Studies on the drying of wood

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Introduction

In drying of wood many equations have been developed on the moisture movement through wood. Among them, hitherto the diffusion equations 1),7),13),15) or the differential equations 4),10),21),27),29) similar to the heat conduction equation had been used, and recently yet many other equations 10),35) are established.

In the range of high moisture content, the movement of free water is mainly due to the capillary force, nevertheless the diffusion equation by the moisture gradient or the differential equation are used. In hygroscopic range besides the above mentioned equations, Voigl 37) explained it by the capillary movement in cell membrane near the fiber saturation point and by the diffusion equation similar to Stefan’s law, Stamm 25) considering it as a diffusion and an electric stream in wood structure, calculated the drying diffusion constant and Kroll 10) explained it as a capillary movement in cell membrane near the fiber saturation point and Knudsen’s molecular stream in pit membrane pore in the lower moisture content.

It may be distinctly an error to apply the differential equation in high moisture range, but it comes into problem in hygroscopic range. According to Martley 17), Ludwig 11), and Egner 39), the diffusion coefficient varies with the moisture content, so that the coefficient of differential equation i.e. drying diffusion constant K becomes the function of the moisture content and the actual calculation by its solution is difficult. On the other hand, according to Batemann 17), Schlueter 30), Stamm 25), and Kroll 10) it may be considered that the drying diffusion constant is independent upon the moisture content at least from 30 to 10% moisture content.

Thus, for the application of the differential equation some deny it clearly and some have the different opinions even in hygroscopic range as well as in high moisture range, nevertheless the differential equation is often used for the convenience that (1) it represents directly the moisture content of wood (2) other equations can not be directly connected with the relative humidity and the velocity of the drying air, but it can be easily
connected with them (3) at least from 30 to 10% moisture content, it coincides well with the drying process (4) it is convenient for the calculation of the drying time.

At the above mentioned standpoint, the present author treated of the differential equation from the drying process and the moisture gradient, and obtained the influences of the drying air conditions to the drying coefficient. In drying, the heat- and moisture-movement take place together but in the present study only the latter was researched.

1. Differential equation of drying of wood

The general differential equation of the moisture movement through wood is given as the same as that of the heat conduction, that is, when \( u \) is the moisture content based on dry weight, the amount of moisture \( dW \) which diffuses through the area \( A \) in time \( dt \), is

\[
dW = -k \frac{du}{dx} A \, dt
\]

where \( k \) is the diffusion coefficient (g/cm\( h \)) and \( x \) the direction of moisture flow. Using equation \( V_n = V_o (1 + 0.84 \, r_n \, u) \) \((V_n, V_o \) is a wet and dry volume, \( r_o \) is the specific gravity) the differential equation is written in the form

\[
\begin{align*}
\frac{\partial u}{\partial t} &= \frac{1+0.84 \, r_n \, u}{r_o} \left( k_r \frac{\partial^2 u}{\partial x^2} + k_t \frac{\partial^2 u}{\partial y^2} + k_l \frac{\partial^2 u}{\partial z^2} \right) \\
&= K_{rt} \frac{\partial^2 u}{\partial x^2} + K_{rt} \frac{\partial^2 u}{\partial y^2} + K_{rt} \frac{\partial^2 u}{\partial z^2} \\
&= (1 + 0.84 r_n u) \left( K_{rt} \frac{\partial^2 u}{\partial x^2} + K_{rt} \frac{\partial^2 u}{\partial y^2} + K_{rt} \frac{\partial^2 u}{\partial z^2} \right)
\end{align*}
\]

where \( K (=k/r_o) \) is the drying diffusion constant (cm\(^2\)/h) and subscript \( r, t, I, \) and \( u \) and 0 represent the radial, tangential, longitudinal direction and moisture content \( u \), and 0, respectively; \( u \) in \((1+0.84 r_n u)\) is 0-0.3 in hygroscopic range and 0.3 in higher moisture range.

Obviously from equation (2), drying diffusion constant \( K \) is the function of the moisture content for a given wood species but its value is approximately equal to 1 in ordinary wood specimen as shown in Fig. 1. Hereafter the present author makes in unidirection the investigation whether \( K \) is constant or not.
11. Differential equation under steady drying conditions

When we assume that the amount of moisture evaporated from the wood surface to the drying air through the fluid film is proportional to the difference between the moisture content at the wood surface and the equilibrium moisture content of the drying air, at the boundary $x = -a$ (ref. Fig. 2),

$$dW = -k \frac{\partial u}{\partial x} A \, dh \quad \text{(from the wood surface)}$$
$$dW = -a \,(u_s - u_{eq}) \, A \, dh \quad \text{(through the fluid film)}$$

so that

$$\frac{\partial u}{\partial x} = \frac{a}{k} (u_s - u_{eq}) = h \,(u_s - u_{eq})$$

where $a$ is the coefficient of surface evaporation (g/cm²h) and $h = a/k$.

When the initial moisture gradient in wood is $f(x)$, then
\[
\begin{align*}
\frac{\partial u}{\partial t} &= K \frac{\partial^2 u}{\partial x^2} \quad \ldots \quad (4) \\
\theta &= 0 : u = f(x) \quad \ldots \quad (5) \\
x &= \pm a : \frac{\partial u}{\partial x} + \frac{h}{2} (u_x - u_{eq}) = 0 \quad \ldots \quad (6) \\
x &= 0 : \frac{\partial u}{\partial x} = 0 \quad \ldots \quad (7)
\end{align*}
\]

If we write \( v = u - u_{eq} \), equation (4)-(7) are transformed into
\[
\begin{align*}
\frac{\partial v}{\partial t} &= K \frac{\partial^2 v}{\partial x^2} \quad \ldots \quad (4)' \\
\theta &= 0 : v = f(x) - u_{eq} = F(x) \quad \ldots \quad (5)' \\
x &= \pm a : \frac{\partial v}{\partial x} + h v = 0 \quad \ldots \quad (6)' \\
x &= 0 : \frac{\partial v}{\partial x} = 0 \quad \ldots \quad (7)'
\end{align*}
\]

the particular solution, thus
\[
v = \frac{1}{a} \sum \frac{\partial u}{\partial u} e^{-K \left( \frac{\delta_n}{a} \right)^2 \theta} \times \cos \frac{\delta_n}{a} x \int_{-\delta_n}^{\delta_n} F(\lambda) \cos \frac{\delta_n}{a} - \delta_n \, d\lambda \quad \ldots \quad (8)
\]

where \( \delta_n \) is \( n \)th real root of \( \cot u = u/a \).

111. Case where the initial moisture distribution \( U_0 \) is uniform

Let \( F(\lambda) = U_0 - u_{eq} = V_0 \), from solution (8) we have
\[
\frac{v}{V_0} = 2 \sum e^{-K \left( \frac{\delta_n}{a} \right)^2 \theta} \cos \frac{\delta_n}{a} x \frac{\sin \delta_n}{\delta_n + \sin \delta_n \cos \delta_n} \quad \ldots \quad (9)
\]

for average moisture content \( v_{av} \) at time \( \theta \),
\[
\frac{v_{av}}{V_0} = 2 \sum e^{-K \left( \frac{\delta_n}{a} \right)^2 \theta} \frac{\sin^2 \delta_n}{\delta_n (\delta_n + \sin \delta_n \cos \delta_n)} \quad \ldots \quad (10)
\]

The water evaporated
\[
W = V_0 G_{eq} (1 - v_{av}/V_0) = V_0 G_{eq} \left( 1 - 2 \sum e^{-K \left( \frac{\delta_n}{a} \right)^2 \theta} \frac{\sin^2 \delta_n}{\delta_n (\delta_n + \sin \delta_n \cos \delta_n)} \right) \quad \ldots \quad (11)
\]

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where $G_m$ is the oven dry weight of wood, then

$$
\frac{dW}{d\theta} = V G_m \sqrt{x} e^{-K \left( \frac{\delta_m}{a} \right)^2} K \left( \frac{\delta_m}{a} \right)^2 - \frac{2 \sin^2 \theta_m}{\delta_m \left( \frac{\delta_m}{a} + \sin \delta_m \cos \theta_m \right)} \cdots \cdots \cdots \cdots (12)
$$

Fig. 3 shows the dimensionless diagram of equation (10).

