# A Study of Fracture of Wood Based on the Theory of Stochastic Process

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角谷和男：木材の破壊に関する確率過程論的考察

## Introduction

A great number of problems in breaking strengths of any material comes out essentially to the mechanism of its fracture.

One of the most important problems in strength of wood is the wide fluctuation of its values. This fluctuation has been considered as a result of differences of specific gravity, annual ring breadth and so on —structure-insensitive characters— *between* each specimen and then its origin have not yet been traced deeply. But inhomogeneity *in* a specimen should play an important role in this fluctuation, too. Because in the previous experiment[1] the fluctuation of bending strengths of many specimens which were selected from one timber of Hinoki (*Chamaecyparis obtusa* ENDL.) and Buna (*Fagus crenata* BLUME) and whose specific gravity and average annual ring breadth affecting on their strength was equal to that of specimens which were still more selected from them to have the same mean of specific gravity, moisture content and average annual ring breadth as them and to eliminate the effect of specific gravity and average annual ring breadth on their strength. Furthermore, strengths of wood are dependent on size, temperature and moisture content of specimens and on loading rate of tests. Hereupon it comes into question how these phenomena in strengths are related with the mechanism of fracture of wood.

In order to answer this question sufficiently, it needs to grasp the fracture in a primary fashion. It would be true to consider that the weakest and/or the most stress-concentrated microscopic element is broken at the start when specimens break down. On the other hand, the observed value of the breaking strength of a cellulose fiber under a uniform tensile stress parallel to the fiber direction is certainly smaller than the values calculated on the assumption that it has a uniform crystal constitution[2]. Even when many specimens of concrete or metal which are produced under the same condition and have almost the same specific gravity and elastic constant are tested under bending or tensile load, their strengths or times elapsed before fracture are scattered beyond the extent of experimental errors[3)4)]. These facts show that the microscopic weak elements in specimens have an important effect upon their strengths, that is, fracture is "structure-sensitive". Therefore, it would be essential to start on the standpoint of statistics or probability for the analysis of fracture, because various weak elements will be statistically distributed in each specimen.

In the present time, there are two ways to do so. One is the method based on the weakest link theory and the other is based on the theory of stochastic process. The weakest link theory takes as a starting point GRIFFITH's theory[5], which states that the reason for the difference between the calculated strengths of materials and the actually observed values resides in the fact that there exist *a priori* cracks in the body which will weaken it. And the dependency of the strengths on their volume can be well explained by this theory as follows: if it is assumed that the least strength of creacks which are distributed uniformly with various strengths in a specimen decides its strength, the mean strength of a group of specimens will be the mode of the least strength of $n$ cracks which are chosen from the distribution function of strengths of these cracks[6]. I explained qualitatively the size effects in strength of wood by using GAUSS's function as this distribution function[7]. On the other hand, it would not be doubted that the thermal motion of molecules constituting wood has some connection with the mechanism of its fracture, because of the dependency of strengths on temperature[8]. Then it may be quite whithin the bounds of possibility for fracture of wood to start from this thermal motion of molecules without the assumption of the existance of *a priori* cracks and the fluctuations of strengths may be caused by the thermal fluctuations of molecules as well. Furthermore, the strengths of wood depend on the rate of loading[9][10] or there are delayed fracture in creep tests[1], that is, fracture of wood is "time-dependent". Therefore, the theory of stochastic process will become more effective for its analysis.

In this report, I deal with the fluctuation of the time elapsed before fracture in bending creep tests from the standpoint of the theory of stochastic process and find the rule controlling it, and then I analyse the breaking strength of wood with this rule and at last discuss the mechanism of fracture of wood on the standpoint of the rate process.

## 1. Stochastic Process Model for Fracture

Now consider a group of many specimens which can be regarded as statistically homogeneous and each of which is under the same dead load. If this load is large enough, they will be surely broken after the lapse of a certain time. But all of them are not always broken at the same time.

If the numbers of specimens which remain to be unbroken at a time $t$ after loading are $N(t)$, the number of specimens broken during the next infinitesimal time $dt$, that is $-dN$, will be expressed by the following equation:

$$-dN=N(t)m(t)dt \qquad \cdots\cdots(1)^{11)}$$

where, $m(t)$ = the probability of occurrence of fracture in unit time at $t$.

If the initial numbers of specimens are $N_0$, from eq. (1)

$$-dN/N_0=(N/N_0)mdt \qquad \cdots\cdots(2)$$

$N/N_0$ is the probability in which fracture does not occur before $t$. If I represent it by $P(t)$, $-dN/N_0=-dP$ and eq. (2) is written by the following expression:

$$-dP=P(t)m(t)dt \qquad \cdots\cdots(3)$$

Then,

$$m(t)=-d(\ln P)dt \qquad \cdots\cdots(4)$$

And if $q(t)dt$ is the probability of occurrence of fracture between $t$ and $t+dt$, that is, the frequency distribution of the time elapsed before fracture, $P(t)$ is

$$P(t)=\int_t^\infty q(t)dt \qquad \cdots\cdots(5)$$

From the definition of $P(t)$, we write

$$P(0)=1 \qquad \cdots\cdots(6)$$

When the frequency distribution curve of the time elapsed before fracture is decided by an experiment, we can calculate the value of $m(t)$ at $t$ by using eqs. (5) and (4)

From the definition of $m(t)$, it corresponds to the rate of occurrence of fracture at $t$, but its physical meaning is not cleared up only by the theory of stochastic process. And, therefore, the dependency of this rate on temperature, applied stress and so on must be tested.

## 2. The Rate of Occurrence of Fracture

In order to establish the stochastic process model for fracture of wood, the times elapsed before fracture must be measured by using a great number of specimens.

A bending test under a constant load is used in this experiment, as both making of specimens and experimental equipments is simple and the load is independent of time. Species used are Hinoki and Buna on behalf of Japanese softwood and hardwood, respectively. In a test, ninety specimens are selected at random from about three thousands specimens which are made at random from one timber. But, as mentioned above, it is a noteworthy fact to select specimens which are treated on the standpoint of the theory of stochastic process that they can be regarded as statistically homogeneous, that is, the probability in which a number of specimens has a certain strength is independent of the position where they are selected. However, the strength of wooden specimens, even though they are selected from

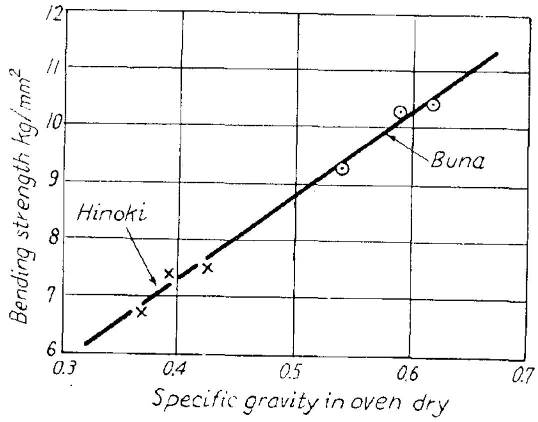

Fig. 1. Dependency of bending strength on specific gravity in Hinoki and Buna.

one timber and are so-called "non-defective", is affected by their specific gravities, average annual ring breadths and moisture contents. So one must make a limitation of their specific gravities, average annual ring breadths and moisture contents to the extent that their fluctuations do not affect the fluctuation of strength. Fig. 1 shows the dependency of the breaking strength of bending loaded at two points at the constant rate in the previous experiment[1], where the size of specimens was the same as in this experiment and fifty specimens were selected at random from the above three thousands specimens, on the specific gravities. It is evident from this figure that the strength of the group having the least specific gravity is extremely weak, in both species. Furthermore, this group has the extremely narrow annual ring breadths. When the specimens which are light —less than 0.36 in Hinoki and 0.56 in Buna in specific gravity in oven dry— and also have a narrow average annual ring breadths —less than 0.9 *mm* in breadth— are taken away, it is ascertained as a result of the analysis of variance that the fluctuations of them in remained specimens do not affect the fluctuation of strength. So, in this experiment, I take away these specimens from the selected ninety specimens.

The experimental apparatus and the size of a specimen are shown in Fig. 2. These specimens are loaded at two points from their outer side with dried sand. The weight of sand $W$ is ajusted for the bending stress $S$ calculated by the following equation to be constant :

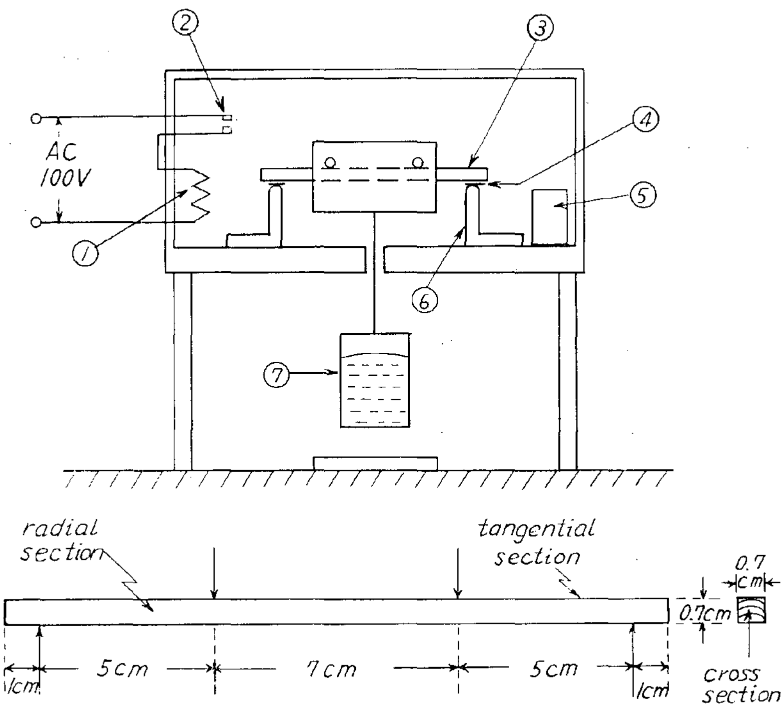

Fig. 2. Experimental apparatus and the size of specimen.
① Heater ② Bimetal ③ Specimen ④ Metal plate
⑤ Saturated solution of salt or water or silica gel
⑥ Support ⑦ Weight

$$S=3W(l-l')2bh^2 \qquad \cdots\cdots(7)$$

where, $b$ = width of specimen

$h$ = thickness of specimen

$l$ = distance between supports

$l'$ = distance between loading points.

