Acoustic Converting Efficiency and Anisotropic Nature of Wood

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It is accepted that both the acoustic converting efficiency and the degree of anisotropy of wood are important factors for the sound board of musical instruments\(^1,2\). In this paper, the relationship between these factors were clarified experimentally and theoretically. The specific dynamic Young's modulus \((E'/\rho)\) and the loss tangent \((\tan \delta_L)\) in the longitudinal \((L)\) direction, and the dynamic shear modulus \((G')\) and the loss tangent \((\tan \delta_S)\) in the LT plane \((T: \text{tangential direction})\) for 101 kinds of woods were measured by using flexural and torsional vibration methods.

There was a negative correlation between \(E'/\rho\) and \(\tan \delta_L\) as shown in Fig. 1. This fact indicates that smaller mean microfibril angles give larger \(E'/\rho\) and lower \(\tan \delta_L\) values\(^3\). Fig. 2 shows the relationships between the ratio of loss tangents \((\tan \delta_S/\tan \delta_L)\) and that of elastic moduli \((E'/G')\). Relatively large \(E'/G'\) and \(\tan \delta_S/\tan \delta_L\) values of wood reflect its anisotropic nature. Fig. 3 shows the relationship between \((E'/G')\) and \((\tan \delta_S/\tan \delta_L)\). The former reflects the degree of anisotropy and the latter relates to the acoustic converting efficiency \((\sqrt{E'/\rho}/\tan \delta_L)\). There was a positive correlation between them. These acoustic properties can be calculated by using a uniaxial cell wall model in which amorphous isotropic matrix is disposed in parallel along the axis of cellulosic fibrils inclining at \(\theta\) to the L direction of wood\(^4\). The \(E'/\rho, \tan \delta_L, G'/\rho\) and \(\tan \delta_S\) can be expressed by

\[
\frac{E'}{\rho} \approx \frac{\nu}{\rho_w} \left( \frac{1}{E_{w1}} + \frac{\theta^2}{G_{w12}} \right)^{-1}, \quad \tan \delta_L \approx \left( \frac{E_{w1}'}{E_{w1}} + \frac{\theta^2}{G_{w12}} \right) \left( \frac{1}{E_{w1}} + \frac{\theta^2}{G_{w12}} \right)^{-1},
\]

\[
\frac{G'}{\rho} \approx \frac{\nu}{\rho_w} \left( \frac{\sin^2 2\theta}{E_{w1}'} + \frac{\cos^2 2\theta}{G_{w12}'} \right)^{-1}
\]

and

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Fig. 1. The relationship between the loss tangent (tan $\delta_L$) and the specific dynamic Young's modulus ($E'/\rho$) in the longitudinal direction of wood. Note: O, Experimental values; solid line, the regression line of experimental values ($r = -0.632$); dotted line, calculated values.

Fig. 2. The relationship between the ratio of loss tangent (tan $\delta_S$/tan $\delta_L$) and that of elastic moduli ($E'/G'$) of wood. Note: O, Experimental values; solid line, the regression line of experimental values ($r = 0.648$); broken line, calculated values.

\[
\tan \delta_S \approx \left( \frac{E_w}{E_w'^2} \sin^2 2\theta + \frac{G_{w12}}{G_{w12}'^2} \cos^2 2\theta \right) \left( \frac{\sin^2 2\theta}{E_w} + \frac{\cos^2 2\theta}{G_{w12}} \right)^{-1},
\]

where $\nu$ is the volume fraction of $S_2$ layer, $E_w$ and $E_w'$ are the Young's moduli of the cell wall in the parallel (1) and perpendicular (2) to the axis of fibrils, $G_{w12}$ is the shear modulus.
Fig. 3. The relationship between the \((E'/G') (\tan \delta_m/\tan \delta_L)\) and the sound velocity divided by the loss tangent \((\sqrt{E'/\rho} / \tan \delta_L)\) of wood. Note: O, Experimental values; line, the regression line of experimental values \((r=0.752)\) broken line, calculated values.

Cell wall in the 1–2 plane, \(\rho_w\) is the density of the cell wall, respectively. Single and double primes indicate the dynamic modulus and loss modulus, respectively. According to the law of mixtures, \(E_{w1}', E_{w1}''\), \(E_{w2}', E_{w2}''\), \(G_{w12}'\) and \(G_{w12}''\) can be expressed by

\[
E_{w1}' = \varphi E_f + (1 - \varphi) E_m, \quad E_{w1}'' = (1 - \varphi) E_m = (1 - \varphi) E_m' \tan \delta_m,
\]

\[
E_{w2}' \approx E_m' \left(1 + \frac{\varphi}{1 - \sqrt{\varphi}}\right), \quad E_{w2}'' \approx E_m' \left(1 + \frac{\varphi}{1 - \sqrt{\varphi}}\right) \tan \delta_m,
\]

\[
G_{w12}' \approx G_m' \left(1 + \frac{\varphi}{1 - \sqrt{\varphi}}\right) \quad \text{and} \quad G_{w12}'' \approx G_m' \left(1 + \frac{\varphi}{1 - \sqrt{\varphi}}\right) \tan \delta_m,
\]

where \(E_f\) is the Young’s modulus of fibrils along the axis, \(\varphi\) is the volume fraction of fibrils, \(E_m', G_m'\) and \(\tan \delta_m\) are the Young’s modulus, shear modulus and loss tangent of the matrix, respectively. Values of \(\nu=0.84\), \(E_f=134\) GPa, \(\varphi=0.5\), \(E_m=2\) GPa, \(G_m=0.77\) GPa, and \(\tan \delta_m=0.015\) were adopted. Dotted lines in Figs. 1–3 show the calculated values. The calculated values in Fig. 3 predicted that smaller microfibril angles give higher values of acoustic converting efficiency as well as higher degrees of anisotropy.

References