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ORBIT SPACES OF HYPERSPACES

SERGEY A. ANTONYAN

A Peano continuum is a connected and locally connected, compact, metrizable space that contains more than one point. By the Hilbert cube we mean the infinite countable power $[0,1]^\infty$ of the closed unit interval. A Hilbert cube manifold is a separable metrizable space that admits an open cover by sets homeomorphic to open subsets of the Hilbert cube.

Let $G$ be a compact Lie group acting (continuously) on a Peano continuum $X$. We denote by $\exp X$ the $G$-space of all nonempty compact subsets of $X$ endowed with the Hausdorff metric topology and the induced action of $G$.

Here we present the following results and some related open problems.

Theorem 0.1. Let $G$ be a compact Lie group acting nontransitively on the Peano continuum $X$. Then the orbit space $(\exp X)/G$ is homeomorphic to the Hilbert cube.

Theorem 0.2. Let $G$ be a compact Lie group acting on the Peano continuum $X$, and let $\exp_0 X = (\exp X) \setminus \{X\}$. Then the orbit space $(\exp_0 X)/G$ is a Hilbert cube manifold.

Conjecture 0.3. Let $G$ be a compact Lie group acting transitively on the Peano continuum $X$. Then the orbit space $(\exp X)/G$ is not homeomorphic to the Hilbert cube.

Recall that for an integer $n \geq 2$, the Banach-Mazur compactum $BM(n)$ is the set of isometry classes of $n$-dimensional Banach spaces topologized by the famous Banach-Mazur metric.

Corollary 0.4. Let $O(n)$ denote the orthogonal group and $S^{n-1}$ the unit sphere of $\mathbb{R}^n$. Then for all $n \geq 2$, the orbit space $(\exp S^{n-1})/O(n)$ is homeomorphic to the Banach-Mazur compactum $BM(n)$.

Below we assume that $n \geq 2$ is an integer. Let $B^n$ be the closed unit ball of $\mathbb{R}^n$ and let $C(B^n)$ denote the subspace of $\exp B^n$ consisting of all nonempty compact convex subsets $A \subset B^n$ such that $A \cap S^{n-1} \neq \emptyset$.

Theorem 0.5. (1) $C(B^n)$ is homeomorphic to the Hilbert cube.

(2) $C(B^n)$ is an $O(n)$-AR.

(3) The orbit space $C(B^n)/O(n)$ is homeomorphic to the Banach-Mazur compactum $BM(n)$.

Let $SO(n)$ be the special orthogonal group. Consider the $SO(n)$-invariant subset $\text{Sym} S^{n-1} \subset \exp S^{n-1}$ consisting of all the sets $A \in \exp S^{n-1}$ such that $A$ is symmetric with respect to an $(n-1)$-dimensional linear subspace $L_A$ of $\mathbb{R}^n$. It is an intriguing problem to understand the topological structure of $\text{Sym} S^{n-1}$. In particular, we ask the following:
Question 0.6. (1) Is $\text{Sym} \mathbb{S}^{n-1}$ homeomorphic to the Hilbert cube?
(2) Is $\text{Sym} \mathbb{S}^{n-1}$ an $SO(n)$-AR? (an AR?)
(3) What is the topological structure of the orbit space $(\text{Sym} \mathbb{S}^{n-1})/SO(n)$?

Of course, similar questions can be asked about the hyperspaces of all the sets $A \in C(\mathbb{B}^n)$ (respectively, $A \in \exp \mathbb{B}^n$) such that $A$ is symmetric with respect to some $(n-1)$-dimensional linear subspace $L_A$ of $\mathbb{R}^n$.

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