The only one investigator measures the time elapsed from the moment of loading to fracture with a stop-watch, so that the individual errors in the measurement of time may be almost eliminated. Furthermore this loading is carried out without striking the specimens. The temperature and the relative humidity before and during the test are kept constant by a heater with a regulator and a saturated solution of KBr, respectively. The temperature of this test is $30°\pm1°C$ and the constant bending stress calculated by eq. (7) is 7.25 $kg/mm^2$ in Hinoki or 10.4 $kg/mm^2$ in Buna, as shown in the Test No. 4 of Table 1(a) and the Test No. 5 of Table 1(b) respectively, which is almost equal to the mean bending strength of the selected specimens[1]. The time elapsed before fracture, specific gravity, moisture content and average annual ring breadth in this experiment are shown in the Test No. 4 of Table 2(a) and the Test No. 5 of Table 2(b). It is ascertained as a result of the analysis of variance that the fluctuation of the moisture content does not affect the time elapsed before fracture.

According to eq. (4), the rate of occurrence of fracture $m(t)$ is clarified by the relation between $\log P$ and $t$. Fig. 3 shows $\log P-t$ diagram in this experiment. As the analytical curve of $q(t)$ can not be decided in this experiment, the value of $P$ at $t$ is not the one calculated by eq. (5) but by the following approximate formula:

$$P(t_\nu)\simeq1-\nu/(N_0+1) \qquad \cdots\cdots(8)^{12)}$$

the right side of which is the $\nu$th mean frequency not to fracture in the case that the times elapsed before fracture of $N_0$ specimens are put in order of $t_1<t_2<\cdots<t_\nu<$

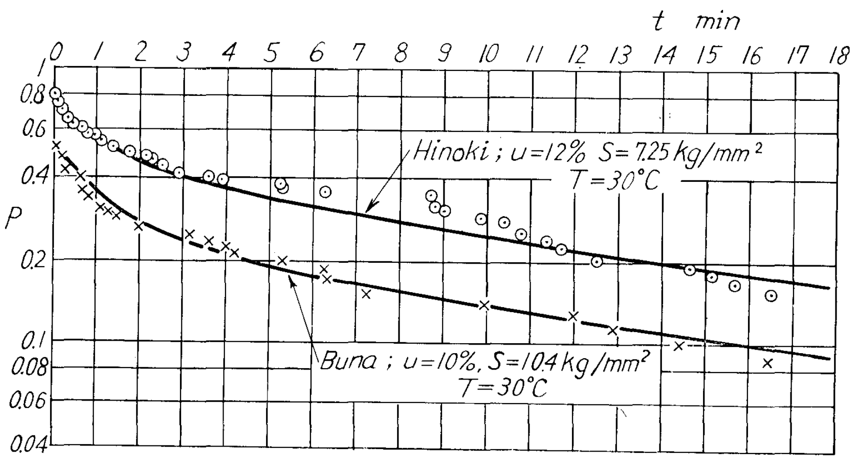


Fig. 3. An example of $\log P-t$ diagram in the solid wood.

$\cdots < t_{N_0}$.

It becomes clear from Fig. 3 that $m(t)$ of wood under a constant load decreases with lapse of time. This inclination is very different from other materials: the rate of glass[13] is constant and the rates of copper[4] and cement[3] become constant after a time within which the fracture does not occur at all. This cause will be discussed in the next section.

It is an interesting problem what change in this rate occurs in the laminated wood, that is, in the existance of the hardened layers — glued lines. With this aim, ninety laminated specimens which have four glued lines at a constant interval and the same size as in the solid specimens mentioned above are tested under a constant bending load[14]. They are also made at random from the same timber as above and are glued at random on the tangential section with urea resin adhesive. The temperature of test is $10°\pm1°C$ and the constant bending stress is $9.5\,kg/mm^2$ in Hinoki and $12.9\,kg/mm^2$ in Buna, which are almost equal to the mean of bending strength of specimens, as shown in Table 1. And the time elapsed before fracture, moisture content and specific gravity in oven dry are shown in Table 2.

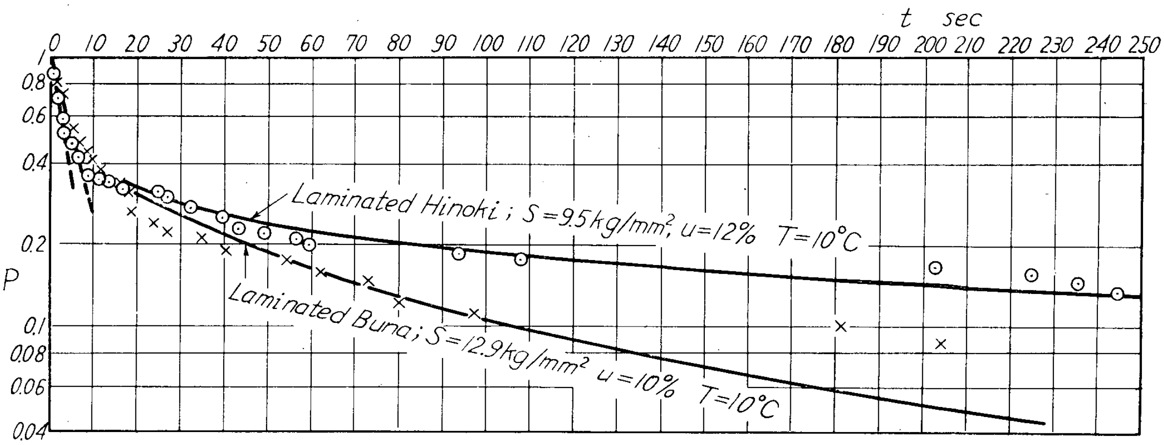


Fig. 4. An example of $\log P-t$ diagram in the laminated wood.

Fig. 4 shows $\log P-t$ diagram in these laminated specimens, which shows that the inclination of $m(t)$ of laminated wood with lapse of time is essentially the same as one of the solid wood.

Now it comes into question by what function of time $m(t)$ or $P(t)$ is indicated. If the following equation exists in wood from the shape of Figs. 3 or 4:

$$\ln P = -At^B \qquad \cdots\cdots\cdots\cdots(9)$$

where $A$ and $B$ are constants, the relation between $\log\log P$ and $\log t$ will be linear. Figs. 5 and 6 show the $\log\log P-\log t$ diagrams in these experiments. According to these figures, it is evident that the above assumption is right except a few moment after loading. Then, from eq. (4) $m(t)$ is

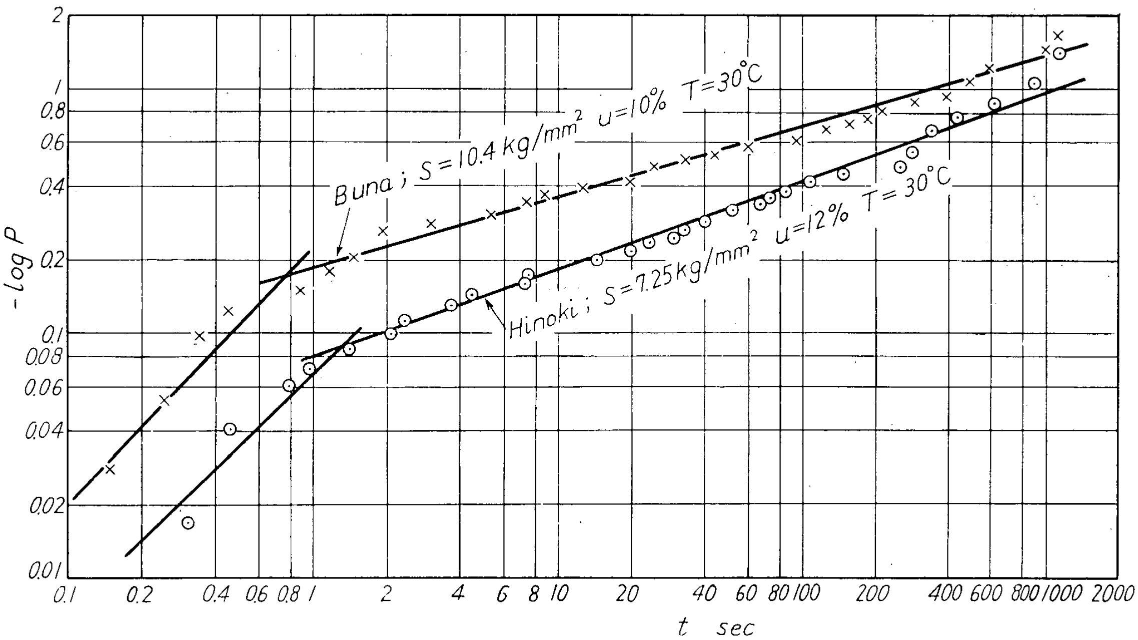


Fig. 5. An example of log log $P-\log t$ diagram in the solid wood.

