta}e^{\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\xi}\varphi(\xi)\,d\xi\right]\quad\cdots\cdots(77)$$

$$t_m=\left(\frac{4}{\pi}\right)^3\sum_n\sum_m\sum_p e^{-\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\theta}$$

$$\times\frac{(-1)^{n+m+p-3}}{(2n-1)(2m-1)(2p-1)}\left[t_0+\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\right.$$

$$\left.\times\int_0^{\theta}e^{\left(\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right)\xi}\varphi(\xi)\,d\xi\right]\quad\cdots\cdots(78)$$

It is considerably convenient for actual calculation to rewrite by using $k=\alpha_{//}/\alpha_{\perp}$ which is obtained in III-4 as follows

$$\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2\right\}+\alpha_{//}\left(\frac{u_p}{c}\right)^2=\alpha_{\perp}\left\{\left(\frac{u_n}{a}\right)^2+\left(\frac{u_m}{b}\right)^2+k\left(\frac{u_p}{c}\right)^2\right\}$$

$$\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2\right\}+\alpha_{//}\left(\frac{\overline{2p-1}\pi}{2c}\right)^2$$

$$=\alpha_{\perp}\left\{\left(\frac{\overline{2n-1}\pi}{2a}\right)^2+\left(\frac{\overline{2m-1}\pi}{2b}\right)^2+k\left(\frac{\overline{2p-1}\pi}{2c}\right)^2\right\}$$

VII. The heat condution of long round timber

1. Heating by hot water or by hot air

a. Case where the temperature of heating medium is constant

When a round timer, with radius a and very large length for the radius, is heated with a heating medium of temperature t_1, the heat conduction in a longitudinal direction can be neglected, and so putting the initial temperature distribution as $f(r)$.

$$\begin{cases}\dfrac{\partial t}{\partial\theta}=\alpha_{\perp}\left(\dfrac{\partial^2 t}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial t}{\partial r}\right) & \cdots\cdots(79)\\ \theta=0:\quad t=f(r) & \cdots\cdots(80)\\ r=a:\quad \dfrac{\partial t}{\partial r}+h(t-t_1)=0 & \cdots\cdots(81)\end{cases}$$

Transforming by $T = t - t_1$, the solution for equation (79)—(81) is

$$t - t_1 = \frac{2}{a^2}\sum_{m=1}^{\infty} e^{-\alpha_1\left(\frac{\lambda_m}{a}\right)^2\theta} J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}\int_0^a \{f(\gamma)-t_1\} J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\, d\gamma \cdots\cdots(82)$$

where λ_m represents the m th positive root of $xJ_1(x) - haJ_0(x) = 0$ and is given in Table 20 for various ha.

Table 20 Roots of eq. $xJ_1(x) - haJ_0(x) = 0$

ha	λ_1	λ_2	λ_3	λ_4	λ_5
0	0.000	3.832	7.016	10.174	13.324
0.001	0.045	3.832	7.016	10.174	13.324
0.002	0.063	3.832	7.016	10.174	13.324
0.005	0.100	3.833	7.016	10.174	13.324
0.01	0.141	3.834	7.017	10.175	13.324
0, 02	0.200	3.837	7.019	10.176	13.325
0.05	0.314	3.845	7.023	10.178	13.327
0.1	0.442	3.858	7.030	10.183	13.331
0.2	0.617	3.884	7.044	10.193	13.338
0.5	0.941	3.959	7.086	10.222	13.361
1	1.256	4.079	7.156	10.271	13.398
2	1.599	4.292	7.288	10.366	13.472
5	1.990	4.713	7.617	10.622	13.679
10	2.180	5.034	7.957	10.936	13.959
20	2.288	5.257	8.253	11.268	14.296
50	2.357	5.411	8.484	11.562	14.643
∞	2.405	5.520	8.653	11.792	14.931

If the temperature distribution of the timber is uniform, t_0, then let $f(\gamma) = t_0$

$$\frac{t_1 - t}{t_1 - t_0} = 2\sum_{m=1}^{\infty} e^{-\alpha_1\left(\frac{\lambda_m}{a}\right)^2\theta} J_0\left(\frac{\lambda_m}{a}r\right)\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}} \cdots\cdots\cdots(83)$$

The temperature t_m of the central axis is, putting $r = 0$

$$\frac{t_1 - t_m}{t_1 - t_0} = 2\sum_{m=1}^{\infty} e^{-\alpha_1\left(\frac{\lambda_m}{a}\right)^2\theta} \frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(84)$$

The forms of equation (82)—(84) are well known widely and Figure 36 is Gurney-Lurie diagram for equation (83).

The time θ_{eq} is, putting $e^{-\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2\theta}=0.01$

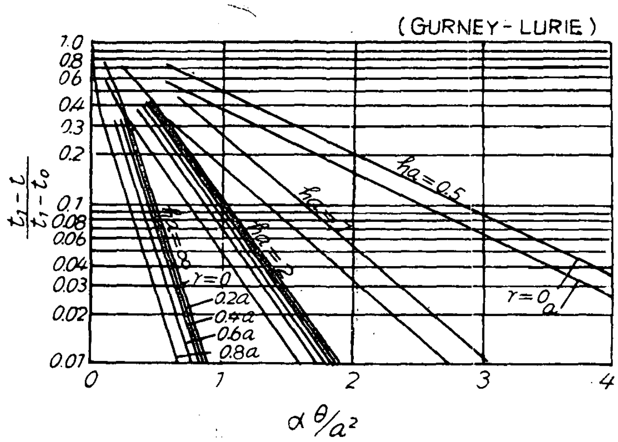

Fig. 36 Diagram of eq. (83)

$$\theta_{eq}=\frac{4.6a^2}{\alpha_\perp\lambda_1^2}\cdots\cdots(85)$$

b. Case where the temperature of heating medium changes with time

$$\begin{cases}\frac{\partial t}{\partial\theta}=\alpha_\perp\left(\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}\right) & \cdots\cdots(79)\\ \theta=0:\quad t=f(r) & \cdots\cdots(80)\\ r=0:\quad \frac{\partial t}{\partial r}+h\{t-\varphi(\theta)\}=0 & \cdots\cdots(86)\end{cases}$$

This solution is obtained by putting

$$\begin{cases}t(\theta,r)=\sum_{m=1}^{\infty}A_m J_0\left(\frac{\lambda_m}{a}r\right)\\ A_m=\frac{2}{a^2}\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}\int_0^a t(\theta,\gamma)\,J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\,d\gamma\end{cases}$$

and by replacing t by $\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}$

$$t=\frac{2}{a^2}\sum_{m=1}^{\infty}e^{-\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2\theta}J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}$$

$$\times\left[\int_0^a f(\gamma)\,J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\,d\gamma+a^2\frac{J_1(\lambda_m)}{\lambda_m}\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2\int_0^\theta e^{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots(87)$$

when the initial temperature distribution is t_0

$$t=2\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}J_0\left(\frac{\lambda_m}{a}r\right)\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}$$

$$\times\left[t_0+\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\int_0^{\theta}e^{\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots(88)$$

the temperature t_m of the central axis is

$$t_m=2\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}\left[t_0+\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\int_0^{\theta}e^{\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots(89)$$

