Symbolic-Numeric Optimization for Estimation of Parameters in a Biological Kinetic Model

折居 萃夫
SHIGE0 ORII
富士通株式会社
FUJITSU LTD *

堀本 勝久
KATSUHA ORIMOT0
産業技術総合研究所・生命情報科学研究センター
COMPUTATIONAL BIOLOGY RESEARCH CENTER, AIST †

穴井 宏和
HIROKAZU ANAI
(株)富士通研究所/(独)科学技術振興機構
FUJITSU LABORATORIES LTD / CREST, JST ‡

Abstract
We have been studying a symbolic-numeric optimization for estimation of parameters in biological kinetic models by quantifier elimination (QE), in combination with numerical simulation methods. The optimization method was applied to a model for the inhibition kinetics of HIV proteinase with ten parameters and nine variables. We apply this optimization procedure to three sets of observed data and obtain kinetic parameters by using only one point of each set of the data.

1 Introduction
Many methods for local and global optimization have been developed to model and simulate the global network of biological molecules in a cell [1, 2], and some simulators based on various optimization methods have also been designed (e.g. [3]). In the optimization methods, the estimation of kinetic parameters plays a key role in the development of kinetic models, which, in turn, promotes functional understanding at the system level, for example, in several biological pathways [4, 5]. An answer to the estimation of kinetic parameters is our symbolic-numeric optimization which combines symbolic QE with numerical simulation [6, 7]. In this paper, firstly, we show our procedure of the optimization for the inhibition kinetics of HIV proteinase [8], which includes an enhanced procedure of the offset computation. Secondly, we show that the kinetic parameters for three sets of observed data can be estimated by using only one point of each set of the data.

*ori0@strad.seg.fujitsu.com
†horimoto@cbrc.jp
‡anai@jp.fujitsu.com
2 MATERIALS AND METHODS

2.1 Mathematical Framework

Problem: In this paper, we consider the following fitting problem: the biological kinetic model analyzed here is of the form:

\[ \dot{x}_i = u_i(X, K) \]  

where \( X = \{x_1, \ldots, x_n\} \) is a set of variables, and \( K = \{k_1, \ldots, k_n\} \) is a set of parameters. The problem is to fit the parameters \( K \) of the model to the observed data \( \tilde{X} = \{\tilde{x}_i^t\} \) for \( i = 1, \ldots, n_x, \) \( t = 0, 1, \ldots, n_t \), under the following additional conditions:

(i) Conservation laws: \( h_i(X) = 0 \)

(ii) Variable ranges: \( x_i \in D_i \), where \( D_i = [a, b], a, b \in \mathbb{R} \cup \{\infty\} \).

Basic Formula Here we set up the leading formula of this paper. As mentioned above, we have the following constraints \( \Psi \) with error variables \( \epsilon \) from kinetic models: \( \Psi = \bigwedge_{\epsilon} \psi_{i} \), where \( \psi_{i} = \epsilon_{i} = 0 \). For the error variables we introduce a new variable, \( \epsilon_{\text{max}} \), which means the magnitude of the error variables: \( |\epsilon_{i}| \leq \epsilon_{\text{max}} \). Moreover, for the variables whose observed data is given, we consider the following objective conditions: \( X_i^{(o)} - \tilde{X}_i^{(o)} = 0 \), to achieve fitting. Then the "basic formula" is given as

\[ F(\tilde{X}, X, K, \epsilon_{\text{max}}, \epsilon_i) = (\Psi \wedge h_i(X) = 0 \wedge x_i \in D_i \wedge |\epsilon_{i}| \leq \epsilon_{\text{max}}) \wedge X_i^{(o)} - \tilde{X}_i^{(o)} = 0. \]  

We apply our symbolic-numeric approach to formulas derived by slightly modifying the basic formula according to various purposes.

2.2 Optimization Procedure

We explain the concrete procedure of symbolic-numeric optimization, which consists of six parts as illustrated in Figure 1:

1. Numerical simulation First we prepare simulation data for \( \dot{x}_i \) and \( x_i \), for which we lack observed data, by performing a numerical simulation of the kinetic models.