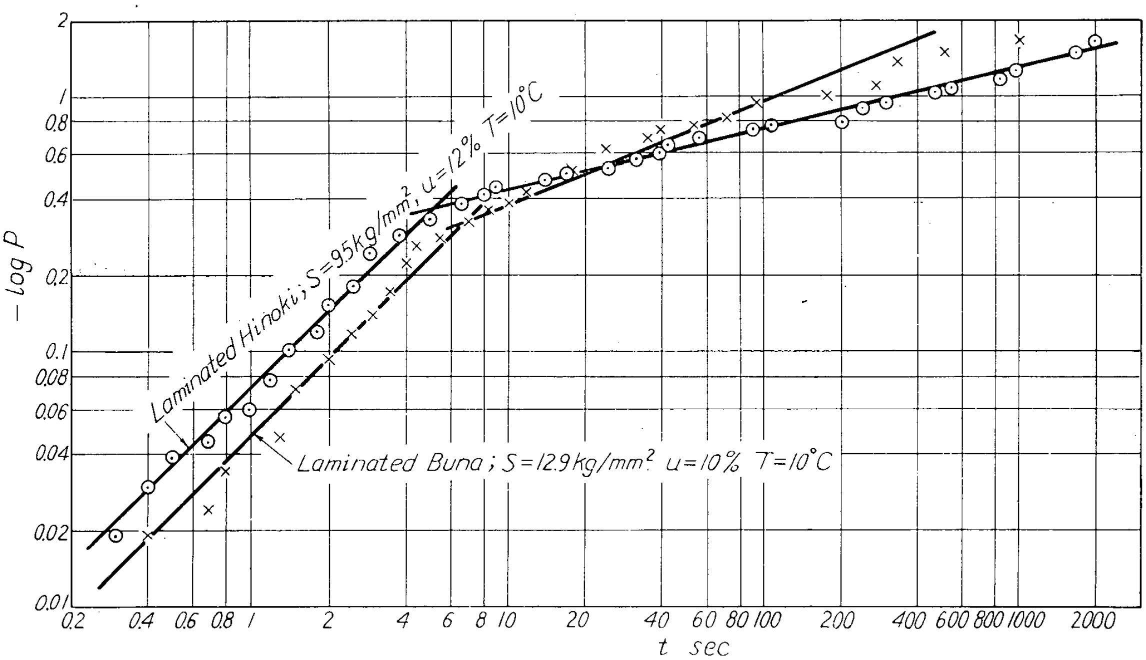


Fig. 6. An example of log log $P-\log t$ diagram in the laminated wood.

$$m(t)=ABt^{-(1-B)} \qquad \cdots\cdots(10)$$

On the other hand, $m(t)$ may be independent of time for a few moments after loading, because the relation between $\log\log P$ and $\log t$ is approximately linear with unit direction coefficient, and then

$$\ln P = -m_0 t \qquad \cdots\cdots(11)$$

where, $m_0$ = constant.

The values of $m_0$, $A$ and $B$ calculated from log log $P$ − log $t$ diagrams are shown in the Test No. 4 and the last line in Table 3(a) and the Test No. 5 and the last line in Table 3(b).

From these results, it seems to be sure that $m(t)$ is constant for a few moments after loading but begins to decrease in accordance with eq. (10) with lapse of time after that moment. And this inclination of $m(t)$ will be observed in a great portion of the experiments described in the next section. As compared Fig. 5 with Fig. 6, the time region where $m(t)$ is constant in the laminated wood seems to be wider than in the solid wood.

### 3. Physical Meaning of the Rate of Occurrence of Fracture

The relation between constant stress $S$ and the mean time elapsed before fracture $\bar{t}$ seems to be indicated generally in wood by the equation

$$S = a - b \cdot \ln \bar{t} \qquad \cdots\cdots\cdots\cdots(12)$$

where, $a$, $b$ = constant. See Fig. 7[15].

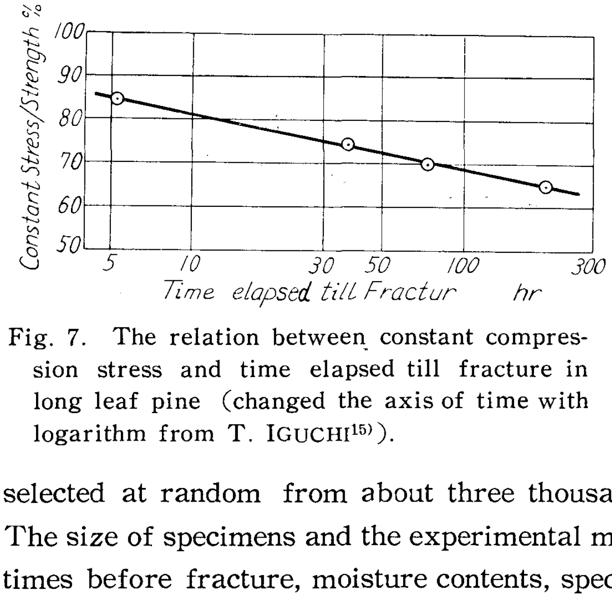


Fig. 7. The relation between constant compression stress and time elapsed till fracture in long leaf pine (changed the axis of time with logarithm from T. IGUCHI[15]).

Furthermore, it is almost certain that $\bar{t}$ is affected by the moisture content and temperature, even if it is under the same stress. Therefore, the dependency of $m(t)$ on moisture content, stress and temperature is investigated in this section.

The testing conditions are shown in Table 1 and the specimens which are selected at random from about three thousands specimens described above are used. The size of specimens and the experimental method are the same as above. The elapsed times before fracture, moisture contents, specific gravities in oven dry, average annual ring breadths and the numbers of specimens in every test are shown in Table 2. The accuracy of the temperatures is ±1°$C$ and the moisture contents are adjusted by silica gel or the saturated solution of $NaNO_2$, KBr, $K_2Cr_2O_7$ and $NaNO_3$ or water. The moisture content, specific gravity and average annual ring breadth of every specimen are measured in accordance with JIS Z2102 immediately after every test.

Figs. 8∼12 show log $P$ − $t$ diagrams which are arranged so as to indicate the effect of moisture content, stress and temperature, respectively. It seems to be recognized from these figures that the time region where $m(t)$ is independent of time may be about a few seconds after loading.

Table 3 shows the values of $m_0$, $A$ and $B$ calculated by log log $P$ − log $t$ diagrams.

Table 1. Testing conditions.

(a) Hinoki

| Test No. | Moisture content (%) | Stress ($kg/mm^2$) | Temperature (°C) |
|---|---|---|---|
| 1 | 4 | 7.25 | 30 |
| 2 | 12 | 7.00 | 30 |
| 3 | 12 | 7.25 | 20 |
| 4* | 12 | 7.25 | 30 |
| 5 | 12 | 7.25 | 50 |
| 6 | 12 | 7.50 | 30 |
| 7 | 12 | 8.00 | 30 |
| 8 | 16 | 7.25 | 30 |
| 9 | saturated | 4.30 | 30 |
| 10 | saturated | 5.00 | 30 |
| 11 | saturated | 5.80 | 30 |
| 12 | saturated | 7.25 | 30 |
| 13 | saturated | 5.80 | 10 |
| 14 | saturated | 5.80 | 20 |
| 15 | saturated | 5.80 | 50 |
| laminated wood* | 12 | 9.50 | 10 |

(b) Buna

| Test No. | Moisture content (%) | Stress ($kg/mm^2$) | Temperature (°C) |
|---|---|---|---|
| 1 | 4 | 10.4 | 30 |
| 2 | 10 | 9.6 | 30 |
| 3 | 10 | 10.0 | 30 |
| 4 | 10 | 10.4 | 15 |
| 5* | 10 | 10.4 | 30 |
| 6 | 10 | 10.4 | 50 |
| 7 | 10 | 11.5 | 30 |
| 8 | 16 | 10.4 | 30 |
| 9 | saturated | 5.0 | 30 |
| 10 | saturated | 5.8 | 30 |
| 11 | saturated | 6.5 | 30 |
| 12 | saturated | 7.5 | 30 |
| 13 | saturated | 10.4 | 30 |
| 14 | saturated | 7.5 | 10 |
| 15 | saturated | 7.5 | 20 |
| 16 | saturated | 7.5 | 50 |
| laminated wood* | 10 | 12.9 | 10 |

* described in section 2

Table 2. Time elapsed before fracture and some physical properties of specimens.