![Figure 3: Dimensionless diagram of eq. (10)](image)

For the drying period $\theta_{eq}$ which required till the wood equilibrate with the drying air, we can take $e^{-K \left( \frac{\delta_m}{a} \right)^2} \approx 0.01$

then

$$
\theta_{eq} = \frac{4.6a^2}{K \delta_m^2} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots 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and

\[ V_{ar} = 2 \gamma e - K(\frac{\delta_n}{a})^2 \frac{\sin \delta_n}{\partial_n + \sin \delta_n \cos \delta_n} \times \left[ V_s \sin \delta_n - 2(V_m - V_s) \left( \frac{\cos \delta_n}{\partial_n} - \frac{\sin \delta_n}{\partial_n^2} \right) \right] \quad (16) \]

where \( V_s = U_s - u_{eq} \), \( V_m = U_m - u_{eq} \); the former is the moisture content at the wood surface and the latter is one in the mid-plane.

If the initial moisture distribution satisfies the boundary condition (6)', then

from (6)'

\[ \left( \frac{dV}{dx} \right)_{x_0 - a} = h V_s \]

from (14)

\[ \left( \frac{dV}{dx} \right)_{x_0 - a} = \frac{2(V_m - V_s)}{\alpha} \]

therefore

\[ V_s = \frac{2V_m}{2 + ha} \]

from parabola

\[ V_{nr} = \frac{2V_m + V_s}{3} \]

therefore

\[ V_s = -\frac{3}{3 + ha} V_{sr}, \quad V_m = \frac{3(2 + ha)}{2(3 + ha)} V_{sr} \]

Using these, equation (16) is transformed into

\[ \frac{v_{nr}}{V_{nr}} = 2 \gamma e - K(\frac{\delta_n}{a})^2 \frac{\sin \delta_n}{\partial_n + \sin \delta_n \cos \delta_n} \times \left[ \sin \delta_n - ha \left( \frac{\cos \delta_n}{\partial_n} - \frac{\sin \delta_n}{\partial_n^2} \right) \right] \quad (18) \]

and the moisture evaporated

\[ W = V_{nr} G_{ns} (1 - v_{nr} / V_{nr}) \]

\[ = V_{nr} G_{ns} \left( 1 - 2 \gamma e - K(\frac{\delta_n}{a})^2 \frac{\sin \delta_n}{\partial_n + \sin \delta_n \cos \delta_n} \times \left[ \sin \delta_n - ha \left( \frac{\cos \delta_n}{\partial_n} - \frac{\sin \delta_n}{\partial_n^2} \right) \right] \right) \quad (19) \]

then

\[ \frac{dW}{d\delta} = V_{nr} G_{ns} \gamma e - K(\frac{\delta_n}{a})^2 \frac{2\sin \delta_n}{\partial_n + \sin \delta_n \cos \delta_n} \times \left[ \sin \delta_n - ha \left( \frac{\cos \delta_n}{\partial_n} - \frac{\sin \delta_n}{\partial_n^2} \right) \right] \quad (20) \]
12. Differential equation under unsteady drying conditions

When the drying air conditions vary with time, we may use \( u_{\infty} = \varphi(\theta) \), but in this case drying diffusion constant \( K \) may be not always constant for the change of the drying conditions or the moisture content. Few experimental results have been reported about the relation between \( K \) and the drying conditions. KAMEI (1937), BATEMAN (1939), and STAMM (1946) reported the agreeable conclusions that \( K \) varies with the temperature, however, there have been an heterodoxy about the relation of \( K \) to the relative humidity of the drying air and the moisture content of wood. According to the present author, \( K \) varies with the change of the temperature and the relative humidity, but in the lower temperature or the higher humidity the variation is very small and for the change of the moisture content of wood, also, \( K \) can be regarded as a constant in practice, and changes of \( K \) by some degree give not so large influence on moisture content therefore under the confined unsteady drying conditig assuming \( K \) as constant

\[
\begin{aligned}
\frac{\partial u}{\partial \theta} &= K \frac{\partial^2 u}{\partial x^2} \\
\theta = 0 : u &= f(x) \\
x = +a : \frac{\partial u}{\partial x} + h \{ u - \varphi(\theta) \} &= 0
\end{aligned}
\]

(4)

(5)

(21)

the particular solution thus
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\[ u = \frac{1}{a} \sum_{n} e^{-K \left( \frac{\delta_n}{a} \right)^2} \cos \theta \frac{\delta_n}{a} x \frac{\delta_n}{\delta_n + \sin \delta_n \cos \delta_n} \]

\[ \times \left\{ f(\xi) \cos \frac{\delta_n}{a} \lambda d\lambda + 2a \sin \frac{\delta_n}{a} K \left( \frac{\delta_n}{a} \right)^2 \int_{0}^{\theta} e^{-K \left( \frac{\delta_n}{a} \right)^2} \phi(\xi) d\xi \right\} \quad \cdots \cdots (22) \]

121. Case where the initial moisture distribution is uniform

Let \( f(x) = U_0 \)

\[ u = 2 \sum_{n} e^{-K \left( \frac{\delta_n}{a} \right)^2} \cos \theta \frac{\delta_n}{a} x \frac{\sin \delta_n}{\delta_n + \sin \delta_n \cos \delta_n} \]

\[ \times \left\{ U_0 + K \left( \frac{\delta_n}{a} \right)^2 \int_{0}^{\theta} e^{-K \left( \frac{\delta_n}{a} \right)^2} \phi(\xi) d\xi \right\} \quad \cdots \cdots (23) \]

\[ u_{on} = 2 \sum_{n} e^{-K \left( \frac{\delta_n}{a} \right)^2} \frac{\sin^2 \delta_n}{\delta_n (\delta_n + \sin \delta_n \cos \delta_n)} \]

\[ \times \left\{ U_0 + K \left( \frac{\delta_n}{a} \right)^2 \int_{0}^{\theta} e^{-K \left( \frac{\delta_n}{a} \right)^2} \phi(\xi) d\xi \right\} \quad \cdots \cdots (24) \]