   1. Set initial conditions \( \tilde{X}^{(0)} \) and starting values for unknown parameters \( \tilde{K}^{(0)} \) as follows: \( \tilde{X}^{(0)} = \{\tilde{x}_i^{(0)}|i = 1, \ldots, n_x\} \) and \( \tilde{K}^{(0)} = K_1^{(0)} \cup K_2^{(0)} \), where \( K_1^{(0)} \equiv \{k_1^{(0)}, \ldots, k_j^{(0)}\} \) are starting values, and \( K_2^{(0)} \equiv \{k_{j+1}^{(0)}, \ldots, k_n^{(0)}\} \) are given fixed parameters.

   2. By numerical simulation of the kinetic model (1), we obtain a time series for \( x_i \) and \( \dot{x}_i \): \( X_i^{(t)} = \{x_i^{(t)}|i = 1, \ldots, n_x, t = 0, 1, \ldots, n_t\} \) and \( \dot{X}_i^{(t)} = \{\dot{x}_i^{(t)}|i = 1, \ldots, n_x, t = 0, 1, \ldots, n_t\} \).

2. Formulation After choosing some variables from \( X \), we call them "focusing variables", \( Y \), and substitute observed/simulated data into the remaining variables:

   1. Choose a subset \( Y \) of \( X : Y \subset X \).

   2. Substitute \( \tilde{X}, X \setminus Y \) in \( F \) by the values of \( \tilde{X}, \tilde{X} \) at a time point \( t \): \( \tilde{X}_t \leftarrow \tilde{X}_t^{(t)}, \tilde{X}_t \leftarrow \tilde{X}_t^{(t)} \), where \( x_t \in \tilde{X}, X \setminus Y \). Then we denote the new formula as \( F'(Y, K_1, \epsilon_{\text{max}}, \epsilon_i) \). We note by performing a QE computation for the formula, \( \exists Y \exists K_1 \forall \epsilon_{\text{max}} \exists \epsilon_i (F') \),
Figure 1: Flowchart of symbolic-numeric optimization. The variables and values are enclosed by the boxes, and the procedures are numbered corresponding to the description in the text.

(3) Computation of offset by QE Observed data often contain an offset. Therefore, we must first determine the offset value. Here we consider the case that the offset appears linearly. For the sake of simplicity, we assume that only $\tilde{x}_1$ has an offset. Let $F'_{offset}$ be the formula obtained by putting $\tilde{x}_1' - offset$ into $\tilde{x}_1^{(\ell)}$ of $F'$, where offset is a variable for offset. By performing QE for $\exists X \exists K_1 \exists e_{max} \exists e_{i}(F'_{offset})$, we obtain the quantifier-free formula $\pi(offset)$, which stands for the feasible ranges of offset. Then we substitute the minimum value of the offset for the variable offset in $F''$, and we denote it again by $F'(Y, K_1, e_{max}, e_{i})$.

(4) Estimation of $e_{max}$ by QE First, we use QE to find the magnitude of $e_{max}$ as small as possible. By computing QE for $F'(Y, K_1, e_{i})$, we obtain a quantifier-free formula $\pi(e_{max})$ describing the feasible ranges of $e_{max}$. Next, we put the minimum value of $e_{max}$ into $e_{max}$ in $F''$, and denote the resulting formula as $F''(Y, K_1, e_{i})$.

And Estimation of $K_1$ by QE We obtain a quantifier-free formula $\pi(K_1)$ describing the feasible ranges of $K_1$ by computing QE for $Y \exists e_{i}(F'')$. Actually, the feasible ranges of $K_1$ are usually sufficiently narrow intervals (e.g., about $10^{-6}$) to choose an appropriate specific value of $K_1$.

(5) Computation of sum of squares ($SSq$) by numerical simulation We estimate the goodness-of-fit for the obtained parameter values $K_1$ from the feasible ranges of $K_1$ in terms of $SSq$.

1. Set initial conditions $\tilde{X}^{(0)}$ and $K_1$.
2. Perform numerical simulation of kinetic model (1).
3. Compute $SSq: SSq = \sum_{t}(x_1^{(t)} - \tilde{x}_1^{(t)})^2$. 

1. Set initial conditions $\tilde{X}^{(0)}$ and $K_1$.
(6) Termination If $SSq$ is smaller than a specific level $\theta$, output $K$. Otherwise, set new initial values and go to (1).