(a) Hinoki

| Test No. | Time elapsed before fracture | | | | Moisture content (%) | | | Specific gravity in oven dry | | | Average annual ring breadth (*mm*) | | | Number of specimens |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | max. | mean | min. | standard deviation | max. | mean | min. | max. | mean | min. | max. | mean | min. | |
| 1 | 59*hr* 1.2*min* | 8*hr* 59.9 *min* | 49.6*sec* | 12*hr* 28.1 *min* | 6.3 | 4.3 | 3.1 | 0.454 | 0.401 | 0.358 | 1.6 | 1.0 | 0.6 | 42 |
| 2 | 40.0*min* | 5.61*min* | 0.0*sec* | 10.0 *min* | 14.2 | 12.5 | 11.0 | 0.496 | 0.413 | 0.361 | 2.0 | 1.3 | 0.6 | 74 |
| 3 | 7*hr* 10.7*min* | 1*hr* 5.7 *min* | 0.0*sec* | 1*hr* 40.5 *min* | 11.4 | 10.5 | 9.6 | 0.478 | 0.423 | 0.365 | 1.7 | 1.2 | 0.6 | 63 |
| 4* | 45.9*min* | 7.68*min* | 0.5*sec* | 11.4 *min* | 13.2 | 11.6 | 10.0 | 0.454 | 0.408 | 0.346 | 2.1 | 1.1 | 0.6 | 78 |
| 5 | 51.2*min* | 2.50*min* | 0.2*sec* | 13.9 *min* | 13.1 | 11.6 | 10.0 | 0.482 | 0.414 | 0.368 | 2.2 | 1.3 | 0.6 | 75 |
| 6 | 11.7*min* | 45.0 *sec* | 0.3*sec* | 1.68*min* | 14.1 | 12.2 | 10.8 | 0.455 | 0.406 | 0.376 | 1.9 | 1.1 | 0.7 | 74 |
| 7 | 41.3*min* | 1.89*min* | 0.0*sec* | 6.72*min* | 13.7 | 12.7 | 11.8 | 0.496 | 0.416 | 0.363 | 2.2 | 1.1 | 0.6 | 90 |
| 8 | 2.2*min* | 8.3 *sec* | 0.0*sec* | 19.3 *sec* | 19.8 | 15.8 | 15.3 | 0.459 | 0.399 | 0.352 | 1.9 | 1.1 | 0.6 | 75 |
| 9 | 44*hr* 3.3*min* | 31.5 *min* | 0.4*sec* | 42.0 *min* | 31.3 | 24.5 | 20.9 | 0.477 | 0.405 | 0.360 | 2.2 | 1.4 | 0.6 | 69 |
| 10 | 6*hr* 4.8*min* | 19.5 *min* | 0.4*sec* | 52.7 *min* | 27.3 | 23.0 | 20.8 | 0.468 | 0.396 | 0.359 | 2.1 | 1.1 | 0.7 | 70 |
| 11 | 9.0*sec* | 0.9 *sec* | 0.2*sec* | 1.6 *sec* | 25.2 | 22.6 | 22.0 | 0.450 | 0.401 | 0.364 | 1.8 | 1.2 | 0.6 | 80 |
| 12 | 2.9*sec* | 0.6 *sec* | 0.2*sec* | 1.0 *sec* | 27.8 | 25.4 | 21.9 | 0.440 | 0.399 | 0.361 | 2.0 | 1.1 | 0.6 | 70 |
| 13 | 27.7*min* | 1.86*min* | 0.1*sec* | 5.13*min* | 28.1 | 25.1 | 21.3 | 0.449 | 0.411 | 0.364 | 2.2 | 1.1 | 0.6 | 73 |
| 14 | 22.6*sec* | 1.1 *sec* | 0.0*sec* | 2.7 *sec* | 29.7 | 25.9 | 23.1 | 0.483 | 0.412 | 0.351 | 2.2 | 1.2 | 0.6 | 78 |
| 15 | 1.8*sec* | 0.6 *sec* | 0.1*sec* | 1.0 *sec* | 24.0 | 20.4 | 18.6 | 0.476 | 0.406 | 0.399 | 2.5 | 1.2 | 0.7 | 76 |
| laminated wood* | 43.3*min* | 2.57*min* | 0.1*sec* | 7.26*min* | 14.1 | 12.2 | 10.1 | 0.487 | 0.440 | 0.388 | — | — | — | 90 |

(b) Buna

| Test No. | Time elapsed before fracture | | | | Moisture content (%) | | | Specific gravity in oven dry | | | Average annual ring breadth (*mm*) | | | Number of specimens |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | max. | mean | min. | standard diviation | max. | mean | min. | max. | mean | min. | max. | mean | min. | |
| 1 | 174*hr* 29.0 *min* | 31*hr* 7.2 *min* | 43.9*min* | 33*hr* 9.6 *min* | 6.6 | 4.3 | 2.3 | 0.686 | 0.637 | 0.552 | 3.0 | 1.8 | 1.0 | 42 |
| 2 | 60.0 *min* | 20.4 *min* | 0.4*sec* | 19.7 *min* | 10.8 | 9.2 | 7.5 | 0.643 | 0.594 | 0.535 | 1.6 | 1.2 | 0.8 | 75 |
| 3 | 30.7 *min* | 2.56*min* | 0.2*sec* | 5.65*min* | 11.9 | 10.0 | 7.5 | 0.659 | 0.576 | 0.517 | 1.8 | 1.2 | 0.9 | 69 |
| 4 | 22*hr* 4.2 *min* | 3*hr* 41.6 *min* | 0.0*sec* | 5*hr* 0.3 *min* | 12.5 | 10.8 | 9.1 | 0.686 | 0.624 | 0.549 | 2.8 | 1.8 | 1.0 | 43 |
| 5* | 39.7 *min* | 3.76*min* | 0.1*sec* | 8.22*min* | 11.4 | 9.5 | 7.2 | 0.648 | 0.584 | 0.516 | 2.0 | 1.3 | 0.8 | 80 |
| 6 | 34.7 *sec* | 2.0 *sec* | 0.0*sec* | 5.4 *sec* | 13.0 | 11.0 | 7.9 | 0.632 | 0.576 | 0.516 | 2.0 | 1.3 | 0.9 | 77 |
| 7 | 1.79*min* | 2.3 *sec* | 0.0*sec* | 12.0 *sec* | 15.0 | 12.3 | 10.3 | 0.689 | 0.602 | 0.533 | 1.9 | 1.1 | 0.6 | 80 |
| 8 | 11.3 *sec* | 1.9 *sec* | 0.3*sec* | 2.7 *sec* | 17.7 | 16.5 | 15.9 | 0.619 | 0.585 | 0.509 | 1.9 | 1.2 | 0.9 | 79 |
| 9 | 15*hr* 18.4 *min* | 3*hr* 25.9 *min* | 21.2*sec* | 3*hr* 12.8 *min* | 34.3 | 27.6 | 22.1 | 0.662 | 0.594 | 0.531 | 2.7 | 1.7 | 1.0 | 83 |
| 10 | 1*hr* 21.6 *min* | 2.26*min* | 0.2*sec* | 9.23*min* | 31.9 | 25.1 | 23.4 | 0.690 | 0.589 | 0.538 | 2.2 | 1.4 | 0.8 | 86 |
| 11 | 20.3 *min* | 1.17*min* | 0.5*sec* | 2.50*min* | 30.3 | 27.7 | 25.8 | 0.657 | 0.605 | 0.516 | 2.8 | 1.7 | 0.9 | 80 |
| 12 | 1.2 *sec* | 0.3 *sec* | 0.0*sec* | 0.3 *sec* | 33.8 | 28.8 | 25.3 | 0.649 | 0.576 | 0.514 | 1.8 | 1.3 | 0.9 | 74 |
| 13 | 1.3 *sec* | 0.2 *sec* | 0.1*sec* | 0.5 *sec* | 30.7 | 29.3 | 26.9 | 0.628 | 0.569 | 0.522 | 1.8 | 1.3 | 0.9 | 73 |
| 14 | 7.70*min* | 12.7 *sec* | 0.1*sec* | 52.0 *sec* | 30.7 | 27.0 | 25.9 | 0.640 | 0.572 | 0.534 | 1.8 | 1.3 | 0.8 | 80 |
| 15 | 15.8 *sec* | 1.8 *sec* | 0.1*sec* | 2.3 *sec* | 34.9 | 30.3 | 24.6 | 0.685 | 0.608 | 0.548 | 2.9 | 1.7 | 1.0 | 78 |
| 16 | 1.7 *sec* | 0.4 *sec* | 0.0*sec* | 0.3 *sec* | 24.7 | 22.1 | 20.6 | 0.680 | 0.591 | 0.514 | 2.1 | 1.4 | 0.9 | 83 |
| laminated wood* | 32.3 *min* | 1.32*min* | 0.0*sec* | 4.00*min* | 10.4 | 9.5 | 8.3 | 0.724 | 0.666 | 0.609 | — | — | — | 90 |

* described in section 2

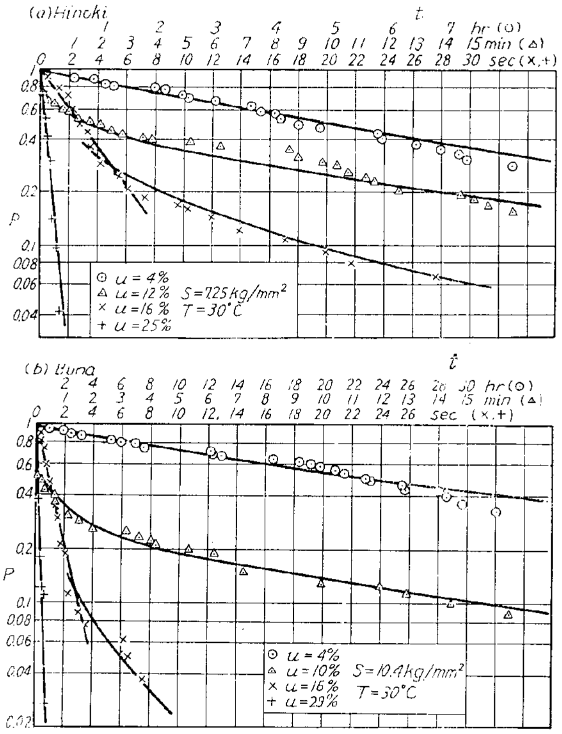


Fig. 8. Log $P-t$ diagram—the effect of moisture content.