122. Case where the initial moisture distribution is parabolic

Let \( f(x) = U_m - (U_m - U_x) \left( \frac{X}{a} \right)^2 \)

\[ u = 2 \sum_{n} e^{-K \left( \frac{\delta_n}{a} \right)^2} \cos \theta \frac{\delta_n}{a} x \frac{1}{\delta_n + \sin \delta_n \cos \delta_n} \]

\[ \times \left\{ U_m \sin \delta_n - 2(U_m - U_x) \left( \frac{\cos \delta_n}{\delta_n} - \frac{\sin \delta_n}{\delta_n^2} \right) \right\} \]

\[ + \sin \delta_n K \left( \frac{\delta_n}{a} \right)^2 \int_{0}^{\theta} e^{-K \left( \frac{\delta_n}{a} \right)^2} \phi(\xi) d\xi \cdots \cdots (25) \]

\[ u_{on} = 2 \sum_{n} e^{-K \left( \frac{\delta_n}{a} \right)^2} \frac{\sin \delta_n}{\delta_n (\delta_n + \sin \delta_n \cos \delta_n)} \]

\[ \times \left\{ U_m \sin \delta_n - 2(U_m - U_x) \left( \frac{\cos \delta_n}{\delta_n} - \frac{\sin \delta_n}{\delta_n^2} \right) \right\} \]

\[ + \sin \delta_n K \left( \frac{\delta_n}{a} \right)^2 \int_{0}^{\theta} e^{-K \left( \frac{\delta_n}{a} \right)^2} \phi(\xi) d\xi \cdots \cdots (26) \]

2. Experimental discussion

21. Experimental method
The schema of the experimental apparatus is shown in Fig. 5, the dryer body was made of galvanized steel plate and was covered with insulating material of 5 cm thick.

**Heating system.** The air was warmed by two unit of Nichrome wire, the one was used to warm the dryer quickly and to maintain a temperature slightly below the desired operating temperature and the other was connected to an automatic dry bulb temperature controller.

**Humidifying system.** Damper placed in the exhaust pipe and the heater placed in hot water bath were used, the heater was connected to an automatic wet bulb temperature controller and used to maintain the desired relative humidity.

**Drying air velocity.** The air circulation was regulated by the rotating speed of fan to maintain the desired air velocity and was evenly distributed by the baffle net installed ahead of the test section.

The weight of test pieces were directly measured by the balance placed on the dryer body.

Test pieces used in this study were cut from the same stock as 10×20 cm² area and 0.2, 0.5, 1.0, and 3.0 cm thickness for the transverse direction (edge grain or flat grain pieces) and 10×20 cm² area and 1.3, 2.2, 3.0 and 5.0 cm thickness for the longitudinal direction (end grain pieces) and 15×15 cm² or 20×20 cm² area for drying of veneer or plywood. Previous to the test, the side surfaces of pieces were covered with synthetic resin for dampproof and one part was soaked in water with the vacuum and pressure treatment and the other saturated in water vapour until they attained the fiber saturation.
point. The latter particularly, had been kept to the operating temperature previous to the test in order to avoid the error caused by the temperature difference between wood and the drying air at the beginning of the experiment.

The test pieces for moisture determinations were timely taken out from the apparatus and the small piece (about 3×4 cm) was taken from each centre and cut into several thin layers for the moisture gradient (Fig. 6), on the same time, for the purpose of check, the distribution was measured by the electric moisture-meter. In case of need, the temperature gradient was measured by the thermocouple.

22. Drying of timber

221. Drying in the range of high moisture content

The test pieces of *Thujaopsis* (Hiba) whose initial moisture distribution was approximately uniform, were dried under the steady air condition of 55°C, 40% and 0.4-0.5 m/s.

The relation between the drying rate and the moisture content were the most typical as shown in Fig. 7. Obviously, in the timber of 0.2 and 0.5 cm thick, the three stages of drying, that is, the constant rate zone, the first and second zone of falling rate appeared, but with increase of thickness the former two disappeared gradually and in the timber of 3 cm thick only the second zone took place from the beginning of the drying. This phenomenon, as Hawley had pointed out, shows the fact that the movement of free water is active only in the confined distance and is explained by the temperature-
drying rate diagram in Fig. 8 and 9, that is, in the timber of 0.2 cm thick, as Keylwerth reported on drying of veneer, the temperature of wood during the constant drying rate keeps the wet bulb temperature of the drying air and the constant surface evaporation takes place during this period. In the timber of 3 cm thick, however, this correlation between the drying rate and the temperature of wood disappears. This shows that the movement of free water from the interior to the surface is not so easy as in veneer, and the temperature rising process indicated by Keylwerth for timber may
not occur. The temperature gradient of *Thujaopsis (Hiba)* of 3 cm thick is shown in Fig. 10.

As obvious from the above mentioned result, the mechanism of the drying varies with thickness of wood, and so we cannot apply the differential equation in all range from the high moisture content to hygroscopic range and moreover to make it applicable, the plots of $v_{oa}/V_o$ vs. $K\theta/a^2$ i.e. $\theta/a^2$ must be all straight lines on a semi-log scale as shown in Fig. 3. In Fig. 11 $v_{oa}/V_o$ is plotted against $\theta/a^2$ for *Cryptomeria (Sugi)* and
Thujopsis (Hiba) of 1 and 3 cm thick, respectively. The results have not so a good agreement with the differential equation as that of Sherwood and Kamei, and it seems that the differential equation is not applicable in general, to say nothing of high moisture range, even in the case where the second zone of falling rate appears from the beginning of the drying.

222. Drying in hygroscopic range

222.1 In the transverse direction

222.11 Determinations of the drying diffusion constant $K$

In the case where drying begins from the fiber saturation point, there is the great possibility to apply the differential equation. The present author used the rectangular slab of Chamaecyparis (Sawara, $r_0 = 0.4$ and Hinoki, $r_0 = 0.41$), Fraxinus (Tamo, $r_0 = 0.56$), and Prunus (Sawara, $r_0 = 0.63$) in this study. As shown in Fig. 12, the relation between $v_{av}/V_0$ and $\theta/a^2$ agrees well with the differential equation diagram in the range from the fiber saturation point to 10% moisture content, but below 10% moisture content, as Kröll stated, the molecular stream may take place instead of the diffusion and no differential equation can be applicable.

As the value of $h$, i.e., $ha$ has never been determined hitherto, the value of $K$ cannot be directly obtained here. The present author has determined $ha$ and $K$ by the
Fig. 12 $v_{as} / V_0 - \theta / a^2$ diagram in hygroscopic range

following manner.

As shown in Table 1, $K$ and $K_a$ for various $h_a$ were calculated from the drying data and as the most probable value of $h_a$, the one for which the coefficient of variation of $K_a$ was minimum was determined (ref. Table 2). On the other hand the moisture distribution was measured and the results were shown as an approximate curve in Fig. 13, from which the moisture content at the wood surface and the mid-plane can be calculated.