2 Heating with steaming

a. Case where the temperature of steaming is constant

In the similar manner as in the previous chapter $J_0(x)=0$ must be satisfied in $xJ_1(x)-haJ_0(x)=0$ for $ha\to\infty$ and by using this results equations (82)—(84) become

$$t-t_1=\frac{2}{a^2}\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{J_1^2(\lambda_m)}\int_0^a\{f(\gamma)-t_1\}J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\,d\gamma\cdots\cdots(90)$$

$$\frac{t_1-t}{t_1-t_0}=2\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{\lambda_mJ_1^2(\lambda_m)}\cdots\cdots(91)$$

$$\frac{t_1-t_m}{t_1-t_0}=2\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}\frac{1}{\lambda_mJ_1^2(\lambda_m)}\cdots\cdots(92)$$

θ_{eq} for this case is from equation (85) by the use of $\lambda_1=2.405$ from Table 20

$$\theta_{eq}=\frac{0.795a^2}{\alpha_{\perp}}\cdots\cdots(93)$$

Figure 37 is, similarly as in the foregoing chapter, the nomograph for the time required to reach $t_1-t_m=2.5, 5, 10$, and 20°C at the central axis in round timber using the first term of equation (92), and Figure 38 is the one for equation (93).

b. Case where the temperature of steaming changes with time

Into equation (87)-(89) the condition $J_0(x)=0$ for $h\to\infty$ is introduced, and thus

$$t=\frac{2}{a^2}\sum_{m=1}^{\infty}e^{-\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\theta}J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{J_1^2(\lambda_m)}\left[\int_0^a f(\gamma)\,J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\,d\gamma\right.$$

$$\left.+a^2\frac{J_1(\lambda_m)}{\lambda_m}\alpha_{\perp}\left(\frac{\lambda_m^2}{a}\right)\int_0^{\theta}e^{\alpha_{\perp}\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right]\cdots\cdots(94)$$

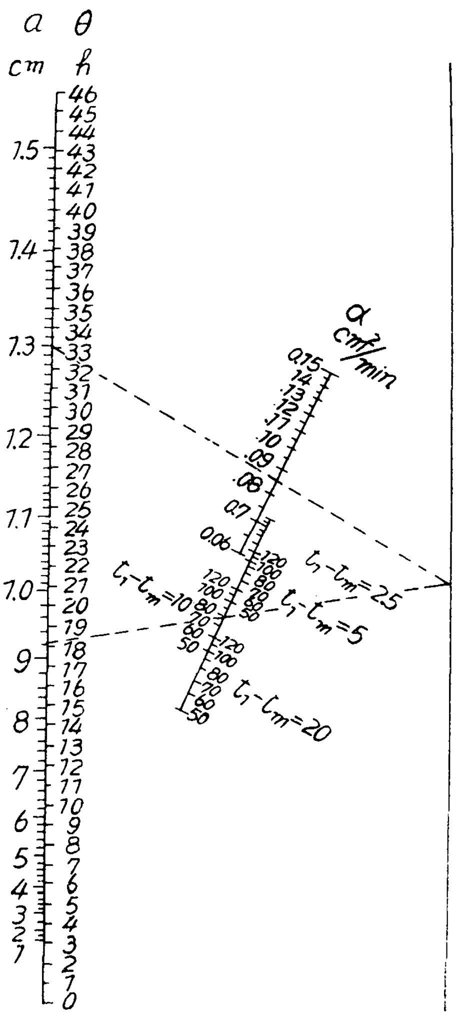

Fig. 37 Nomograph of $\frac{t_1 - t_m}{t_1 - t_0} = 2e^{-\alpha\left(\frac{\lambda_1}{a}\right)^2\theta} \frac{1}{\lambda_1 J_1^2(\lambda_1)}$

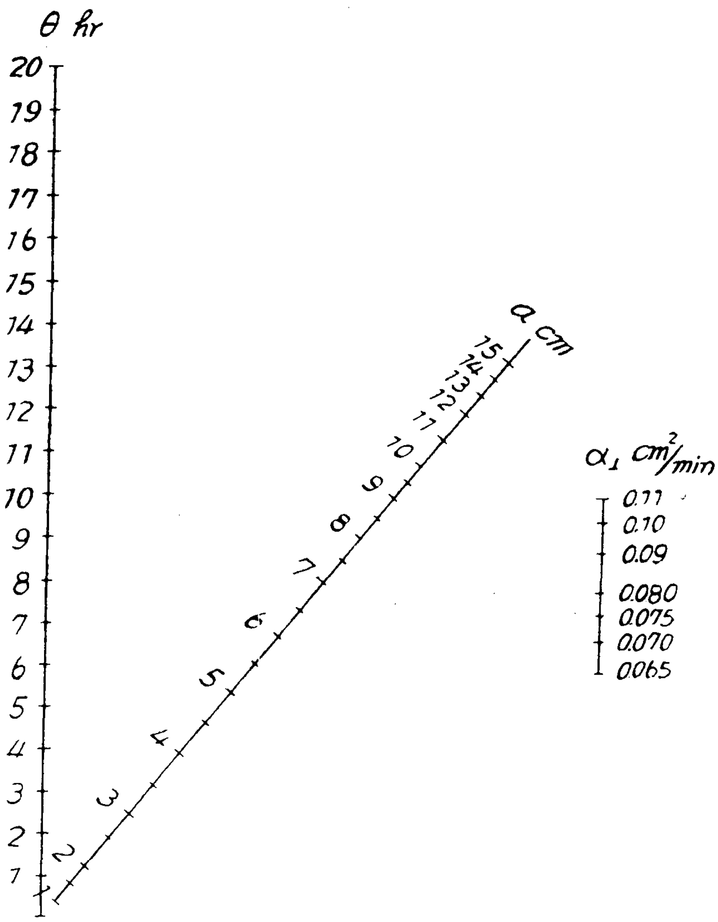

Fig. 38 Nomograph of $\theta_{eq} = \frac{0.795a^2}{\alpha}$

$$t = 2\sum_m e^{-\alpha_1\left(\frac{\lambda_m}{a}\right)^2\theta} J_0\left(\frac{\lambda_m}{a}r\right)\frac{1}{\lambda_m J_1(\lambda_m)}\left[t_0 + \alpha_1\left(\frac{\lambda_m}{a}\right)^2\int_0^\theta e^{\alpha_1\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right] \cdots (95)$$

$$t_m = 2\sum_m e^{-\alpha_1\left(\frac{\lambda_m}{a}\right)^2\theta} \frac{1}{\lambda_m J_1(\lambda_m)}\left[t_0 + \alpha_1\left(\frac{\lambda_m}{a}\right)^2\int_0^\theta e^{\alpha_1\left(\frac{\lambda_m}{a}\right)^2\xi}\varphi(\xi)\,d\xi\right] \cdots\cdots\cdots\cdots (96)$$