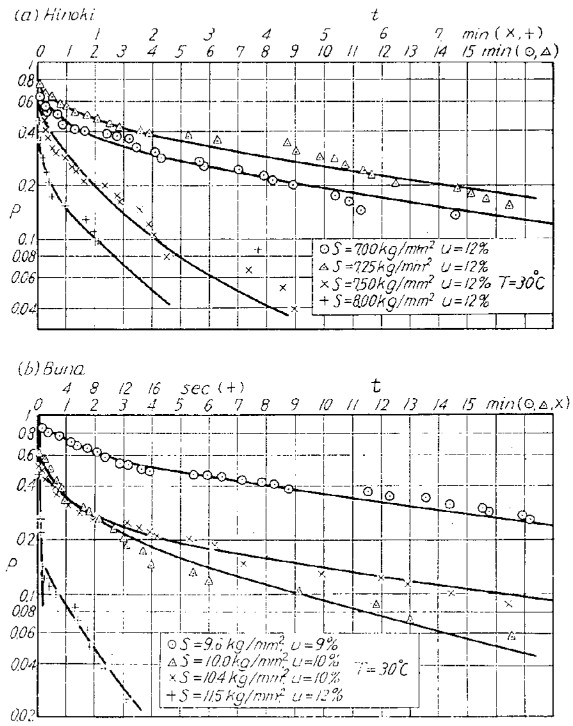


Fig. 9. Log $P-t$ diagram—the effect of applied stress in dry condition.

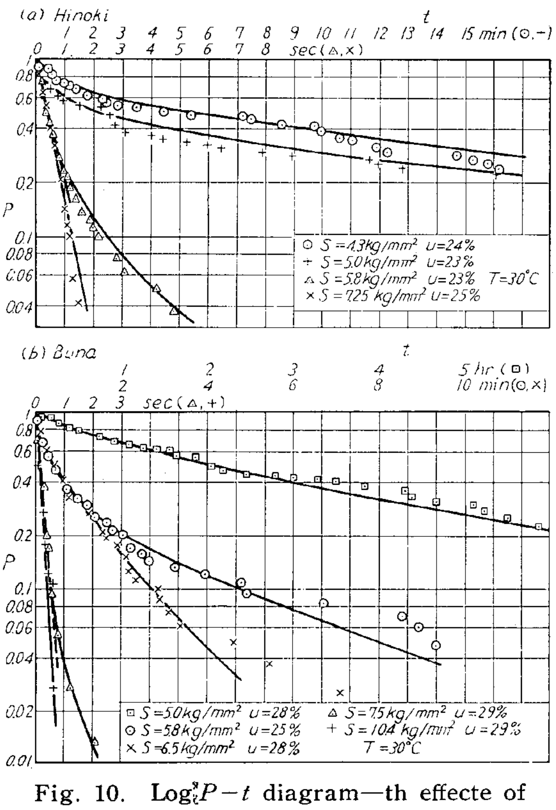


Fig. 10. Log $P-t$ diagram—th effecte of applied stress in wet condition.

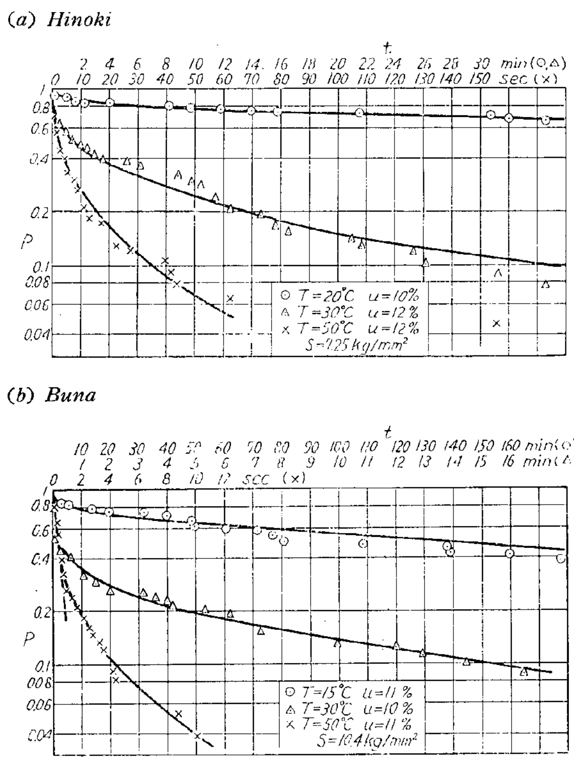


Fig. 11. Log $P-t$ diagram—the effect of temperature in dry condition.

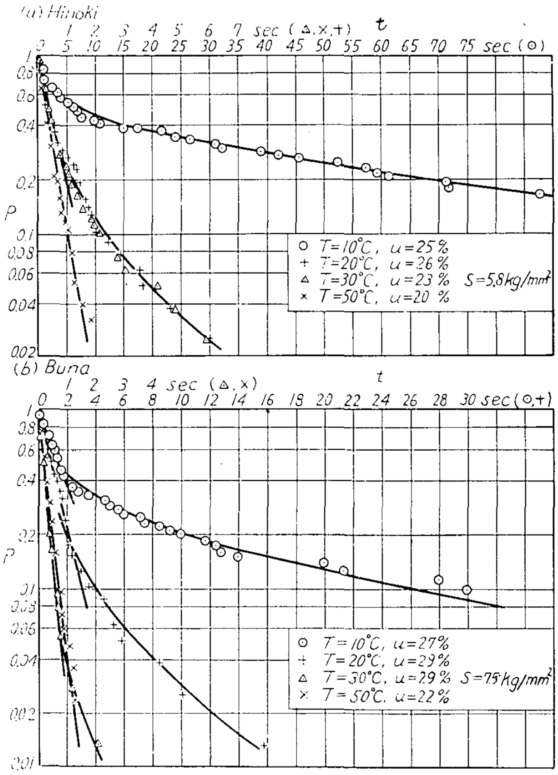


Fig. 12. Log $P-t$ diagram—the effect of temperature in wet condition.

Hereupon, the value of the probability of occurrence of fracture in unit time $m(t)$ is allowed to exceed unit with the unit of time. The real lines in Figs. 8~12 are drawn by substituting these values of $m_0$ for eq. (11) or the values of $A$ and $B$ for eq. (9).

In the high-stressed and moisture-saturated conditions, the dependency of $m(t)$ on time disappears as shown in Figs. 10 and 12. This cause will be discussed below.

The relations between $m_0$, $A$ or $B$ and moisture content $u$, stress $S$ or absolute temperature $T$ are shown in Figs. 13~21. In Figs. 16 and 19, the values of $m_0$ where the mean of moisture content of specimens are very different from others, as in the moisture-saturated condition in high temperature, are adjusted by the straight line in Fig. 13 to the moisture content shown in these figures.

According to these figures, it may be considered that $\log m_0-u$, $-S$, $-1/T$ and $\log A-S$, $-1/T$ are generally linear except in the hight-stressed and moisture-saturated condition, but $B$ scarcely has any connection with $u$, $S$ and $T$. The value of $B$ in high-dried condition, however, seems to be near unit.

Moreover, it will be reasonable to assume that $\log A-u$ is linear except in the high-dried condition, because the time region where $m(t)$ is constant may be fixed as mentioned above. It seems to be that the shift from the linear relation of $\log A-$

Table 3. The values of $m_0$ in eq. (11), $A$ and $B$ in eq. (9), when the unit of time is second.

(a) Hinoki

| Test No. | $m_0$ | $A$ | $B$ |
|---|---|---|---|
| 1 | cannot be decided | 0.000094 | 0.91 |
| 2 | 0.099 | 0.20 | 0.34 |
| 3 | 0.027 | 0.034 | 0.33 |
| 4* | 0.11 | 0.14 | 0.37 |
| 5 | 0.30 | 0.48 | 0.44 |
| 6 | 0.32 | 0.25 | 0.46 |
| 7 | 0.80 | 0.67 | 0.31 |
| 8 | 0.25 | 0.64 | 0.44 |
| 9 | 0.035 | 0.041 | 0.50 |
| 10 | 0.069 | 0.12 | 0.37 |
| 11 | 1.3 | 1.4 | 0.52 |
| 12 | 1.9 | cannot be decided | cannot be decided |
| 13 | 0.25 | 0.30 | 0.41 |
| 14 | 1.7 | 1.4 | 0.56 |
| 15 | 2.1 | cannot be decided | cannot be decided |
| laminated wood* | 0.14 | 0.55 | 0.24 |

(b) Buna

| Test No. | $m_0$ | $A$ | $B$ |
|---|---|---|---|
| 1 | cannot be decided | 0.000030 | 0.89 |
| 2 | 0.030 | 0.046 | 0.50 |
| 3 | 0.17 | 0.21 | 0.38 |
| 4 | 0.053 | 0.034 | 0.33 |
| 5* | 0.28 | 0.37 | 0.27 |
| 6 | 1.3 | 1.3 | 0.40 |
| 7 | 1.9 | 1.6 | 0.33 |
| 8 | 0.74 | 1.4 | 0.43 |
| 9 | cannot be decided | 0.00065 | 0.78 |
| 10 | 0.16 | 0.13 | 0.52 |
| 11 | 0.053 | 0.065 | 0.70 |
| 12 | 3.5 | 3.2 | 0.43 |
| 13 | 5.3 | cannot be decided | cannot be decided |
| 14 | 0.47 | 0.64 | 0.39 |
| 15 | 0.67 | 1.2 | 0.49 |
| 16 | 3.2 | cannot be decided | cannot be decided |
| laminated wood* | 0.085 | 0.35 | 0.41 |

* described in section 2

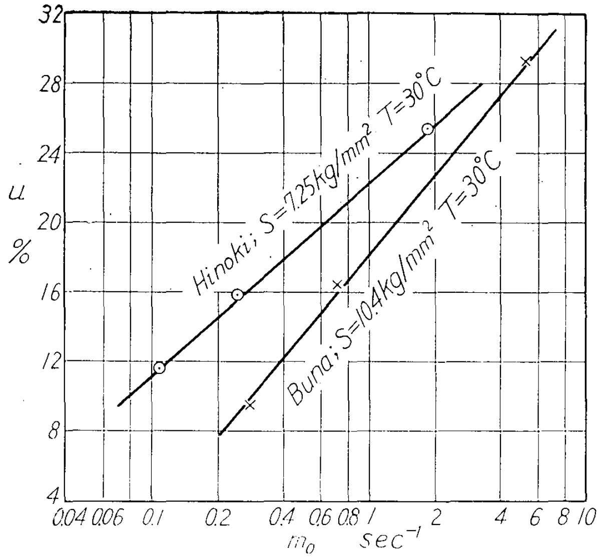


Fig. 13. Relation between moisture content and $m_0$ in eq. (11).