### Table 1 Fraxinus (TAMO) 3 cm thickness

<table>
<thead>
<tr>
<th>time $\theta$ content $\nu_{as} / V_0$</th>
<th>$\theta / a^2$</th>
<th>$K_a / a^2$</th>
<th>$K_a$</th>
<th>$K_m - K_a$</th>
<th>$K_m - K_a)^2$</th>
<th>$K_{as} / a^2$</th>
<th>$K_{as}$</th>
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<td>0.925</td>
<td>0.222</td>
<td>0.0040 0.018</td>
<td>0.0158</td>
<td>0.0059</td>
<td>0.348</td>
<td>0.0040 0.018</td>
<td>0.0158</td>
</tr>
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<td>0.3</td>
<td>30.0</td>
<td>0.844</td>
<td>0.888</td>
<td>0.0200 0.0240</td>
<td>0.0198</td>
<td>0.0019</td>
<td>0.0361</td>
<td>0.0200 0.0240</td>
<td>0.0198</td>
</tr>
<tr>
<td>2.0</td>
<td>26.0</td>
<td>0.770</td>
<td>1.776</td>
<td>0.0430 0.0420</td>
<td>0.0216</td>
<td>0.0001</td>
<td>0.0460 0.0259</td>
<td>0.0232</td>
<td>0.0018</td>
</tr>
<tr>
<td>4.0</td>
<td>23.9</td>
<td>0.687</td>
<td>3.122</td>
<td>0.0720 0.0230</td>
<td>0.0208</td>
<td>0.0009</td>
<td>0.0820 0.0295</td>
<td>0.0266</td>
<td>0.0016</td>
</tr>
<tr>
<td>7.0</td>
<td>22.0</td>
<td>0.613</td>
<td>4.450</td>
<td>0.1160 0.0260</td>
<td>0.0236</td>
<td>0.0019</td>
<td>0.1360 0.0281</td>
<td>0.0292</td>
<td>0.0042</td>
</tr>
<tr>
<td>10.0</td>
<td>19.8</td>
<td>0.527</td>
<td>7.111</td>
<td>0.1730 0.0430</td>
<td>0.0222</td>
<td>0.0005</td>
<td>0.2110 0.0296</td>
<td>0.0270</td>
<td>0.002</td>
</tr>
<tr>
<td>16.0</td>
<td>18.0</td>
<td>0.456</td>
<td>8.900</td>
<td>0.2330 0.0252</td>
<td>0.0242</td>
<td>0.0025</td>
<td>0.2750 0.0281</td>
<td>0.0284</td>
<td>0.0034</td>
</tr>
<tr>
<td>20.0</td>
<td>16.5</td>
<td>0.398</td>
<td>11.571</td>
<td>0.2910 0.0252</td>
<td>0.0234</td>
<td>0.0017</td>
<td>0.3460 0.0300</td>
<td>0.0279</td>
<td>0.0029</td>
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<tr>
<td>33.0</td>
<td>14.8</td>
<td>0.332</td>
<td>14.7</td>
<td>0.3580 0.0244</td>
<td>0.0228</td>
<td>0.0011</td>
<td>0.4140 0.0282</td>
<td>0.0264</td>
<td>0.0014</td>
</tr>
<tr>
<td>43.0</td>
<td>13.0</td>
<td>0.262</td>
<td>19.1</td>
<td>0.4580 0.0240</td>
<td>0.0226</td>
<td>0.0009</td>
<td>0.5200 0.0272</td>
<td>0.0256</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

$K_{as} = 0.0217$  C. V. = 0.107  $K_{as} = 0.0250$  C. V. = 0.161
### T. MAKU: Studies on the drying of wood

<table>
<thead>
<tr>
<th>$ha = 10$</th>
<th>$ha = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K_{m0} - K_0)^2$, $K_0/a^2$</td>
<td>$K_0$</td>
</tr>
<tr>
<td>$ \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>0.846</td>
<td>0.0290</td>
</tr>
<tr>
<td>0.270</td>
<td>0.0190</td>
</tr>
<tr>
<td>0.0324</td>
<td>0.0640</td>
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<tr>
<td>0.0266</td>
<td>0.1180</td>
</tr>
<tr>
<td>0.176</td>
<td>0.1740</td>
</tr>
<tr>
<td>0.04</td>
<td>0.2510</td>
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<tr>
<td>0.116</td>
<td>0.3190</td>
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<tr>
<td>0.0841</td>
<td>0.3970</td>
</tr>
<tr>
<td>0.0196</td>
<td>0.4710</td>
</tr>
<tr>
<td>0.0036</td>
<td>0.5980</td>
</tr>
</tbody>
</table>

$K_{in} = 0.0168$ C.V. = 0.0548   

<table>
<thead>
<tr>
<th>$ha = 2$</th>
<th>$ha = 1$</th>
<th>$ha = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$</td>
<td>$K_{m0} - K_0$</td>
<td>$K_0/a^2$</td>
</tr>
<tr>
<td>$ \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.154</td>
<td>0.0706</td>
<td>4.98</td>
</tr>
<tr>
<td>0.113</td>
<td>0.0296</td>
<td>8.76</td>
</tr>
<tr>
<td>0.0976</td>
<td>0.014</td>
<td>1.96</td>
</tr>
<tr>
<td>0.0844</td>
<td>0.0008</td>
<td>0.0064</td>
</tr>
<tr>
<td>0.0788</td>
<td>0.0048</td>
<td>0.23</td>
</tr>
<tr>
<td>0.0667</td>
<td>0.0165</td>
<td>2.86</td>
</tr>
<tr>
<td>0.0568</td>
<td>0.0165</td>
<td>2.82</td>
</tr>
<tr>
<td>0.0612</td>
<td>0.0224</td>
<td>5.02</td>
</tr>
<tr>
<td>0.0580</td>
<td>0.0256</td>
<td>6.55</td>
</tr>
<tr>
<td>0.0547</td>
<td>0.0286</td>
<td>8.35</td>
</tr>
</tbody>
</table>

$K_{in} = 0.0636$ C.V. = 0.351   

$K_{in} = 0.198$ C.V. = 0.463   

$K_{in} = 0.299$ C.V. = 0.501

* $K_0 = (1 + 0.84r_n)t$

** Coefficient of variation of $K_0$
Table 2 The values of $h_a$ determined from drying process and the moisture gradient (Transverse direction)

<table>
<thead>
<tr>
<th>Wood species</th>
<th>$h_a$ at C. V. minimum from eq. (27)</th>
<th>$K_{h_a=10}$</th>
<th>$K_{h_a=2.5}$</th>
<th>$K_{h_a}$</th>
<th>$h_a$ for $K_{h_a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Chamaecyparis</em></td>
<td>10</td>
<td>2.3</td>
<td>0.02</td>
<td>0.045</td>
<td>0.033</td>
</tr>
<tr>
<td>(SAWARA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot; (HINOI)</td>
<td>2~10</td>
<td>6</td>
<td>0.034</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Fraxinus</em></td>
<td>10</td>
<td>12</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(TAMO)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Prunus</em></td>
<td>10</td>
<td>11</td>
<td>0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SAKURA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* C.V. : coefficient of variation of $K_a$

Now, take the first term of equation (19) for approximate using, $v_x$ and $v_{ax}$ is given as

\[ \frac{v_x}{v_a} = 2e - K \left( \frac{\delta_1}{a} \right)^2 \frac{\sin \delta_1 \cos \delta_1}{\delta_1 + \sin \delta_1 \cos \delta_1} \]

\[ \frac{v_{ax}}{V_a} = 2e - K \left( \frac{\delta_1}{a} \right)^2 \frac{\sin \delta_1}{\delta_1 + \sin \delta_1 \cos \delta_1} \]

therefore

\[ \frac{v_x}{v_{ax}} = \cos \delta \]  

(27)

---

Fig. 13 Gradient of moisture content in transverse direction
(Drying conditions: 55°, 40%, 0.4--0.5m/s)
Using \( v_s \) and \( v_m \) in Fig. 13, \( \cos \beta \); i.e. \( ha \) were determined by equation (27). The values of \( ha \) thus obtained, as shown in Table 2, have a good agreement with previous one excepting \textit{Chamaecyparis} (SAWARA), and may seem to vary with wood species as will be observed later in the longitudinal direction but the details could not be clarified in here.