VIII. The heat conduction in short round timber

1. Heating by hot water or by hot air

a. Case where the temperature of heating medium is constant

When a timber has a short length the longitudinal heat conduction from the end surface can not be neglected and the following equations are obtained when the timber whose initial temperature distribution is $f(r, z)$ is heated by a heating medium having a constant temperature t_1

$$\begin{cases} \dfrac{\partial t}{\partial \theta}=a_\perp\left(\dfrac{\partial^2 t}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial t}{\partial r}\right)+a_{//}\dfrac{\partial^2 t}{\partial z^2} & \cdots\cdots(97) \\ \theta=0 \ : \ t=f(r,z) & \cdots\cdots(98) \\ r=a \ : \ \dfrac{\partial t}{\partial r}+h(t-t_1)=0 & \cdots\cdots(81) \\ z=\mp c: \ \dfrac{\partial t}{\partial z}\mp h(t-t_1)=0 & \cdots\cdots(62) \end{cases}$$

The solution satisfying these equations is obtained by transforming by $T=t-t_1$, simiarly

$$t-t_1=\frac{2}{a^2c}\sum_m\sum_p e^{-\left\{a_\perp\left(\frac{\lambda_m}{a}\right)^2+a_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{u_p}{c}z\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}\frac{u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\int_0^a\int_{-c}^c\left\{f(\gamma,\nu)-t_1\right\}J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\cos\frac{u_p}{c}\nu\ d\gamma\ d\nu \quad\cdots\cdots(99)$$

In which λ_m is the m th positive root of $xJ_1(x)-ha\,J_0(x)=0$ and u_p p th real root of $\cot u=u/hc$.

When the initial temperature distribution is t_0, putting $f(\gamma,\nu)=t_0$

$$\frac{t_1-t}{t_1-t_0}=4\sum_m\sum_p e^{-\left\{a_\perp\left(\frac{\lambda_m}{a}\right)^2+a_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{u_p}{c}z\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}$$

$$\cdots\cdots(100)$$

For the temperature t_m of the center, putting $r=0$, $z=0$

$$\frac{t_1-t_m}{t_1-t_0}=4\sum_m\sum_p e^{-\left\{a_\perp\left(\frac{\lambda_m}{a}\right)^2+a_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta}\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}\cdots\cdots(101)$$

The time θ_{eq} for this case is, putting $e^{-\left\{a_\perp\left(\frac{\lambda_m}{a}\right)^2+a_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta}=0.01$

$$\theta_{eq}=\frac{4.6}{a_\perp\left(\frac{\lambda_1}{a}\right)^2+a_{//}\left(\frac{u_1}{c}\right)^2}\cdots\cdots(102)$$

b. Case where the temperature of heating medium changes with time

$$\begin{cases} \dfrac{\partial t}{\partial \theta}=a_\perp\left(\dfrac{\partial^2 t}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial t}{\partial r}\right)+a_{//}\dfrac{\partial^2 t}{\partial z^2} & \cdots\cdots(97) \\ \theta=0 \ : \ t=f(r,z) & \cdots\cdots(98) \\ r=a \ : \ \dfrac{\partial t}{\partial r}+h\left\{t-\varphi(\theta)\right\}=0 & \cdots\cdots(86) \\ z=\mp c: \ \dfrac{\partial t}{\partial z}\mp h\left\{t-\varphi(\theta)\right\}=0 & \cdots\cdots(68) \end{cases}$$

By the Stoke's method, assuming the solution as

$$\begin{cases} t(\theta,r,z)=\sum_m\sum_p A_{mp} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{u_p}{c}z \\ A_{mp}=\frac{2}{a^2c}\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}\frac{u_p}{u_p+\sin u_p\cos u_p}\int_0^a\int_{-c}^{c} t(\theta,\gamma,\nu)J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\cos\frac{u_p}{c}\nu\, d\gamma\, d\nu \end{cases}$$

and then replacing t with $\alpha_\perp\left(\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}\right)+\alpha_{//}\frac{\partial^2 t}{\partial z^2}$ as in the previous cases

$$t=\frac{2}{a^2c}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{u_p}{c}z\frac{1}{J_0^2(\lambda_m)+J_1^2(\lambda_m)}\frac{u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\left[\int_0^a\int_{-c}^{c} t(\theta,\gamma,\nu)J_0\left(\frac{\lambda_m}{a}\gamma\right)\cos\frac{u_p}{c}\nu\, d\gamma\, d\nu\right.$$

$$\left.+\frac{2a^2cJ_1(\lambda_m)\sin u_p}{\lambda_m u_p}\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right] \cdots\cdots\cdots(103)$$

If the initial temperture distribution t_0 is uniform

$$t=4\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{u_p}{c}z\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}\frac{\sin u_p}{u_p+\sin u_p\ \mathrm{con}\ u_p}$$

$$\times\left[t_0+\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots(104)$$

the temperature of the center is

$$t_m=4\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\theta}\frac{J_1(\lambda_m)}{\lambda_m\{J_0^2(\lambda_m)+J_1^2(\lambda_m)\}}\frac{\sin u_p}{u_p+\sin u_p\cos u_p}$$

$$\times\left[t_0+\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right] \cdots\cdots\cdots\cdots\cdots\cdots(105)$$

2. Heating with steaming

a. Case where the tempeature of steaming is constant

By introducing into equation (99)—(101) the conditions for $h\to\infty$ i. e. $J_0(x)=0$, $u_p=\frac{2p-1}{2}$, $\sin u_p=(-1)^{p-1}$ and $\cos u_p=0$

$$t-t_1=\frac{2}{a^2c}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{(2p-1)\pi}{2c}z\frac{1}{J_1^2(\lambda_m)}$$

$$\times\int_0^a\int_{-c}^{c}\{f(\gamma,\nu)-t_1\}J_0\left(\frac{\lambda_m}{a}\gamma\right)\gamma\cos\frac{(2p-1)\pi}{2c}\nu\, d\gamma\, d\nu \cdots\cdots\cdots\cdots\cdots\cdots\cdots(106)$$

$$\frac{t_1-t}{t_1-t_0}=\frac{8}{\pi}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{(2p-1)\pi}{2c}z\frac{1}{\lambda_m J_1(\lambda_m)}\frac{(-1)^{p-1}}{2p-1} \quad \cdots\cdots\cdots\cdots(107)$$

$$\frac{t_1-t_m}{t_1-t_0}=\frac{8}{\pi}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\pi}{2c}\right)^2\right\}\theta}\frac{1}{\lambda_m}\frac{(-1)^{p-1}}{J_1(\lambda_m)} \quad \cdots\cdots\cdots\cdots(108)$$

in which λ_m is the mth positive root of $J_0(x)=0$. θ_{eq} is obtained by putting $\lambda_1=2.405$ and $u_1=\frac{\pi}{2}$ into equation (102).