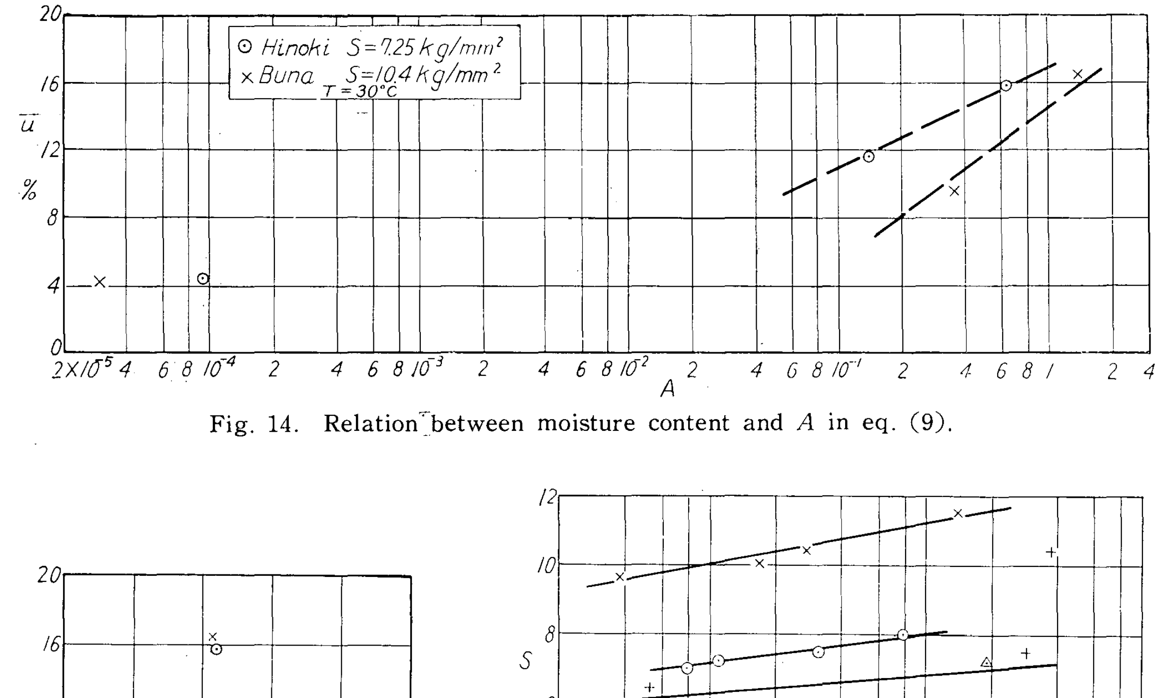


Fig. 14. Relation between moisture content and $A$ in eq. (9).

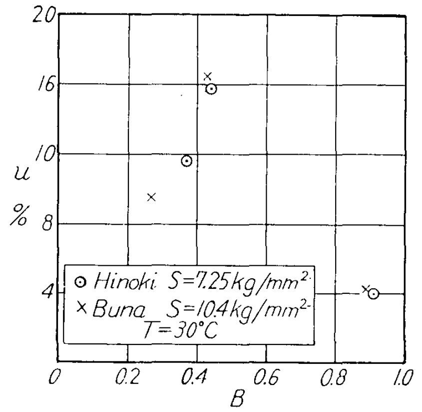


Fig. 15. Relation between moisture content and $B$ in eq. (9).

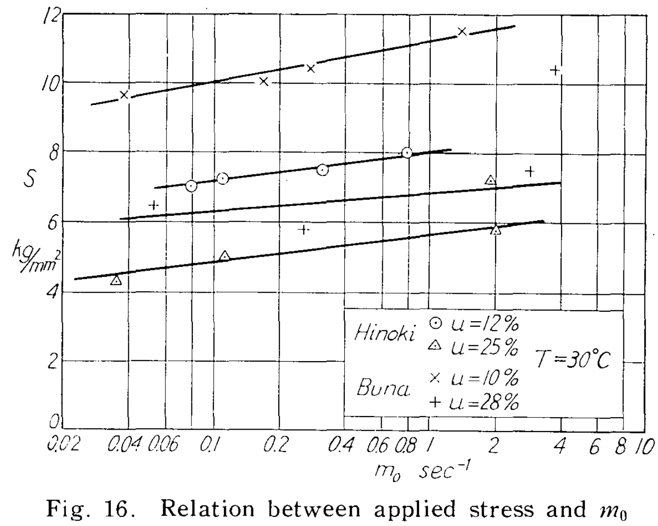


Fig. 16. Relation between applied stress and $m_0$ in eq. (11).

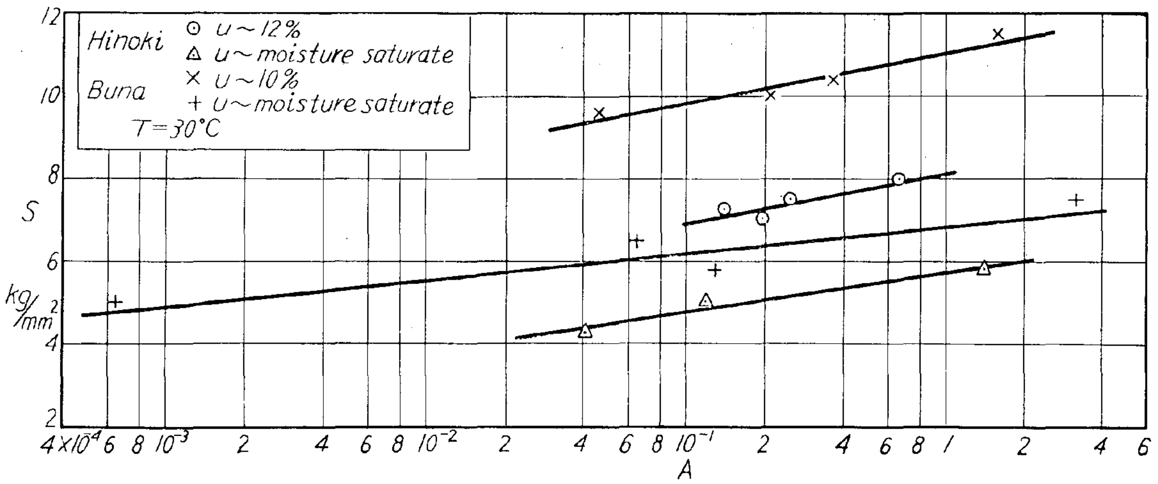


Fig. 17. Relation between applied stress and $A$ in eq. (9).

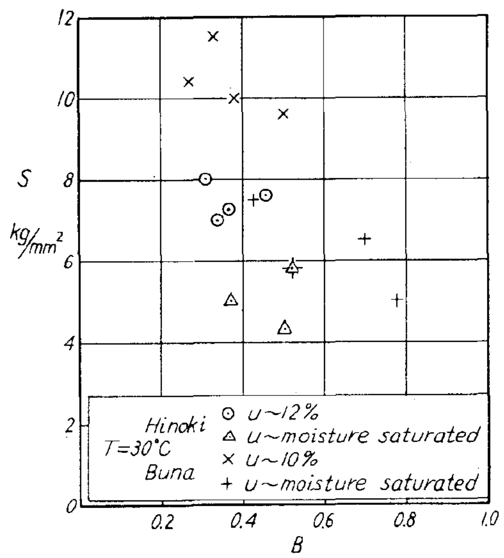


Fig. 18. Relation between applied stress and $B$ in eq. (9).

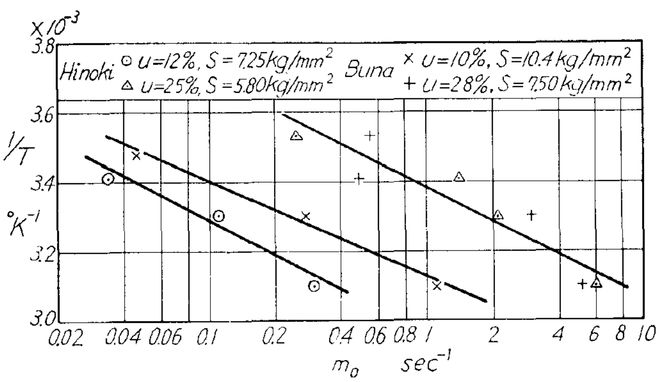


Fig. 19. Relation between temperature and $m_0$ in eq. (11).

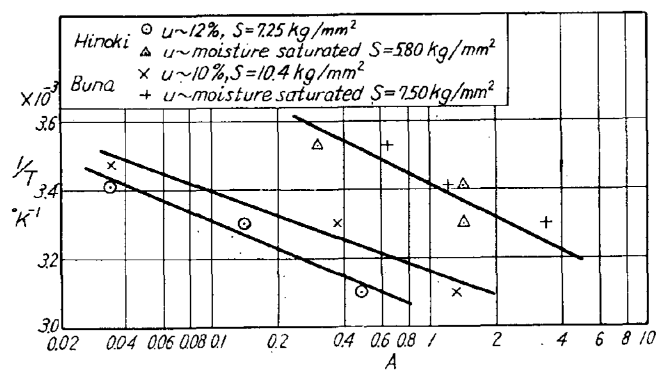


Fig. 20. Relation between temperature and $A$ in eq. (9).

