

# Stability, Chaos and Multiple Attractors: A Single Agent Makes a Difference\*

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## Abstract

This paper shows that heterogeneity can be one of the most important factors that generate complex dynamics of the market. Particular stress is laid on how heterogeneity generates qualitative and quantitative differences of dynamics. We consider the standard cobweb market where all suppliers share a quadratic cost function and naive price expectations, and the market demand function is isoelastic.

If all suppliers produce the profit-maximizing amount immediately (henceforth 'naive optimizer') and the price elasticity is greater than one, the market is stable and converges to an equilibrium price. If the price elasticity shrinks due to higher advertising expenditures, e.g., and gets smaller than one, the price trajectory becomes unstable and explodes. Suppose that there appears a different-type of supplier who adjusts adaptively his last period's production in the direction of the production amount that is profit-maximizing under naive expectations (henceforth 'cautious adapter'). Then the market will not explode but begin to behave chaotically. As the number of cautious adapters increases, the amplitude of chaotic behavior of the market becomes smaller. Adaptive behavior stabilizes the cobweb market that is unstable without an adaptive agent. A single adaptive supplier may change the behavior of the market.

On the contrary, as we showed in the previous paper\*\*, if there are exclusively cautious adapters, the unstable cobweb market is either in a stable equilibrium,

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a periodic oscillation or a chaotic situation, depending on the constellation of parameters. In the case of periodic oscillation, there exists the only periodic attractor. If there appears a naive supplier among adaptive ones, then the market may exhibit infinitely many coexisting periodic attractors. A single naive supplier may change the behavior of the market.

*JEL classification:* D21; E32

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