Now, using \( k = \frac{r^2}{100} \cdot \frac{\partial^2}{\partial \theta^2} \int du \cdot dx \) by Egner's method the values of \( K_E \) were obtained graphically from the moisture gradient as follows (ref. Fig. 14)

![Fig. 14 Relation between the drying diffusion constant \( K \) calculated by Egner's k and the moisture content](image)

For \textit{Chamaecyparis} (SAWARA) \( K_E = 0.034 \text{ cm}^2/\text{h} \)

\[ \approx \] (HINOKI) \( \approx 0.027 \)

\[ \approx \] \textit{Fraxinus} (TAMO) \( \approx 0.032 \)

\[ \approx \] \textit{Prunus} (SAKURA) \( \approx 0.0197 \)

Somewhat smaller as \( K_E \) is for \textit{Chamaecyparis} (HINOKI), \( K_E \) have a good agreement with \( K_o \) for \textit{Fraxinus} (TAMO) and \textit{Prunus} (SAKURA) and with the mean value of \( K_o \) at \( ha = 2.3 \) and 10 for \textit{Chamaecyparis} (SAWARA) (ref. Table 2), therefore, according to these results the drying diffusion constant is regarded as a constant independently of the moisture content. The dotted line in Fig. 13 is the one calculated by \( K = 0.034, 0.032, \) and 0.020 for \textit{Chamaecyparis} (HINOKI), \textit{Fraxinus}, and \textit{Prunus} respectively.

222.12 Determinations of the coefficient of the surface evaporation \( \alpha \)

From equation (3) the coefficient of surface evaporation is given as

\[ \alpha = \frac{dW}{dW_A v_s} \]
Then, by the diagram of dimensionless equation (12) \( \alpha \) can be obtained as in Table 3, as

Table 3 Transverse coefficient of surface evaporation \( \alpha \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \theta/\theta^2 )</th>
<th>( K_l/\theta^2 )</th>
<th>( \Sigma ) of eq. (12)</th>
<th>( dW/\theta )</th>
<th>( A )</th>
<th>( dW/\theta A )</th>
<th>( \nu )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.222</td>
<td>0.00705</td>
<td>0.0504</td>
<td>3.84</td>
<td>401</td>
<td>0.00957</td>
<td>0.119</td>
<td>0.0804</td>
</tr>
<tr>
<td>1</td>
<td>0.444</td>
<td>0.0141</td>
<td>0.0456</td>
<td>3.48</td>
<td>399</td>
<td>0.00872</td>
<td>0.0969</td>
<td>0.0900</td>
</tr>
<tr>
<td>2</td>
<td>0.888</td>
<td>0.0282</td>
<td>0.0388</td>
<td>2.96</td>
<td>388</td>
<td>0.00744</td>
<td>0.0730</td>
<td>0.102</td>
</tr>
<tr>
<td>5</td>
<td>2.22</td>
<td>0.0706</td>
<td>0.0280</td>
<td>2.14</td>
<td>394</td>
<td>0.00543</td>
<td>0.0496</td>
<td>0.109</td>
</tr>
<tr>
<td>10</td>
<td>4.44</td>
<td>0.141</td>
<td>0.0204</td>
<td>1.56</td>
<td>389</td>
<td>0.00401</td>
<td>0.0396</td>
<td>0.110</td>
</tr>
<tr>
<td>20</td>
<td>8.88</td>
<td>0.282</td>
<td>0.0153</td>
<td>1.17</td>
<td>385</td>
<td>0.00304</td>
<td>0.0253</td>
<td>0.120</td>
</tr>
<tr>
<td>30</td>
<td>13.32</td>
<td>0.424</td>
<td>0.0106</td>
<td>0.809</td>
<td>382</td>
<td>0.00212</td>
<td>0.0188</td>
<td>0.113</td>
</tr>
<tr>
<td>40</td>
<td>17.79</td>
<td>0.566</td>
<td>0.0080</td>
<td>0.610</td>
<td>379</td>
<td>0.00161</td>
<td>0.0141</td>
<td>0.114</td>
</tr>
</tbody>
</table>

\[ \alpha = 0.105 \]

obvious from this table the values of \( \alpha \) are nearly constant independently of the moisture content excepting the short period at the beginning of drying. Calculating \( ha \) from \( \alpha \),

\[ ha = \alpha \theta / K = \alpha \theta / K \theta = 0.105 \cdot 1.5 / 0.0318 \cdot 0.56 \approx 9 \]

this value of \( ha \) is nearly agreeable with previously determined \( ha \) (=10).

From the results of 222, 11, 222, 12, the drying process in hygroscopic range coincides well with the differential equation from the view of the drying rate and the moisture gradient occurred in practice. Consequently, even in hygroscopic range the surface moisture content does not reach the equilibrium moisture content \( u_{eq} \) given by the drying air and the moisture movement is occurred by the correlation between the surface evaporation based on this moisture difference and the internal diffusion. So that the drying diffusion constant \( K \) is to say nothing of the temperature, directly influenced by the relative humidity and the velocity of the drying air.

Many results hitherto gave \( \nu = 0 \) i.e. \( u_\nu = u_{eq} \) in hygroscopic range, in these cases, the thinner the timber is, the steeper the moisture gradient becomes for the same \( u_\nu \), and this is inconsistent with the fact that the thinner the timber is the easier the drying becomes, on the contrary definite value of \( ha \) gives above fact the appropriateness. Using \( K = 0.034, 0.032, \) and \( 0.020 \) for Chamaecyparis (HINOKI), Fraxinus (TAMO), and Prunus (SAKURA) respectively, the calculated moisture amount based on equation (11) and the measured values are shown in Fig. 15.
222,13 Relation of $K$ and $a$ to the thickness of timber

The similar experiments were done on *Thujopsis* (Hiba) and *Cryptomeria* (Sugi) of 3, 1, 0.5, and 0.2 cm thick from the high moisture content. As the differential equation is applicable after the moisture content of midplane reached the fiber saturation point, determining the time by means of $V_{in}$ for $U_{in} = 0.3$ and putting $ha = 6$ for 3 cm thick timber (ref. Table 2), consequently $ha = 2, 1,$ and 0.4 for 1, 0.5, and 0.2 cm thick respectively, the values of $K$ were obtained as shown in Fig. 16. Obviously in timber of below 1 cm thick for which 3 stages of the drying rate appeared the value of $K$ decreases a little with decreasing of thickness. The same inclination is recognized in plate of below 4 cm thick in the longitudinal direction, too (ref. Fig. 24). In both cases this relation between $ha$ and $K$ is shown as Fig. 17 in general. If the drying is controlled by
the differential equation the values of $K$ must be independently of thickness, that is, smaller value $a'$ must be used instead of $a$. Now, in the porous material as wood, the effective surface $a_e$ must be somewhat inner than $a$ (ref. Fig. 18). As there are hardly reported on the relation between $a_e$ and $a'$, it may be concluded at least that $a_e$ is connected with dimension of cell. The thickness of $aa'$ obtained from the experiment are about 0.3 - 0.8 mm in radial direction and 2 - 3.5 mm in fiber direction (the former is as 10 - 25 times as diameter of tracheid and the latter nearly equal to the length of tracheid). From this result, the present author assumes the surface phenomenon similar to the fluid film which may occur in the surface of wood with above order of dimension. The existence of this anomalous thin layer is confirmed a certain extent by the fact that when the wet timber which seems to be pretty dried at the first glance is planed the new wet surface appears again and the fact that when the well dried timber absorbs moisture, after reaching nearly the equilibrium moisture content it shows the remarkable absorption again when it is planed, namely, hygroscopically anomalous thin layer exists at the surface of wood. It is easily imagined that the effect of this layer on $K$ disappears gradually with increase of wood thickness. As to this phenomenon, however, the author can not conclude without more detailed experiments and this may be one of the most interested problems on drying of wood in future.