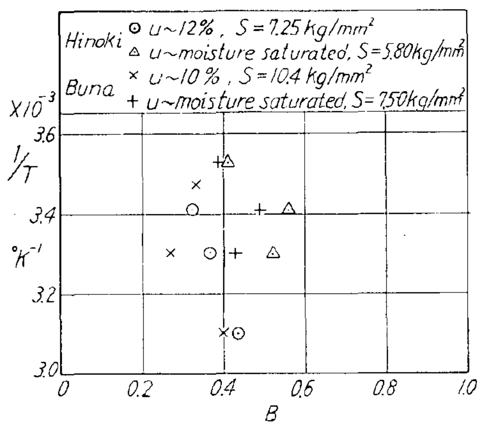


Fig. 21. Relation between temperature and $B$ in eq. (9).

$u$ and the value of $B$ in high-dried condition is essentially concerned with the mechanical properties of wood, as described below, but the the shift from the linear relation of $\log m_0 - S$ in moisture-saturated condition results from the accuracy of the stop-watch (0.1 *sec*) and the technique at the moment of loading.

According to these results, the rate of occurrence of fracture of wood will be indicated by the following equation except in high-dried condition:

$$m(t) = C(t) \cdot \exp\{(-F_0 + \lambda u + aS)/kT\} \quad \cdots\cdots(13)$$

where, $F_0$, $\lambda$, $a$, $k$ = constant.

And $C(t)$ is the function of time only which is indicated by the following representation:

$$C(t) = \text{constant} \quad \cdots\cdots(14)$$

in the region where $m(t)$ is independent of time, or

$$C(t) = Dt^{-(1-B)} \quad \cdots\cdots(14')$$

in the region where $m(t)$ is dependent on time.

On the other hand, the absolute reaction rate $I$ is

$$I = Z(kT/h) \cdot \exp(-F^*/kT) \quad \cdots\cdots(15)^{16)}$$

where, $Z$ = numbers of molecules which come in the reaction

$F^*$ = activation energy

$T$ = absolute temperature

$k$ = BOLTZMANN's constant: $1.38 \times 10^{-16}$ *erg/mol°K*

$h$ = PLANK's constant: $6.62 \times 10^{-27}$ *erg·sec*

When stress acts on a material, the potential energy of molecule becomes to be a function of stress and the activation energy is shown as a monotonous decreasing function of stress[17]. It will be considerable, furthermore, that the activation energy decreases according to the increase of moisture content, because the moistures which penetrate into the cell walls break hydrogen bonds between the fibers and weaken their cohesion. Now, if it is assumed that the activation energy of wood is a monotonous decreasing function of first degree of them:

$$F^* = F_0 - \lambda u - aS \quad \cdots\cdots(16)$$

where, $F_0$ = activation energy of wood which is in the oven dried condition under no stress,

eq. (15) will be almost the same as eq. (13) by substituting eq. (16) for eq. (15). Therefore, it will be considerable that $m(t)$ of wood is a rate of the reaction rate process.

As compared eqs. (13) and (14') with eq. (15), it should be considered that the dependency of the rate of fracture on time, which is observed only in wood, deals with the decrease of numbers of molecules taking part in fracture with lapse of time. This decrease of numbers of molecules will result from the restraint of the thermal motion of molecules caused by the change of internal structure, for example from amorphous to micellous. This change of structure will be delayed because molecules constituting wood are highpolymers, so that the dependency of $m(t)$ on time will appear after the lapse of a few moment.

If the applied stress is too high, the fracture will be finished untill the effect of this structure change on fracture appears, and then the depedency of $m(t)$ on time disappears, as in the high-stressed and moisture-saturated tests. As this structure change will be delayed in the laminated wood by the restraint caused by hardened layers, the time dependency of $m(t)$ appears later than in the solid wood. In the very low moisture content, it will be considered that this thermal motion of molecules will be small and then it will be difficult that this structure change occurs, so that the value of $B$ in eq. (9) will be near unit.

Hereupon, an attention must be given to the fact that $S$ in eq. (14) is the stress calculated by eq. (7)—the uniform stress on the surface of the specimen under the bending load. As there will be many stress-concentrated portions in wood and in the bending test an inclination of stress in a specimen exists, one must integrate the probability density function of a point over whole region to calculate $m(t)$ of wood. But, as almost all of specimens in these experiments ruptured on the tensile side and these ruptures will start, as mentioned above, from the weakest element, it will be right to use the value calculated by eq. (7) as $S$, including the coefficient of concentrated stress into $\alpha$ in eq. (14).

## 4. Analysis of Breaking Strength

In this section, I consider the breaking strength on the standpoint of the theory of stochastic process. To simplify the problem, the test in the constant rate of loading which is the usual test in Japan will be treated here.

The stress $S(t)$ at any time $t$ is in this case

$$S(t)=vt \qquad \cdots\cdots(17)$$

where, $v$ = loading velocity.

On the other hand, from eq. (4) $P$ is

$$P=\exp(-\int_0^t mdt) \qquad \cdots\cdots(18)$$

From eq. (5), that is, $q\cdot dt=(-dP/dt)dt$, then

$$q(t)dt=m\cdot\exp(-\int_0^t mdt)dt \qquad \cdots\cdots(19)$$

or

$$q(S)dS=(m/v)\exp\{(1/v)\int_0^S mdS\}dS \qquad \cdots\cdots(19')$$

Now consider that the strength $\sigma$ is the mode of eq. (19′), and then $\sigma$ will be given by substituting the value of $t$ which satisfies $\partial q/\partial t=0$ *i.e.*

$$dm/dt=m^2 \qquad \cdots\cdots(20)$$

for eq. (17).

It will be possible to use eq. (13) as $m(t)$ of wood in calculation of eq. (20), because it is probable that the mechanism of fracture in the increasing load with lapse of time compares well with those in a constant load. But the time elapsed before fracture in the former is far from a few seconds except the impact test, so that it needs to use eq. (14′) as $C(t)$ in eq. (13). Putting eqs. (17), (14′) in eq. (13) and calculating eq. (20), the following equation is gained :

$$\frac{a\sigma}{kT}-(1-B)=D\left(\frac{\sigma}{v}\right)^B\exp\left(\frac{-F_0+\lambda u+a\sigma}{kT}\right) \qquad \cdots\cdots(21)$$

or

$$\frac{a\sigma}{kT}-\ln\left(\frac{(a\sigma/kT)-(1-B)}{\sigma^B}\right)=B\ln v-\ln D-\frac{-F_0+\lambda u}{kT} \qquad \cdots\cdots(21')$$

Generally $\frac{(a\sigma/kT)-(1-B)}{\sigma^B}\gg 1$, so that $\ln\left(\frac{(a\sigma/kT)-(1-B)}{\sigma^B}\right)\simeq$ constant $(\equiv M)$.

Therefore,

$$\frac{a\sigma}{kT}-M\simeq B\ln v-\ln D-\frac{-F_0+\lambda u}{kT} \qquad \cdots\cdots(22)$$

Eq. (22) shows that the breaking strength increases in proportion to logarithm of the loading velocity and decreases in proportion to the moisture content and absolute temperature. These results are in good agreements with the experimental results[9)10)18)] in a certain region of them. But the tensile or bending strength seems to be indepedent on moisture content or increase in proportion to it near 0% moisture content[18)] and the shift from linearity between $m(t)$ and moisture content is also found at 4%, as shown in Fig. 10. Therefore, near 0% moisture content, the assumption of eq. (16) is not realized, but it may be considered that the activation energy is independent of moisture content or increases in proportion to it.

As $D$ in eq. (22) is in proportion to the numbers of molecules taking part in fracture and the fracture of wood will be affected by the weak defects as shown by GRAF *et al.* in the size effect of tensile strength of wood[19)], these numbers of molecules are in proportion to the numbers of weak defects and the weak defects increase in proportion to the volume of the specimen. So that eq. (22) also shows that the strength decreases in proportion to logarithm of the volume of specimen. Fig. 22 shows my previous results of the size effect of strength[7)20)] plotted in logarithm of volume $V$ *versus* strength $\sigma$. These results agree qualitatively well with eq. (22).

Now consider the mean time elapsed before fracture $\bar{t}$ on the standpoint of stoch-

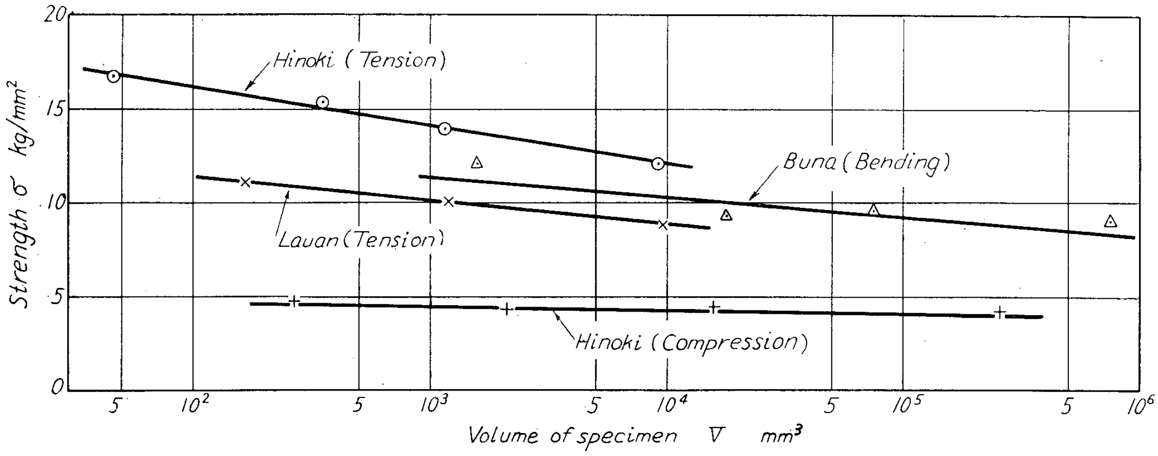


Fig. 22. Relation between strength and volume of specimen.

astic process. Then,

$$\bar{t}=\int_0^{\infty} tq(t)dt \qquad \cdots\cdots(23)$$

From eqs. (13), (14′) and (19),

$$\bar{t}=\int_0^{\infty} D\Omega t^{B} \exp\{-\int_0^{t} D\Omega t^{-(1-B)}dt\}dt \qquad \cdots\cdots(24)$$

where, $\Omega \equiv \exp[(-F_0+\lambda u+\alpha S)/kT]$.