b. Case where temperature of steaming changes with time

Introducing the conditions for $h\to\infty$ into equation (103)–(105), the solutions are

$$t=\frac{2}{a^2c}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{2p-1\ \pi}{2c}z\frac{1}{J_1^2(\lambda_m)}$$

$$\times\left[\int_0^a\int_{-c}^c f(\eta,\nu)J_0\left(\frac{\lambda_m}{a}\eta\right)\eta\cos\frac{2p-1\ \pi}{2c}\nu\, d\eta\, d\nu\right.$$

$$\left.+\frac{4}{\pi}a^2c\frac{J_1(\lambda_m)}{\lambda_m}\frac{(-1)^{p-1}}{2p-1}\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\xi}\varphi(\xi)d\xi\right] \quad \cdots\cdots\cdots\cdots(109)$$

$$t=\frac{8}{\pi}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\theta} J_0\left(\frac{\lambda_m}{a}r\right)\cos\frac{2p-1\ \pi}{2c}z\frac{1}{J_1^2(\lambda_m)}\frac{(-1)^{p-1}}{2p-1}$$

$$\times\left[t_0+\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right] \quad \cdots\cdots(110)$$

$$t_m=\frac{8}{\pi}\sum_m\sum_p e^{-\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\theta}\frac{1}{J_1^2(\lambda_m)}\frac{(-1)^{p-1}}{(2p-1)}$$

$$\times\left[t_0+\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\int_0^\theta e^{\left\{\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}\xi}\varphi(\xi)\,d\xi\right] \quad \cdots\cdots(111)$$

In the same manner as in VI, putting $\alpha_{//}=k\alpha_\perp$ in the above equations so as to rewrite coefficient as follows for convenience of practical calculation

$$\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{u_p}{c}\right)^2=\alpha_\perp\left\{\left(\frac{\lambda_m}{a}\right)^2+k\left(\frac{u_p}{c}\right)^2\right\}$$

$$\alpha_\perp\left(\frac{\lambda_m}{a}\right)^2+\alpha_{//}\left(\frac{2p-1\ \pi}{2c}\right)^2=\alpha_\perp\left\{\left(\frac{\lambda_m}{a}\right)^2+k\left(\frac{2p-1\ \pi}{2c}\right)^2\right\}$$

IX. Application to hotpressed plywood[30]

In I-VIII, no flow of heat is considered except the heating surface, and there discussions have been made on the common case where the heat thus conducted is entirely used to raise the temperature of wood. In manufacturing hotpressed plywood there come other facters which influence on the heat conduction. For example, high water resistant plywood employs phenol resin, resorcinol resin, or melamine resin so that the temperature of hot plates becomes often 130–150°C, and when a number of thin veneer are inserted between hot plates, even below 100°C some parts of heat radiate from the sides of the assembly or is consumed in the form of latent heat of evaporation or consumed by a kind of thermal resistance caused by microscopically incomplete contact between many of glue lines and veneers. These factors work to give anomalous process of temperature rise in veneer assembly. Therefore in these cases the general basic differential equation for the heat conduction should has some additional terms, but these are influenced by a great number of factors so that even if the theoretical solutions are obtained their actual calculation is difficult and it seems convenient for practical purpose to use the theoretical solutions with modification by determining experimentally appropriate coefficients. The present chapter contains a few considerations for these particular cases.

1. Lumber-core plywood

In lumber-core plywood the percentage of glue lines to plywood thickness is comparatively small, and so its influence can be neglected and this case is treated quite similarly as that of wood plate concerning to the heat conduction. Thus as shown in Fig. 15 in oven dried wood experimental result gives a good agreement with the theoretical solution above 100°C as well as below 100°C, and even in wet wood gives fairly a good agreement if the heating temperature is below 100°C. An example is given in Fig. 39, which indicates that whether moisture-proofing treatment is made on the side-surfaces of plate or not the process of temperature rise is almost similar ; the curve in the figure represents of the result obtained by calculation taking $a_{\perp}=0.0865$ for $u=0.245$ from Fig. 21.

However, in the case where wood contains moisture and the heating temperature is above 100°C the theoretical solution from equation (27) shows a good agreement with the experimental values only before the temperature of the mid-plane reaches nearly 100°C as shown in Fig. 40 and after that the latter shows remarkably low temperature-rising so that it deviates from the normal process. This is obviously due to the fact that above

100°C the larger parts of heat supplied are consumed as latent heat and also a part radiates from the side-surface. In this case the theoretical solution will, as mentioned

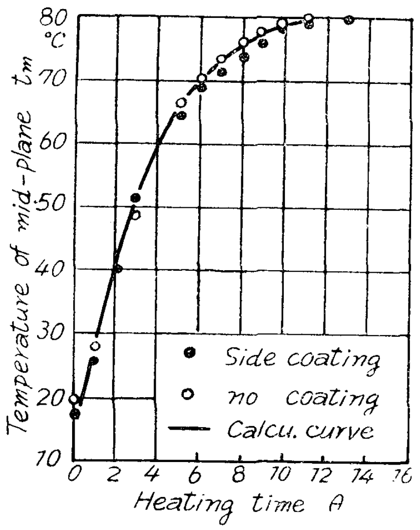

Fig. 39 Change of temperature of wet wood (*Cryptomeria*)

	initial moisture content	end 〃 〃	Thick-ness
Coating	25, 4%	23, 6	1, 5cm
no 〃	25, 9	22, 9	1, 47

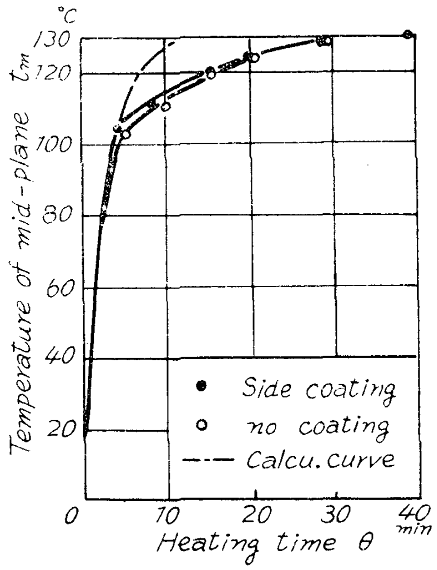

Fig. 40 Change of temperature of wet wood when heated at 130°C (*Cryptomeria*), thickness 1, 4—1, 45cm.

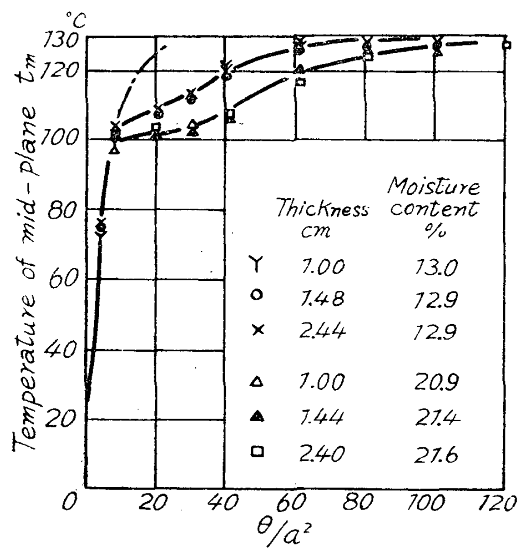

Fig. 41 Change of temperature of wet wood (*Fagus*)