Now, in wood of below 1 cm thick the apparent value of $K$ varies with the thickness and decreases with the ratio of about 16.7 % / cm to the value of $K$ of 1 cm thick timber
and as obvious from Fig. 19 and 24, this ratio is the same as in the coefficient of surface evaporation and in the longitudinal direction below 4 cm thickness. Therefore at 55°C, 40% relative humidity and 0.4 - 0.5 m/s

\[ K_z = K_z \{1 + 0.167(2a_z - 1)\} \] ................................. (28)

where \( K_z \) is the drying diffusion constant of 1 cm thick timber. The thicker the timber is, the more negligible the effect of anomalous thin layer on \( K \) becomes and \( K \) shows almost constant value for the timber of over 1 cm thick.

For the coefficient of the surface evaporation \( a_z \) also, as obvious from 222,12, the same relation is given

\[ a_z = a_z \{1 + 0.167(2a_z - 1)\} \] ................................. (29)

where \( a_z \) is the value of 1 cm thick timber.

As for the relation between the drying time and the thickness under the same drying condition there have been \( \frac{\theta_2}{\theta_1} = \left(\frac{a_z}{a_z'}\right)^n \), TUOMALA and EGNER \(^7\) gave \( n = 1.7 \) and MOLL and KOLLMANN \( n = 1.5 \).

As obvious from equation (10), (16) and etc. (use the first term approximately), for different thickness \( 2a_z \) and \( 2a_z' \)

\[
\frac{\theta_z}{\theta_1} = K_z \left(\frac{\dot{a}_{z+1}}{a_z}\right)^2 \left(\frac{a_z}{a_z'}\right)^n = \left(\frac{a_z}{a_z'}\right)^n
\]

.................................(30)
therefore, \( n = 2 \) can exist only in the case where \( \delta_{11} = \delta_{12} \), that is, \( 2a_{1} \) and \( 2a_{2} \) are quite large, and in the practical range of thickness \( n \) is somewhat smaller than 2 and below 1 cm \( n \) becomes more and more smaller by the decreasing of \( K \). Fig. 20 shows the

![Graph showing values of exponent \( n \) of eq. (30), from 1 cm to each thickness.](image)

Fig. 20 Values of exponent \( n \) of eq. (30), from 1 cm to each thickness

![Graph showing relation of transverse drying diffusion constant to the drying air conditions.](image)

Fig. 21 Relation of transverse drying diffusion constant to the drying air conditions (*Fagus*, for timber of over 1 cm thick)
value of \( n \) calculated from 1 cm thickness to different thickness. The figure shows that \( n \) is 1.3 – 1.8 in the range of 0.2 – 10 cm thick and the value of Tuomola and Moli are all contained in above equation.

222.14 Relation of \( K \) and \( a \) to the drying air conditions

Fig. 21 and 22 show the relations of \( K \) and \( a \) to the various drying conditions for timber of over 1 cm thick which were converted by equation (28) and (29) from the data of veneer of Fagus (Buna, \( 2a = 1.5 \text{ mm} \)) dried at various, steady conditions.

![Diagram](image)

Fig. 22 Relation of transverse coefficient of surface evaporation to the drying air conditions (Fagus, for timber of over 1 cm thick)

Namely, the drying diffusion constant has the same inclination as the diffusion coefficient which increases generally with increase of the absolute temperature and is in inverse proportion to pressure, and as for the air velocity \( u \) its influence is slight and \( K \) seems to increase in proportion to \( u^{0.3 - 0.4} \) in range of \( u = 0.5 - 2.0 \text{ m/s} \).

Thus, the values of \( K \) obtained from the definite \( ha \) is naturally higher than those which
many researchers obtained under the condition $ha = \infty$, and so the both may not be compared, however, for reference, calculated values of STAMM and KRÖLL were converted into those of the same specific gravity and plotted in Figure.

222,15 Relation of $K$ to the specific gravity

The relation between specific gravity and $K$ converted by equation (28) from the drying data of veneer to the plate of over 1 cm thick are shown later in Fig. 36. Drying diffusion constant decreases linearly in inverse proportion to the increase of specific gravity, but the difference is slight.

222,21 Drying diffusion constant $K_i$

The drying data of *Chamaecyparis* (SAWARA and HINOKI), *Fagus* (BUNA), and *Fraxinus* (TAMO), (each initial moisture content is about 50%) give a good agreement with the dimensionless diagram of Fig. 3 only in hygroscopic range (the moisture content 30-10%) as similar as in 222,1 and probable $ha$ obtained from the rate of the drying and the moisture gradient shown in Fig. 23 are given in Table 4. According to these results, it is concluded also in longitudinal direction that the differential equation can be applied in hygroscopic range and the surface evaporation based on the moisture difference $(u_s - u_w)$ occupies a certain extent. From Table 4 the present author adopted hereafter the value of $ha$ determined from the minimum coefficient of variation of $K_o$ and showed $K_i$ corresponded to these $ha$ in Table.
Table 4  The values of $h_a$ determined from drying process and the moisture content (Longitudinal direction)

<table>
<thead>
<tr>
<th>Wood species</th>
<th>$h_a$ at C.V. minimum</th>
<th>$h_a$ from eq. (27)</th>
<th>$K_3$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Chamaecyparis</em> (SAWARA)</td>
<td>2</td>
<td>2.1</td>
<td>0.661</td>
<td>Drying condition</td>
</tr>
<tr>
<td>&quot; (HINOKI)</td>
<td>2~5</td>
<td>6</td>
<td>0.457</td>
<td>50, 55,</td>
</tr>
<tr>
<td><em>Fagus</em> (BUNA)</td>
<td>5</td>
<td>11</td>
<td>0.257</td>
<td>50, 62,</td>
</tr>
<tr>
<td><em>Fraxinus</em> (TAMO)</td>
<td>5</td>
<td>7.2</td>
<td>0.257</td>
<td>50, 62,</td>
</tr>
</tbody>
</table>

222.22 Relation of $K_i$ and $a_i$ to the thickness of timber

Test pieces of *Chamaecyparis (HINOKI)* of 5, 3, 2.2, and 1.3 cm thick were dried from about 50% to 10% moisture content. The value of $K_i$ was calculated by equation (18) from the drying data after the moisture content of mid-plane reached 30%, $K_i$ decreases with decreasing of thickness in plate of below 4 cm thick (ref. Fig. 24) and the following equation similar to equation (28) is obtained at 50°C, 55% relative humidity and 0.41 m/s.

\[ K_i = K_{1} \{1 + 0.167(2a_i - 1)\} \]  \hspace{1cm} (31)

where $K_i$ is the drying diffusion constant of 1 cm thick plate. For the coefficient of the surface evaporation $a_i$, similarly

\[ a_i = a_{1} \{1 + 0.167(2a_i - 1)\} \]  \hspace{1cm} (32)

where $a_i$ is $a$ of 1 cm thick plate.
Relation of $K_i$ and $a_i$ to the drying air conditions

To obtain the relation of $K_i$ and $a_i$ to the drying conditions, test pieces of *Fraxinus* (TAMO, thickness 1.3 cm, the initial moisture content 50-60%) were dried. The values of $K_i$ and $a_i$ obtained in hygroscopic range by equation (18) increase, as the same as in transverse direction, with increasing of absolute temperature and are inverse proportional to the relative humidity (Fig. 25, 26), thus

\[
K_i = n_i \left( \frac{273+t}{273} \right)^{a_i} \left( \frac{1}{H} \right) \quad (33)
\]

where $n_i$ and $m_i$ are the experimental coefficient, $H$ is the relative humidity, and $n_i = 0.876-4.0$, $m_i = 5.4-9.95$ in the range of the experiment (air temperature 20–90°C, relative humidity 20–80%).

