Effect of the Degree of Wedge Fixation on the Rotational Behavior of Beam-Column “Nuki” Timber Joint*1

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Introduction

Nuki joint has been used to construct traditional wooden frame structures in Japan. At Nuki joint, beam member that penetrate column, is stiffened by wedges. Rigidity of such Nuki joint mainly depends on the embedment resistance which arises from the contact between the beam and the column, or and the column and the wedges. The compression behavior perpendicular to the grain is rich in ductility, so the wooden structures which have Nuki joints show excellent performance in energy absorbing aspect1). It is reported that the rotational rigidity of Nuki joint, into which wedges were driven strongly, is high2). However, the role of wedges in such Nuki joints has not been clear up yet. In this study, we intended to make the effect of wedge fixation clear theoretically, as well as experimentally.

Estimating Method

From the assumed mechanical model of Nuki joint as shown in Fig. 1, following relationships were obtained.

\[
\sigma_c(x) = k_e(\beta + \theta)x \quad (0 \leq \theta < \theta_y) \quad \text{or} \quad (0 \leq x < X_c, \theta \leq \theta) \\
\sigma_c(x) = k_e(\beta + \theta)X_c + k_{e2}(\beta + \theta)(x-X_c) \\
\sigma_b(x) = k_e(\beta - \theta)x \quad (0 \leq \beta \leq \beta_y) \\
\sigma_b(x) = 0 \quad (\beta > \beta_y) 
\]

If looking at one particular contact point between Nuki and column member standing on the rotation center, the tangential force acting at the point can be separated into two vectors. The vertical force acts as embedment force to the beam, and the horizontal force acts as friction load on contact surface between beam and wedge. This mechanism of load distribution can be expressed as parallel spring model in dependence on a rotational angle.

In a certain rotational angle, two components of moment can be induced from embedment and friction.

\[
M = M_e + M_t \\
M_t = M_{ec} - M_{eo} 
\]

Where

\[
M_{ec} = \frac{1}{12}bh^3k_e(\beta - \theta) \quad (0 \leq \theta < \theta_y) 
\]

Fig. 1. Compressive Stress Distribution in Nuki joint. $k_e$: compressive rigidity of the beam, $k_{e2}$: secondary compressive rigidity of the beam, $\beta$: a slope of initial stress distribution on contact surface of the beam, $\theta$: rotational angle, $\theta_y$: deformation angle when compression yield occurs first, $X_c$: length of the elastic part after compressive yield occurs.

\[
M_{eo} = \frac{1}{24}bh^3k_e(\beta - \theta) \left( \frac{\beta + \theta}{\beta + \theta} \right) \left( \frac{\beta + \theta}{\beta + \theta} \right) \\
+ \frac{1}{12}bh^3k_{e2}(\beta - \theta) \left( 1 - \frac{\beta + \theta}{\beta + \theta} \right) \left( \theta \leq \theta_y \right)
\]

Fig. 2. Column-Beam ('Nuki') cross specimen.
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Experimental - Calculated
Initial Bearing Force 2kN

Fig. 3. Comparison of analytical and experimental moment-deformation angle relationship at each initial bearing force level.

\[ M_{\infty} = \frac{1}{12} bh_c^3 (\beta - \theta) \quad (0 \leq \theta < \beta) \]
\[ M_{\infty} = 0 \quad (\beta \leq \theta) \]
\[ M_t = C \mu (\mu: \text{coefficient of friction}) \]  \hspace{1em} (5)
\[ C = C_c + C_o \]  \hspace{1em} (6)

Where
\[ C_c = \frac{1}{8} bh_c^3 k_c (\beta - \theta) \quad (0 \leq \theta < \theta_y) \]
\[ C_c = \frac{1}{8} bh_c^3 k_c (\beta - \theta) \left[ 2 \left( \frac{\beta + \theta}{2} \right) \left( \frac{\beta + \theta}{2} \right) \right] \left( \frac{\beta + \theta}{2} \right) \left( \frac{\beta + \theta}{2} \right) \]
\[ + \frac{1}{8} bh_c^3 k_c (\beta - \theta) \left[ 1 - \left( \frac{\beta + \theta}{2} \right)^2 \right] \left( \theta_y \leq \theta \right) \]
\[ C_o = \frac{1}{8} bh_c^2 (\beta - \theta) \quad (\theta \leq \theta < \beta) \]
\[ C_o = 0 \quad (\beta \leq \theta) \]

Here, \( \beta \) and \( \theta_y \) are expressed by equation (7).
\[ \beta = \frac{4 C_0}{bh_c^2 k_c}, \quad \theta_y = \frac{2 C_o}{k_c h_c} - \beta \]  \hspace{1em} (7)

Results and Discussion

It was observed that moment-deformation angle relationships were influenced by the magnitude of initial bearing force as shown in Fig. 3. The rotational moment at the primary yield point of specimens under the high initial bearing force was higher than those given low initial bearing force. While initial rotational rigidity of each specimen was almost same and without regard to the amount of initial fixation controlled by wedges penetration depth into the joint. And the significant difference was not found on the amount of energy absorbing under the large deformation.

Fig. 3 also shows the comparison between analytical curve and experimental value on the moment-deformation angle relationship. The analytical initial rigidity was calculated from the elementary beam theory for cantilever beam. From the analytical result, it was shown that the higher the initial bearing force was given to the Nuki joint, the higher the moment was obtained at primary yield point which is derived from friction load. While secondary yield point, caused by the compression yield of the beam, appeared quickly with high initial bearing force. Thus when rotational angle was large, initial bearing force hardly affects upon the moment.

In each initial bearing force level, both results showed good agreement with respect to initial rotational rigidity, rotational moment at primary yield point, slope at plastic stage and especially the locus from the primary yield point to the secondary one. From these results, it was confirmed that the theory based on the assumptions taken the effects of initial bearing force into consideration could expresses the rotational behavior of Nuki joints very well.

References