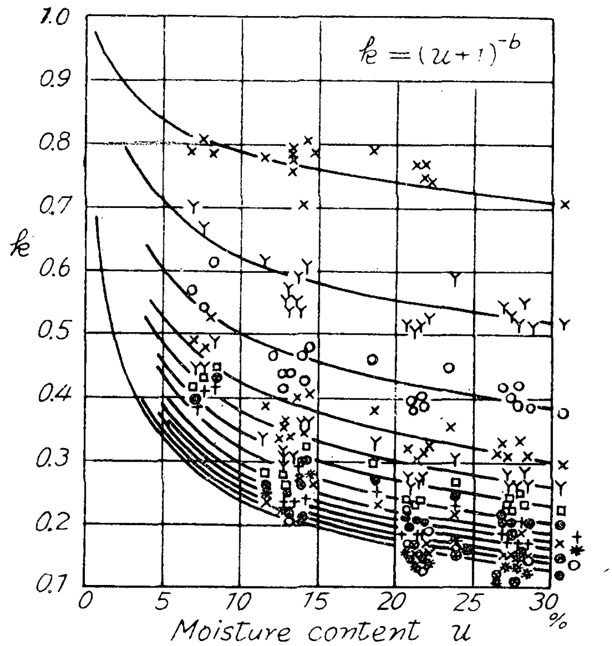

Fig. 42 Relation between *k* and the moisture content

previously, do nothing in actual utilization at all. Fig. 41 represents a part of results of the similar experiments with plate having the same initial moisture content and different thickness, and from this it is obvious that if the inital moisture content is same the temperature t_m of the mid-plane shows nearly the same value for the same θ/a^2 independently on the thickness. In other words, the heat conduction in this case is similar to theoretical solution and its characteristic is not lost. Therefore the present author has tried to obtain the formula determining k experimentally by employing the following experimental formula

$$\frac{t_1 - t_m}{t_1 - t_0} = \frac{4}{\pi} \sum_n e^{-k a_\perp \left(\frac{2n-1}{2a}\pi\right)^2 \theta} \frac{(-1)^{n-1}}{2n-1} \quad \cdots\cdots\cdots\cdots\cdots(112)$$

as similar equation of formula (27) for the temperature of the mid-plane, k is an dimensionless number which is to be regaded as a kind of resistant coefficient of heat transfer, and is determined by the initial moisture content of plate and θ/a^2. For the application of formula (112) a diagram must be obtained, which represents $\frac{t_1 - t_m}{t_1 - t_0}$ and $ka\frac{\theta}{a^2}$ in a quite similar way as the Gurney-Lurie diagram ; it can be obtained by converting the abscissa of the Gurney-Lurie diagram into $ka\frac{\theta}{a^2}$. First, plates having same moisture content and different thickness are heated by hot plates with a constant temperature to estimate t_m and then to determine the relations between θ/a^2 and ka, i. e. k. The relation of θ/a^2

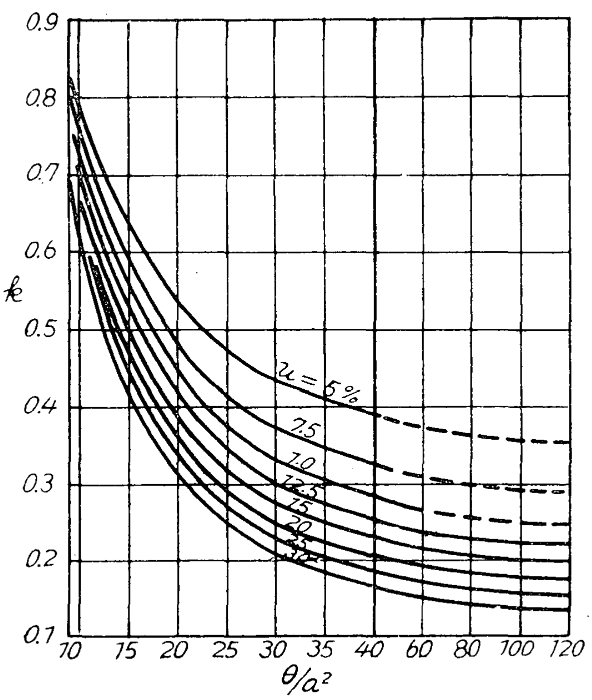

Fig. 43 Relation of k to θ/a^2
For lumber-core plywood
u : initial average moisture content of veneer and core.

Table 21 The values of k at various initial moisture content of wood
(*Chamaecyparis* TAIWANHINOKI, $r_0=0.36$, ${}_{20}\alpha_0=0.109$)

	Initial moisture content 27.4 %						〃 27.2					
	Tkickness 1.0 cm						〃 1.48					
	Initial temperature of wood 23.2°C						〃 20.0					
$\frac{\theta}{a^2}$	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$k\alpha\cdot\frac{\theta}{a^2}$	$k\alpha$	k	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$k\alpha\cdot\frac{\theta}{a^2}$	$k\alpha$	k
0	23.2						20.0					
8												
12	105.0	25.0	0.234	0.690	0.0575	0.527	104.7	25.3	0.230	0.698	0.0581	0.533
16	105.0	25.0	0.234	0.690	0.0431	0.395	104.7	25.3	0.230	0.698	0.0436	0.400
20	105.0	25.0	0.234	0.690	0.0345	0.316	104.7	25.3	0.230	0.698	0.0349	0.320
24	105.0	25.0	0.234	0.690	0.0288	0.264	104.7	25.3	0.230	0.698	0.0291	0.267
28	105.0	25.0	0.234	0.690	0.0246	0.226	104.7	25.3	0.230	0.698	0.0249	0.228
32	105.5	24.5	0.229	0.700	0.0219	0.201	104.9	25.1	0.228	0.700	0.0219	0.201
36	106.0	24.0	0.225	0.705	0.0196	0.180	105.2	24.8	0.225	0.705	0.0196	0.180
40	106.5	23.5	0.220	0.715	0.0179	0.164	106.0	24.0	0.218	0.720	0.0180	0.165
60	117.3	12.7	0.119	0.965	0.0161	0.148	117.1	12.9	0.117	0.970	0.0162	0.149
80	128.0	2.0	0.0187	1.715	0.0214	0.256	127.6	2.4	0.0218	1.650	0.0206	0.189
100												

	〃 28.3					
	〃 2.44					
	〃 18.5					
$\frac{\theta}{a^2}$	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$k\alpha\cdot\frac{\theta}{a^2}$	$k\alpha$	k
0	18.5					
8						
12	103.3	26.7	0.239	0.681	0.0567	0.520
16	103.5	26.5	0.238	0.682	0.0426	0.391
20	103.8	26.2	0.235	0.690	0.0345	0.316
24	103.8	26.2	0.235	0.690	0.0288	0.264
28	103.8	26.2	0.235	0.690	0.0246	0.226
32	103.8	26.2	0.235	0.690	0.0216	0.198
36	104.0	26.0	0.233	0.691	0.0192	0.176
40	104.4	25.6	0.230	0.698	0.0175	0.160
60	114.7	15.3	0.137	0.910	0.0152	0.139
80	125.2	4.8	0.0430	1.350	0.0169	0.155
100	128.7	1.3	0.0116	1.910	0.0191	0.175