The above relation is all the same to $a_i$

\[
a_i = n_i' \left( \frac{273+t}{273} \right)^{a_i'} \left( \frac{1}{H} \right) \quad (34)
\]
where \( n' = 0.85 - 4.0, m' = 4.4 - 8.9 \).

Fig. 27, 28 show the converted value for plate of over 4 cm thick.

Fig. 26 Relation of longitudinal coefficient of surface evaporation to the drying air conditions (Fraxinus, for timber of 1.3 cm thick)
Fig. 27 Relation of longitudinal drying diffusion constant to the drying air conditions (Fraxinus, for timber of over 4 cm thick)

Fig. 28 Relation of longitudinal coefficient of surface evaporation to the drying air conditions (Fraxinus, for timber of over 4 cm thick)

23. Drying of veneer
Studies on the drying of wood

231 Drying in the range of high moisture content

231.1 Relation of constant rate of drying $C$ to the drying air conditions

Test pieces of *Chamaecyparis* (HINOI, $2a=1.6$ mm, $r_0=0.34$), *Tilia* (SINA, $2a=1.39$, $r_0=0.43$), *Shorea* (Laauan $2a=1.24$, $r_0=0.48$), *Fagus* (BUNA $2a=1.54$, $r_0=0.56$), and *Betula* (KABA $2a=1.40$, $r_0=0.62$; $2a=1.34$, $r_0=0.67$) (each initial moisture content is about 50–60%) were used in this study. As for the drying process of veneer, as already explained in Fig. 7 and 8 in 221, the constant rate zone and the first zone of falling rate take place until near the fiber saturation point, however, the gradient of the latter is so gentle that it may be considered as a constant rate in practice.

In drying of veneer, Peck and Keylwerth established the equation $\frac{du}{dh} = -cu^n$ in all moisture content range ($c, u$: experimental coefficient) but the results of the present author did not agree with their results. So hereafter the present author will discuss on the range of high moisture content and on hygroscopic range, respectively.

Now, in Fig. 29 an example of drying process under various drying conditions are shown. From these results

1. The constant rate of drying (%/min) takes place until near the fiber saturation point
2. Constant rate of drying varies with wood species and drying conditions.

Fig. 30 shows the relation between the drying conditions and the average constant drying rate $C$ from initial moisture content to the fiber saturation point for *Fagus*. $C$, as similar as $K$, increases with increasing of temperature and decreases as the relative humidity increases and generally

$$C = n'' \left( \frac{273 + t}{273} \right)^{m''} \left( \frac{1}{H} \right)$$

where $n''$ and $m''$ are experimental coefficient and $n''=16.5-22$, $m''=4.8-6.26$ in the experimented range.

These results are somewhat different from Tuomola's experimental result.

As for the relation between the coefficient of heat transfer $h'$ and the mean air velocity $\bar{u}$, Zürges gave for rough face and $\bar{u}<5$ m/s

$$h' = 5.32 + 3.68\bar{u}$$

and Keylwerth gave

$$h' \propto \bar{u}^{0.5-0.8}$$
Fig. 29 Drying process of veneer under various drying conditions (Betula, KABA, 1.4 mm thick veneer)
If there is a proportional relation between the coefficient of heat transfer and the amount of moisture evaporated, above relation may be established between $C$ and $\bar{u}$, however, in the present experiment $C$ is nearly proportional to $\bar{u}^{0.5-0.8}$ in the extent of $\bar{u}=0.5-2.0$ and the influence of air velocity on $C$ is smaller. This result gives the same inclination with Kamei's experimental result which showed that in below 2 m/s constant drying rate is hardly influenced by the air velocity.

According to present experiment, it may be considered that the effect of air velocity is remarkable in below 0.5 m/s or over 2 m/s, but in the extent of 0.5-2 m/s the effect is slight.

Fig. 31 shows the amount of moisture evaporated in drying of veneer and hatched points on curves show the time when veneer reached the fiber saturation point in mid plane.

231.2 Relation of $C$ to specific gravity

Fig. 32 shows the relation of specific gravity and the constant rate of drying obtained
from various wood species and converted to veneer of 1 mm thick under the assumption that in the thin plate such as veneer drying period is proportional to the thickness, that is, drying velocity is inversely proportional to the thickness. As obvious from the figure, the constant drying rate decreases linearly with increasing of the specific gravity.

232. Drying in hygroscopic range

232,1 Relation of $K$ and $\alpha$ to the drying conditions

The drying process of veneer in hygroscopic range gave a good agreement with the differential equation, that is, similarly as in 222,13 calculating $V_{an}$ for $U_{an} = 0.3$ by equation (18) and determining the time by means of this $V_{an}$, from the drying data in 231
the author obtained a linear relation of $\frac{v_{in}}{V_{ar}}-t/a^2$ diagram.

Then, as similar as in 222,11, the most probable value of $ha$ was determined as about 0.5 from minimum coefficient of variation of $K$. $ha$ was independent on the drying conditions and had a good agreement with one calculated by $h=6.67$ which is determined by $ha=10$ (i. e. $h=6.67$) of Fraxinus and Prunus of 3 cm thick (for Fagus veneer $a=0.075$ cm; $ha=6.67 \times 0.075=0.51$). Therefore, putting $ha=0.5$, the relation between drying diffusion constant $K$ and the drying conditions was obtained in Fig. 33. From the figure, $K$ increases with increasing of the absolute temperature and decreases in inverse proportion to the relative humidity as similarly as the diffusion coefficient. That is

$$K = n\left(\frac{273+t}{273}\right)^n \left(\frac{1}{H}\right)$$

Fig. 33 Relation of transverse drying diffusion constant to the drying conditions (Fagus, for veneer of 1.5 mm thick)
where \( n, m \) are the experimental constant, and in the experimented range \( n = 0.35-0.8, m = 4.5-6.1 \).

As for the relation of the drying diffusion constant to the air velocity, \( K \) increases quickly in the range of \( \bar{u} = 0-0.5 \text{ m/s} \), gradually in the range of 0.5-2.0 m/s i.e. in proportional to \( \bar{u}^{0.5} \) and in over 2 m/s seems to increase again. Fig. 34 shows the relation between the coefficient of surface evaporation \( \alpha \) and the drying conditions, as same as \( K \), the following formula is given for \( \alpha \)

\[
a = n' \left( \frac{273+t}{273} \right)^{m'} \left( \frac{1}{H} \right)
\]

Fig. 34 Relation of transverse coefficient of surface evaporation to the drying conditions (Fagus, for veneer of 1.5 mm thick)

where \( n' = 1.7-2.7, m' = 5.4-5.9 \).

Fig. 21 and 22 show the values converted from Fig. 33 and 34 by equation (28) and
(29). Fig. 35 shows the measured values of the moisture evaporated after the moisture content of mid-plane reached the fiber saturation point (hatched point on curve in Fig. 31) and the values calculated by equation (18) and $K$ which was obtained by the above mentioned method.