If $S$ is constant,

$$\bar{t}=\left(\frac{D\Omega}{B}\right)^{-1/B}\Gamma\left(1+\frac{1}{B}\right) \qquad \cdots\cdots(25)$$[21]

where, $\Gamma(1+1/B)$ is a gamma function.

Therefore,

$$\frac{\alpha S}{kT}=\ln\left(\frac{B}{D}\right)+B\ln\Gamma\left(1+\frac{1}{B}\right)-\frac{-F_0+\lambda u}{kT}-B\ln\bar{t} \qquad \cdots\cdots(26)$$

Eq. (26) is the very same as eq. (12).

According to these results, it is evident that the fracture of wood can be treated by way of a rate process not only in the constant load, but in the increasing load with lapse of time.

## 5. Mechanism of Fracture

Fracture consists generally of various processes, at least two processes—the origination of microscopic cracks and their growth to macroscopic cracks. In the compression failure of wood, it has been observed that the initial slip lines of the same order as fibril appear at first both on the radial and the tangential plane by the locally concentrated stress which will occur on the weak portions of cell walls and they grow to the slip bands and then macroscopic fracture occurs[22][23]. The process of the bending and tensile fracture in which the crack appears may be analogized from compression failure: plastic deformations and then microscopic cracks occur at first at the portion where the greatest stress acts on and sub-microscopic cracks occur by jointing them and then they grow to a macroscopic crack. These processes may be deduced by Photos. 1 and 2 in the present experiments. Photo. 1 shows a microscopic crack which exists near the front of a main crack and Photo. 2 shows that a round sub-microscopic crack is jointed with others by two main cracks which mainly propagate along the grain, as shown in Photo. 3. Moreover, microscopic cracks may occur on and along the boundary of cell, because of the


Photo. 1. A micro-photograph of a microscopic crack. (×150)
A: the tip of main crack
B: a microscopic crack
Species: Hinoki
Testing condition: moisture content 10%
bending stress $7.00kg/mm^2$
temperature 30°C

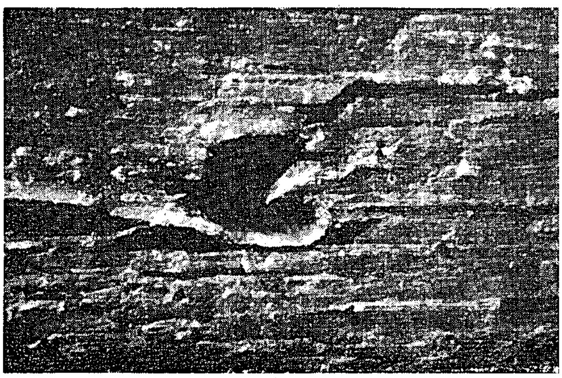

Photo. 2. A micro-photograph of a sub-microscopic crack. (×60)
Species: Buna
Testing condition: moisture content 16%
bending stress $10.4kg/mm^2$
temperature 30°C
Main crack propagates from left to right.

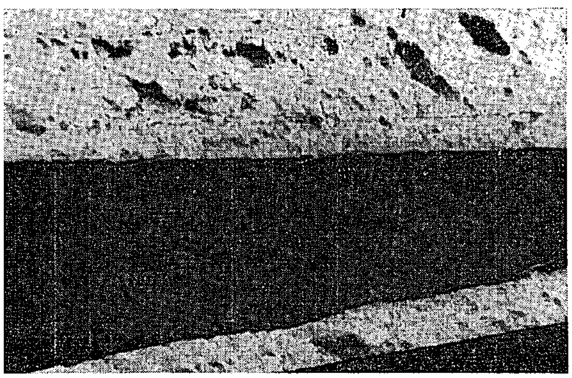

Photo. 3. A typical crack in Buna. (×2)
Testing condition: moisture content 10%
bending stress $10.0kg/mm^2$
temperature 30°C
Crack propagates from left to right

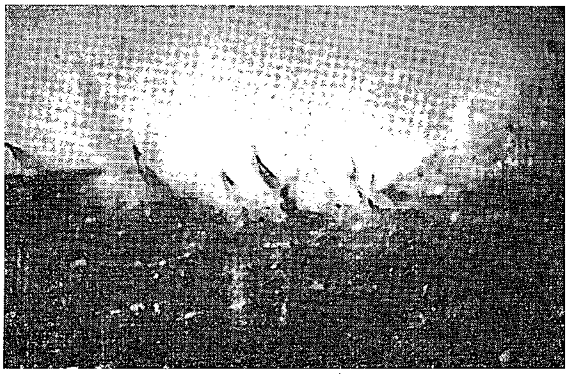

Photo. 4. Separation of cells. (×150)
Species: Hinoki
Testing condition: moisture saturated
bending stress $5.8kg/mm^2$
temperature 10°C
Crack propagates from right to left

difference of the mechanical properties between cell and middle lamella which consists of isotropic materials[23]. The separations of cells on the surface of main crack shown in Photos. 3 and 4 will result from this fact.

In the previous paper[24], I measured the speed of a macroscopic crack propagation parallel to the grain under the tensile load perpendicular to the grain in Hinoki and Buna by an electronic method. In both species, the mode of these speeds were about 1,000 $m/sec$ at 15% moisture content and less than 500 $m/sec$ at water-saturated condition, though their values were scattered very widely—about 70~3,000 $m/sec$ at the former and about 5~3,000 $m/sec$ at the latter.

According to the theory of the absolute rate process, the molecules in the initial state and the activated complex which has the least energy to transfer from the initial state to the last are in equilibrium with each other, so that the velocity of this transfer must be very slow. Therefore, it will be reasonable to consider that a great portion of the time elapsed before fracture in the present experiments is the time required to the origination of microscopic or sub-microscopic cracks which appear at first on the weakest portion of the tensile side of specimens and $m(t)$ described above is the rate of occurrence of these microscopic cracks.

## Summary

In order to clarify the fluctuation of strength and its dependency on moisture content, temperature, loading rate and size of specimens, fracture of wood is treated on the standpoint of the stochastic process.

At first, the fluctuation of the times elapsed before fracture of Hinoki and Buna under a constant bending stress is measuered and it is found that the rate of occurrence of fracture $m(t)$ is constant within a few seconds after loading, but begins to decrease in accordance with eq. (10) with lapse of time after that moment, either in the solid wood or in the laminated wood.

Secondarily, the variations of $m(t)$ under various moisture contents, applied stresses and temperatures are investigated, and then eqs. (13) and (14) in the region where $m(t)$ is independent of time and eqs. (13) and (14′) in the region where $m(t)$ is dependent on time are gained. Therefore, it is considered that $m(t)$ is a rate of the reaction rate process and its dependency on time deals with the decrease of numbers of molecules taking part in fracture, the cause of which will be the restraint of the thermal motion of molecules resulting from the change of internal structure.

Using eqs. (13) and (14′), the dependency of strength under a constant rate of loading on moisture content, temperature, loading rate and size of specimens in a certain region is well explained theoretically. Then it is clarified that the fracture

of wood can be treated by way of a rate process not only in the constant load but in the increasing load with lapse of time.

A great portion of the time elapsed before fracture of wood will be one required to the origination of fracture which will be microscopic or sub-microscopic cracks at the portion of the great stress, because the speed of propagation of macroscopic cracks is very fast. And then $m(t)$ is the rate of occurrence of these microscopic cracks.

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## 摘　　　要

強度におけるいろいろの問題一強度のばらつき，寸法効果，温度，含水率，荷重速度による影響などは本質的には破壊機構の問題に結びついている。これらの間の関係を明らかにするために，木材の破壊をその内部の微細な欠陥より始まると考え，確率過程の立場から取り扱つた。

針葉樹，広葉樹の代表としてヒノキ，ブナを用い，一定曲げ応力下で破壊までに要する時間を測定し，荷重後のある時間 $t$ における単位時間に試片の破壊する確率 $m(t)$ を求めた。木材では荷重後2～3秒の間 $m(t)$ は一定と見なしうるが，その後 (10) 式に従つて時間とともに減少する。この傾向は全く木材特有のものであり，素材，積層材を問わず現われる。

つぎに，種々の含水率，応力，温度における $m(t)$ の変化を調べ，荷重直後 $m(t)$ 一定の領域では $m(t)$ は (13) 式および (14) 式で表わされ，その後は (13) 式および (14′) 式で表わされることを見出した。これらの式は絶対反応速度論における速度と一致することより，$m(t)$ は速度過程の速度と考えられる。また $m(t)$ が時間とともに減少する原因として，内部構造の変化による破壊に関与する分子数の減少が考えられることを指摘した。

(13) 式，(14′) 式を用いて，一定荷重速度下の破壊強度が含水率，温度，荷重速度および試片の寸法に影響される傾向を理論的に導き，実験結果と定性的によく一致する結果をえた。

さらに，木材の破断面の進行速度が非常に速いことより，破壊までに要する時間は主として初期割れ目の発生に要する時間であり，木材に対してはこの初期割れ目の発生過程を速度過程として扱うべきことを指摘した。