	〃 7.4						〃 14.4					
	〃 1.0						〃 1.5					
	〃 22.5						〃 20.0					
$\frac{\theta}{a^2}$	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$k\alpha\cdot\frac{\theta}{a^2}$	$k\alpha$	k	t_m	t_i-t_m	$\frac{t_i-t_m}{t_i-t_0}$	$k\alpha\cdot\frac{\theta}{a^2}$	$k\alpha$	k
0	22.5						20.0					
8	105.8	24.2	0.225	0.705	0.0881	0.801	99.1	30.9	0.281	0.615	0.0769	0.705
12	114.0	16.0	0.149	0.875	0.0729	0.669	105.3	24.7	0.224	0.710	0.0591	0.542
16	116.7	13.3	0.124	0.950	0.0594	0.545	107.3	22.7	0.206	0.740	0.0463	0.425
20	120.0	10.0	0.0930	1.065	0.0533	0.489	108.5	21.5	0.195	0.765	0.0383	0.351
24	122.5	7.5	0.0698	1.180	0.0492	0.451	111.0	19.0	0.173	0.815	0.0340	0.312
28	124.7	5.3	0.0493	1.320	0.04	0.432	113.9	16.1	0.146	0.885	0.0315	0.289
32	126.5	3.5	0.0326	1.490	0.0465	0.427	117.2	12.8	0.116	0.975	0.0305	0.280
36	127.7	2.3	0.0214	1.670	0.0464	0.426	120.5	9.5	0.0864	1.095	0.0274	0.251
40							122.5	7.5	0.0682	1.190	0.0298	0.273
60							128.9	1.1	0.0100	1.967	0.0328	0.301
80												
100												

and k is previously determined by following the same process for various initial moisture contents (Table 21). Then, the temperature t_m after heating for θ minutes is obtained by calculating θ/a^2 and k corresponding to the initial moisture content from the above relation and by seeking $\frac{t_1-t_m}{t_1-t_0}$ by ka, i. e. $ka\ \theta/a^2$ from the diagram. Any standard may be available for the value of a_L and the present author employs one given in Fig. 21.

Fig. 42 shows the relation of θ/a^2, u, and k obtained from the above experiments on some species and on some moisture contents. From the fact that $k=1$ when $u=0$, this relation is represented by the following formula

$$k=(u+1)^{-b}$$

The constant b is 0.102, 0.193, 0.285, 0.354, 0.402, 0.441, 0.468, 0.494, 0.520, 0.550, 0.565, 0.572, and 0.577 for $\theta/a^2=8$, 12, 16, 20, 24, 28, 32, 36, 40, 60, 80, 100, and 120 respectively. For the sake of convenience the relation of k and θ/a^2 of Eig. 42 is represented in the form of a graph, in Fig. 43.

Illustration

What is the temperature of the mid-plane of the lumber-core plywood of ***Fagus*** (BUNA), 2 cm thick, after hotpressing for 12 minutes at 130°C ? The average initial moisture content of plywood assembly is 18% and the initial temperature is 18°C.

The thermal diffusivity of plywood is from Fig. 21 $a_u=0.091$ and the time required for the mid-plane of the plywood to reach 100°C is $\frac{t_1-t_m}{t_1-t_0}=\frac{130-100}{130-18}=0.268$. From the diagram $a_u\ \frac{\theta}{a^2}=0.63$ and $\theta=0.63\cdot 1^2/0.091\fallingdotseq 6.9$ min. Then in Fig. 43, the value of k for $\theta/a^2=12/1^2=12$ is obtained from the line for 18% as $k=0.57$, therefore $ka\ \frac{\theta}{a^2}=0.57\cdot 0.109\cdot 12=0.745$ (The standard value of a is 0.109 from Fig. 21). Thus again from the diagram, $\frac{t_1-t_m}{t_1-t_0}=0.2$, $t_1=130$, and $t_0=18$ so that it is obtained $t_m=107.6$°C.

Fig. 44 shows the heating time required for a plate, 2 cm thick ($2a=2$), which is hotpessed at 130, 140, and 150°C, to reach 120°C at its depth from the surface $0.4a$ ($x=0.6a$), $0.6a$ ($x=0.4a$), $0.8a$ ($x=0.2a$), and at the mid-plane ($x=0$) for various moisture contents. It can do with a plywood of a given thickness by recalculating these standard values in proportion to ratio of square of the thickness. For example, the time θ_1 required to reach 120°C at the position 6 mm deep ($x=0.6a$) from the surface of plywood, which is 3 cm thick and contains average moisture content 10 %, is obtained by seeking the time $\theta_0=7.3$ for the thickness 2 cm, $x=0.6a$ from Fig. 44 and then by calculating from $7.3/\theta_1=1^2/1.5^2$, $\theta_1=16.5$ minutes.

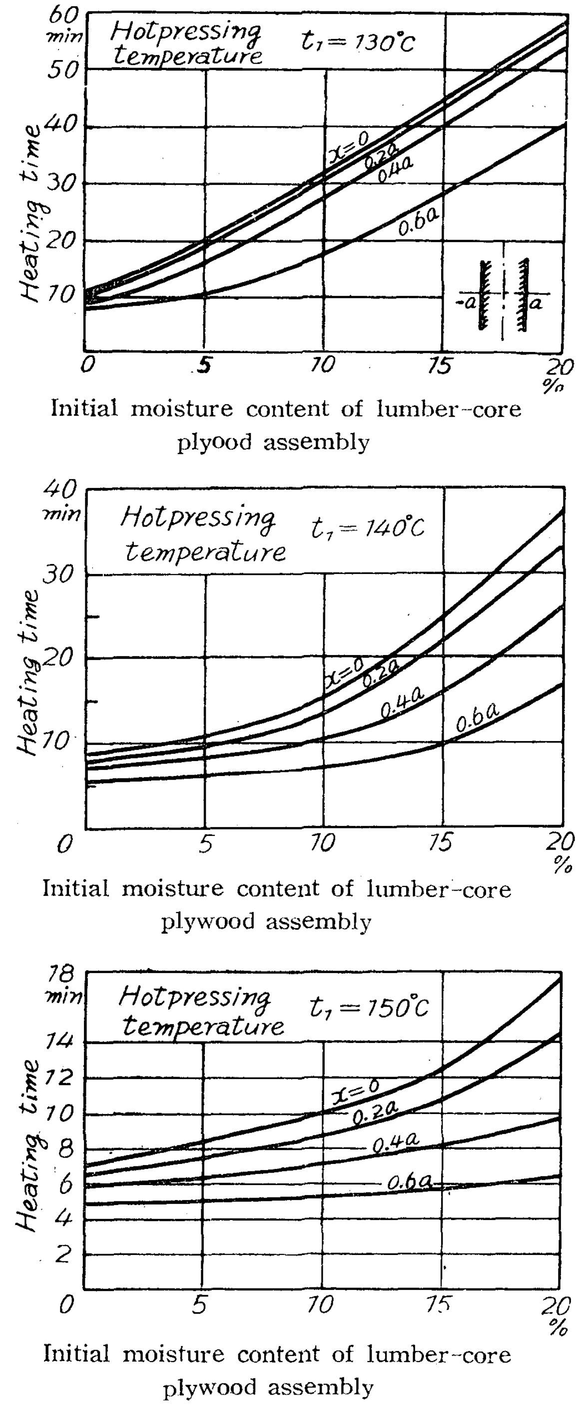

Fig. 44 Standerd diagram of time required to reach 120°C at each line $x=0$, $0, 2a$, $0, 4a$ and $0, 6a$ at total thickness of plywood 2cm ($a=1$).
Initial wood temperature 20°C.