![Fig. 35 Evaporated moisture in hygroscopic range (Shorea, lauan, 1.24 mm thick veneer)](image)

![Fig. 36 Relation of the drying diffusion constant to the specific gravity](image)

1. *Chamaecyparis (HINOKI)*
2. *Tilia (SINA)*
3. *Shorea (lauan)*
4. *Fagus (BUNA)*
5. *Betula (KABA)*

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232.2 Relation of \( K \) to the specific gravity

According to the data of Fig. 32 the relation between the specific gravity and \( K \) are shown in Fig. 36. \( K \) decreases linearly with increase of the specific gravity. The relation between both for timber of over 1 cm thick are calculated by equation (28) and shown in Fig. 36 together.

24. Drying of plywood

Urea resin bonded, 3 ply, *Betula* (Birk) and *Shorea* (Lauaan) plywood (thickness 3.2 and 4.6 mm respectively; initial moisture content 23-24%) were dried under the various drying conditions.

As same as in timber or veneer, \( v_{oc}/V_0 - \theta/a^2 \) diagram shows a straight line and \( h_{a} \) obtained from minimum value of coefficient of variation of \( K_s \) are given as follows.

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Thickness ( 2a )</th>
<th>Spec. gravity ( r_0 )</th>
<th>( h_a ) at C.V. min.</th>
<th>( h ) calculated by ( h_a )</th>
<th>Drying air conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Chamaecyparis</em></td>
<td>3.0</td>
<td>0.4</td>
<td>2~3</td>
<td>1.3~2</td>
<td>55°, 40%, 0.4~0.5m/s</td>
</tr>
<tr>
<td>SAWARA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.41</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.56</td>
<td>10</td>
<td>6.67</td>
<td></td>
</tr>
<tr>
<td><em>Fraxinus</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TAMO</td>
<td></td>
<td>0.63</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Pyrus</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAKURA</td>
<td></td>
<td>0.43</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Timber</td>
<td></td>
<td>1.0</td>
<td>2~3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.2</td>
<td>0.5~1</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td><em>Thujaopsis</em></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HIRA</td>
<td></td>
<td>0.43</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
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<td></td>
<td>0.32</td>
<td>0.32</td>
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<tr>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.32</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td><em>Chamaecyparis</em></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HINOKI</td>
<td></td>
<td>0.34</td>
<td>2</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>SAWARA</td>
<td></td>
<td>0.16</td>
<td>0.5~1</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.14</td>
<td>0.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Shorea</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAUAN</td>
<td></td>
<td>0.124</td>
<td>0.48</td>
<td>0.41</td>
<td>30<del>60°C, 25</del>72%</td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.154</td>
<td>0.56</td>
<td>0.52</td>
<td>26~50°C, 20°~80%</td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Fagus</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BUNA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.14</td>
<td>0.63</td>
<td>0.47</td>
<td>30<del>60°C, 25</del>72%</td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.133</td>
<td>0.67</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td><em>Betula</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KABA</td>
<td></td>
<td>0.46</td>
<td>1</td>
<td>1.5</td>
<td>30<del>50°, 20</del>40%</td>
</tr>
<tr>
<td>Plywood</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>0.32</td>
<td>0.5~1</td>
<td>1.1</td>
<td>0.1~0.6m/s</td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
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</tr>
</tbody>
</table>

-- 116 --
for lauan plywood \( ha = 1.2 \)
for birk plywood \( ha = 0.52 \)

and the value of \( ha \) calculated from \( h = 6.67 \) are

for lauan plywood \( ha = 6.67 \times 0.23 = 1.53 \)
for birk plywood \( ha = 6.67 \times 0.16 = 1.07 \)

Undoubtedly, the values of \( ha \) of both cases have a good agreement.

Table 5 shows summarily the values of \( ha \) on timber, veneer, and plywood. As obvious from the table, \( ha \) obtained from the minimum value of coefficient of variation

![Fig. 37](image-url) Rel. of the drying diffusion constant to the drying conditions (Betula-, Shorea-plywood)
of $K$, $ha$ obtained from the moisture gradient and $ha$ of veneer and plywood which calculated from $h$ of timber coincide well each other. From this fact it seems that in hygroscopic range the value of $h$ varies somewhat with wood species ($ha$ is smaller in light wood and larger in heavy one, so the moisture gradient becomes gentler in former and steeper in latter) but does not vary with the thickness of wood and the drying conditions in the extent of experiment, and even in plywood which has a few glue lines $h$ does not vary with above factors.

Fig. 37 shows the relation between $K$ calculated from the above value of $ha$ and the drying condition and in Fig. 33 the values converted for thickness are shown, namely the influence of the drying conditions on the drying diffusion constant in plywood is similar to that in timber and veneer, but it seems that the absolute value of $K$ is somewhat smaller in plywood than in timber or veneer by the presence of glue lines.

Summary

In drying of wood the differential equation similar to the heat conduction equation is not always adaptable to explain the moisture movement through wood, however, it is often used for the convenience that (1) it represents directly the moisture content (2) the other equations can not be connected directly with the relative humidity and the velocity of drying air, but it can be easily connected with them (3) at least from 30-10% moisture content it coincides well with the drying process (4) it is convenient for the calculation of the drying time and the moisture gradient.

At the above mentioned standpoint, the present author treated of the differential equation and obtained the relations of the drying constant to the drying conditions.

1. Under the assumption that the coefficient of the differential equation i.e. drying diffusion constant $K$ is constant, the solutions were obtained in steady drying condition and in confined unsteady drying condition.

2. In high moisture range, the differential equation can not be used generally, however, in hygroscopic range it coincides well with the drying process.

3. According to the drying process and the moisture gradient in hygroscopic range, drying diffusion constant $K$ and the coefficient of surface evaporation $a$ are independent upon the moisture content of wood both in the transverse direction and in the longitudinal direction, and the moisture content at the wood surface is always higher a little than the equilibrium moisture content of the drying air, and the drying proceeds under the
correlation between the surface evaporation by this moisture difference and the internal diffusion.

4. In the transverse direction the value of $K$ and $a$ decrease somewhat with decreasing of thickness of wood in the extent below 1 cm thick and in the longitudinal direction they decrease similarly in the extent below 4 cm thick. It may be caused by the existence of the thin anomalous layer at the wood surface, but the value of the coefficient becomes constant over the above mentioned thickness because the thin layer does not influence in practice.

5. The relation of the drying air conditions to drying coefficient $K$ and $a$ was searched by the drying data of veneer and this was converted to the value of timber. The value of $K$ and $a$ increase as a exponential function of absolute temperature and decreases inverse proportionally with increase of the relative humidity as similar as the general diffusion coefficient. The influence of the air velocity $u$ on $K$ and $a$ is quite slight in the extent of $u = 0.5$–2.0 m/s and $K$ and $a$ are nearly proportional to $u^{0.5}$.

The value of $K$ thus obtained are larger a little than the value determined by the assumption that the surface moisture content of wood conforms to the equilibrium moisture content of the drying air in hygroscopic range, but, for reference, they are shown in comparison with the value calculated by STAMM and KRÖLL.

6. In drying of veneer the constant rate of drying takes place until near the fiber saturation point in practice and therefore the relation between the constant drying rate and the drying condition was searched.

7. The drying process of plywood coincides well with the differential equation, but the drying constant is somewhat smaller than that of veneer and timber. It may be caused by the existence of the glue lines.

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