2. All-veneer plywood

As described previously when a large number of glue lines exist in plywood the temperature there differs to some degree from that in the case of the foregoing article due to a kind of thermal resistance. Fig. 45 shows an example of such a case, which indicates the relation of θ/a^2, u, and k of the plywood composed of 1 mm thick *Betula* (KABA) and 1.5 mm thick *Chamaecyparis* (HINOKI) veneer, phenol-resin film glue, which is hotpressed at 140°C, by measuring t_m, t_m', and t_m'' respectively for the depth $x=0$, $x=a/3$, and $x=2a/3$, in the similar way as in 1. The values of the coefficient b in $k=(u+1)^{-b}$ are shown in Table 22.

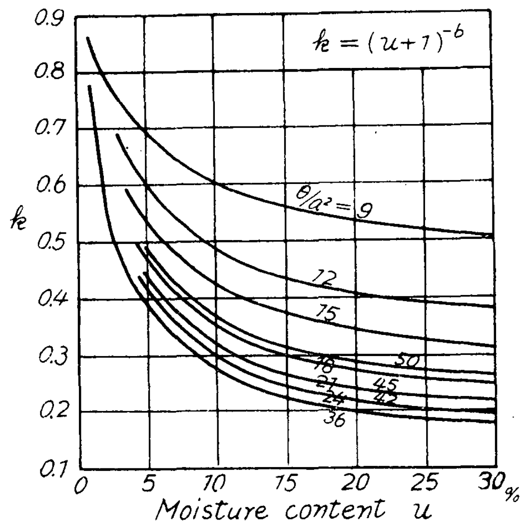

Fig. 45 Relation between k and moisture content. For t_m ($x=0$)

Table 22 The values of b of $(u+1)^{-b}$

θ/a^2	for t_m ($x=0$)	for t_m' ($x=a/3$)	for t_m'' ($x=2a/3$)
6	--	--	0.268
9	0.210	0.239	0.346
12	0.289	0.319	0.382
15	0.349	0.387	0.422
18	0.410	0.433	0.449
21	0.465	0.466	0.470
24	0.492	0.495	0.498
36	0.522	0.520	0.520
42	0.490	0.494	0.503
45	0.464	0.461	0.461
50	0.403	0.439	0.449

In Fig. 46 the relation of k and θ/a^2 is shown for t_m, t_m', and t_m'' for the sake of practical convenience, and calculation by this method is made in quite the same manner as in 1. It should be remarkable in this figure that when $\theta/a^2 < 24$ the values of k differ according to their position in the plywood. This indicates that the temperature distribution is not similar to the theoretical one at the early period of heating so that it becomes more flat in the vicinity of the mid-plane.

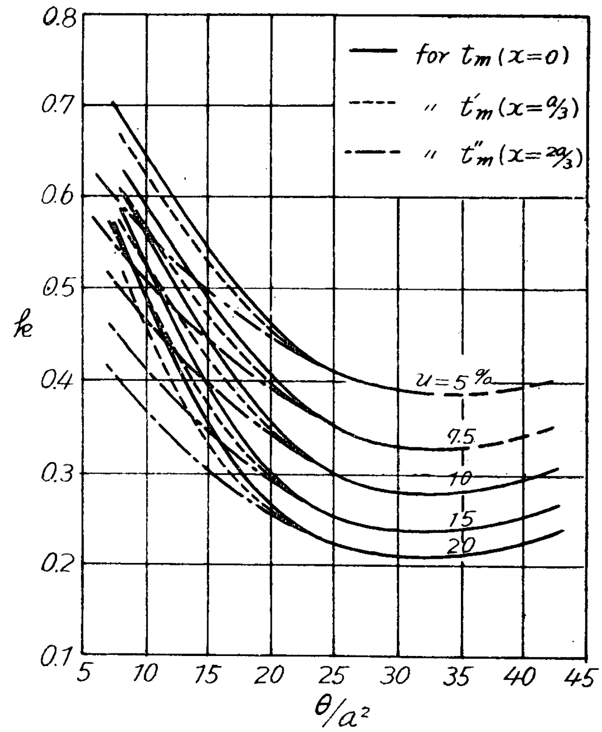

Fig. 46 Relation of k to θ/a^2
For all-veneer plywood
u : initial moisture content of veneer.

In the next place comes the case of the temperature rise in a plywood bonded with soyabeen glue and hotpressed at 100°C, which gives a considerable agreement with the

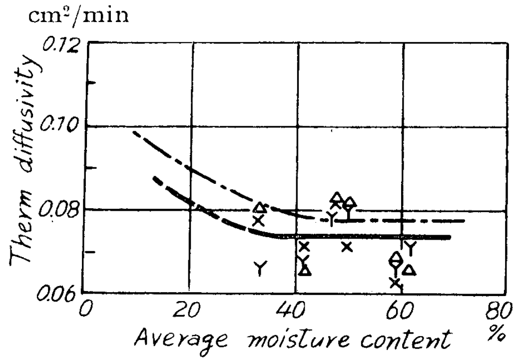

Fig. 47 Relation of $\alpha_\perp$ to the average moisture content of all-veneer plywood (Hotpressing)

theoretical solution. Calculating α from the measured temperature, its relation with the average initial moisture content is shown in Fig. 47, in which the chain line is given for comparison with the curve in Fig. 21. Being obvious therefrom, the appearent thermal diffusivity in this case is somewhat lower than that in plates or in lumber-core plywood so that the temperature rise is slower, the difference, however, being not so remarkable.

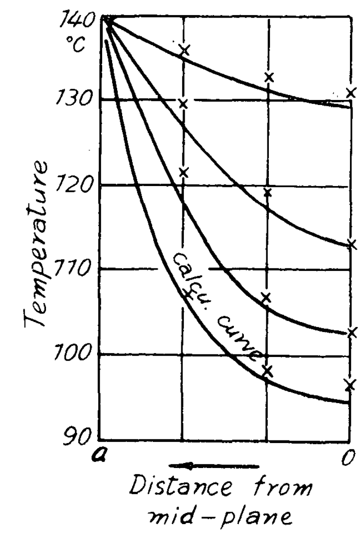

Fig. 48 Temperature gradient in Tego bonded all-veneer plywood (Hotpressing)

One result is shown in Fig. 48, on the calculated values and the corresponding measured values when film-glue is employed.

Summary

In the present study, discussions have been made on the availability of the differential equation of heat conduction in wood particularly on the inflnence of various factors on the thermal diffusivity, and on some representative cases the solutions or the practical method of calculation are given. Finally applications to hotpressed plywood are discussed.

1. Wood is a kind of heterogeneous, anisotropic, and porous material composed of various kinds of cells, but as a industrial material it can be regarded to be homogeneous at least along one direction and the internal flow of heat is done chiefly by conduction. Thus the fundamental equation (1) and (2) is applicable on approximate basis to plate, balk and round timber.

2. The thermal diffusivity obtained from calculation according to numbers of experimental formula previously reported varies to some extent depending on the density and temperature of wood but it can be regarded to be constant for the practical purpose. By

moisture, however, it greatly influenced, particularly in low moisture range, so that it can not be a constant when the moisture content of wood is changed considerably during heating.

3. The values of the thermal diffusivity obtained directly from the measurement of the internal temperature by the use of the theoretical solution (26) or (27) show similar tendency for various factors described above to the calculated ones: (a) in the direction perpendicular to fiber the experimented values agree fairly well with the calculated ones, and (b) in the fiber direction the experimented ones differ markedly from the calculated ones in low moisture content range and there is also considerable difference between conifers and broad-leaved trees (cf. Fig. 25). Therefore employment of the calculated value of α concerning to the heat conduction in the fiber direction produces considerable errors.

4. In the range which is represented by broken line in Fig. 25, the thermal diffusivity is changed during steaming so that the theoretical solution can not be employed.

5. In chapter IV-VIII, the solutions were obtained on a plate, long and short balk timber, round timber, and nomographs were given for particularly important cases in practise.

6. When a plywood is hotpressed at above 100°C the internal temperature shows anomalous manner of rising. For this case the coefficient k was obtained experimentally by the use of similar formula to solution (26) or (27) on lumber-core plywood and all-veneer plywood, and a simplified method of temperature calculation was devised therefrom. When many thin veneers are inserted between hot plates and hotpressed at below 100°C appearent thermal diffusivity of them give somewhat lower than that of wood plate or lumber-core plywood (Fig. 47).

Literature cited

1) Bittner, J.: Furniere, Sperrholz, Schichtholz Teil 1 (1939).
2) Bosch, M. T.: "Wärmeübertragung" (1936).
3) Cammerer, J. S.: Holz als Roh- u. Werkstoff Jg. 1, H. 6 (1938).
4) Carslaw, H. S.: "Introduction to the Mathematical Theory of Conduction of Heat in Solids", (1921).
5) Delmonte, J.: "Technology of Adhesives" (1947).
6) Dunlap, F.: U. S. Dept. Agric. Bull. No. 110, Washington (1912).
7) ——: J. Agric. Res. Bd. 2 (1914).
8) Egner, K.: Forsch. Ber. Holz H. 2 (1934).
9) Forsaith, C. C.: Technology of New York State-timber (1926).

10) Griffiths, E. & Kaye, G. W. C.: Proc. Roy. Soc. Lond. Ser. A Bd. 104 (1923).
11) Gurney, H. P. & Lurie, L.: Ind. Eng. Chem. **15**, (1923).
12) Hartig, R.: Untersuchg. Forstbot. Inst. München Bd. 2 (1882).
13) Hofmeister, W.: Flora Jg. 45, 7 (1862).
14) Huber, B. & Drütx, G.: Holz als Roh- u. Werkstoff Jg. **1**, H. 10 (1938).
15) Hunt, G. M.: Amer. Wood-Preserver's Assoc. Proc. 11 (1915), 19-22 (1923-26), 20 (1924), 22 (1926).
16) Johnston, H. W. & MaaB, O.: For. Prod. Lab. Canad. Res. Notes 11 (1929).
17) KANEHIRA, R.: Bull. Gov. Res. Inst. TAIHOKU, FORMOSA 4 (1926).
18) KAWASITA, K.: "NETU DENDO RON" Jap. (1944).
19) KODAIRA, Y.: "BUTURI SUGAKU" Bd. 2 Jap. (1933).
20) KODAMA, M.: Jour, Soc. Mech. Eng. Jap. **36**, 192 (1933).
21) Koehler, A.: "The Properties and Uses of Wood" (1924).
22) Kollmann, F.: "Technologie des Holzes" (1936).
23) ―――――― : Holz als Roh- u. Werkstoff Jg. **2**, H. 1 (1939).
24) ―――――― : "Technologie des Holzes u. der Holzwerkstoffe" Bd. 1 (1951).
25) MacLean, J. D.: Amer. Wood- Preserer's Assoc. Proc. 23 (1927), 24 (1928), 26 (1930), 28 (1932).
26) ―――――― : U. S. Dept. Agric. Wisc. Pub. No. 224 (1935).
27) MAKU, T.: "Wood Industry" **4**, 6 (1949).
28) ―――――― : This Bull. this No.
29) ―――――― : "Wood Research" **9**, (1952).
30) ―――――― : 〃 **3**, (1949).
31) ―――――― : 〃 **8**, (1952).
32) ―――――― : 〃 **6**, (1951).
33) ―――――― : 〃 **3**, (1949).
34) ―――――― : Trans. 59 th Meet. Jap. For. Soc. Jap. (1951).
35) MATANO, T.: "ŌYO BUTURI" 11 (1942).
36) MATUMOTO, B.: Jour. Jap. For. Soc. **20**, 5 (1938).
37) Mörath, E.: Mitt. Holzforsch.-stelle an der Tech. Hochschule, Darmstadt H. 1 (1932).
38) Mull, W. u. H. Reicher : Beih. 28, Reihe 1 zum Gesundh.-Ing. (1930).
39) NAGASAWA, T.: Jour. Jap. For. Soc. **22**, 9 (1940).
40) Narayanamurti, D. & Ranganathan, V.: Proc. Ind. Acad. Sci. Bd. **13**, No. 4 Ser. A (1941).
41) Nuselt, W.: V. D. I. Bd. **52**, (1908).
42) ŌGA, S.: "DENNETU SYORON TO SO NO ŌYO" (1931).
43) OGURA, T.: Jour. Jap. For. Soc. **22**, 11 (1940).
44) OGURA, IKEDA : Jour. Jap. For. Soc. **25**, 3 (1942).
45) Prioform-Handbuch 2 Aufl. 1 (1930).
46) Perry, T.: "Modern Plywood" (1948).
47) Rowley, F. B.: Heat. Pip. Air Condit. Bd. 5 (1933).
48) Sachs, J.: Arb. Bot. Inst. Würzburg Bd. 2 (1879).
49) Schwappach, A.: Untersuch. über des Raumgewicht (1897).

50) SEKIYA, F.: "MOKUZAI KŌGEI GAKU" Jap. (1933).
51) Stamm, A. J.: Tech. Bull. No. 929 U. S. Dept. Agr. Wash. D. C. (1946).
52) TERASAWA, KOTAIRA.: "ŌYO SUGAKU" Jap. (1928).
53) Tiemann, H. D.: "Wood Technology" (1951).
54) Voigt, H., O. Krischer u. H. Schauss: Holz als Roh- u. Werkstoffs Jg. **3**, H. 10 (1940).
55) WAKASUGI, M.: "HOONZAI" Jap. (1951).
56) Wangaard, F. F.: Heat, Pip. Air Condit. Bd. 12 (1940).
57) Watson, G. N.: "Theory of Bessel Functions" (1922).
58) Wirka, R. M.: Amer. Wood-Preserver's Assoc. Proc, 20 (1924).
59) YAMABAYASI, S.: Bull. For. Exp. Stat. Gov. Gener. CHOSEN. 7 (1928).