2.3 Biological Model

We analyzed a model for the inhibition kinetics of HIV protease [8], as shown in Figure 2. The protease monomer ($M$) is inactive, but the enzyme ($E$) is active in the dimeric form. The dimer catalyzes the conversion of the substrate ($S$) to the product ($P$). The inhibitor ($I$) is competitive for the substrate and the product, and the inhibitor-binding enzyme is irreversibly deactivated ($EJ$). In the model, there are ten parameters and nine variables. According to the previous studies [8, 9], five parameters ($k_{11}, k_{12}, k_{21}, k_{41}, k_{51}$) are given, and the remaining five unknown parameters ($k_{22}, k_{3}, k_{42}, k_{52}, k_{5}$), two initial values ($E_{\text{init}}, S_{\text{init}}$) and the offset of the fluorimeter are estimated by the present method. The experimental data of the product [$P$], which are composed of 300 data points measured from 0 to 3600 seconds, were downloaded from a web site (http://www.gepasi.org/tutorials/opt/hivfit.html).

Figure 2: Kinetic model for the inhibitor of HIV protease. The start values for ten parameters and the initial values for nine variables [9] are as follows: $k_{11} = 0.1, k_{12} = 10^{-4}, k_{21} = 100, k_{22} = 300, k_{3} = 10, k_{41} = 100, k_{42} = 500, k_{51} = 100, k_{52} = 0.1, \text{and} k_{6} = 0.1; x_{1} = 0, x_{2} = 0.004, x_{3} = 25.0, x_{4} = 0, x_{5} = 0, x_{6} = 0, x_{7} = 0.003, x_{8} = 0, \text{and} x_{9} = 0.$

3 RESULTS

First, we will describe the practical procedure for parameter optimization in the kinetic model for HIV protease, and then we will evaluate the optimized parameters by using only one point of the observed data.

3.1 Procedure for Optimizing Parameters in HIV inhibition Model

To perform the numerical simulation (in (1) of 2.2), $K_1$ and $K_2$, are defined as the five unknown parameters and the five given parameters, and the nine variables are allocated to [$P$, $E$, $S$, $[ES]$, $[M]$, $[EP]$, $[I]$, $[EI]$, and $[EJ]$. Then we set the start value $K^{(0)}$ and the initial value $K^{(0)}$. The start values for ten parameters and the initial values for nine variables are cited from the previous study [9] (see the legend in Figure 2). Also, the two initial values, $E_{\text{init}}$ and $S_{\text{init}}$, are changed within a limited range with reference to the previous studies [8, 9]: 31 discrete values for $([E] = 0.00350, 0.00355, \cdots, 0.00500)$ and 13 values for $([S] = 23.0, 23.5, \cdots, 29.0)$. The focusing variables $Y$ (in (2) of 2.2) are simply obtained by the symbolic computation with QE from the relationship between $X$ and $K_1$ in the model. In the inequality $u(X, K) \Delta t + x_{\ast} \geq 0$, the elimination of $\Delta t$ by QE outputs five inequalities including five parameters:

$$100 \ast [E] \ast [I] - k_{52} \ast [EJ] - k_{6} \ast [EI] > 0, 100 \ast [E] \ast [I] - k_{52} \ast [EI] > 0, 100 \ast [E] \ast [P] - k_{42} \ast [EP] - k_{3} \ast [ES] < 0, 100 \ast [E] \ast [P] - k_{42} \ast [EP] > 0, \text{and} 100 \ast [E] \ast [S] - k_{22} \ast [ES] - k_{3} \ast [ES] > 0.$$
Table 1: Goodness of fit with optimized parameters by symbolic-numeric method. (a) is in the case of I=0, which means no inhibition. (b) is in the case of I=0.0015. (c) is in the case of I=0.003. *Ir* is the iterations number of the symbolic-numeric optimization.

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For reference, the values related to the present optimization are also cited from previous studies [8, 9].
From the last two equations, we can obtain 
\[ \text{offset} = \tilde{x}_1 - 3/125 * ([S_{\text{init}}] - [S] - [EP] - [ES]). \]

By considering the properties of the kinetic model, this equation can be approximated with the observed data. In the initial state, [EP] and [ES] are much less than [S], and as the reaction proceeds, [S] decreases steadily. Therefore, \([S_{\text{init}}] >> [S] - [EP] - [ES]\) at a steady state. Thus, we can obtain 
\[ \text{offset} = (\tilde{x}_1)_{\text{steady}} - 3/125 * [S_{\text{init}}], \]
where \((\tilde{x}_1)_{\text{steady}}\) is a value of \(\tilde{x}_1\). In the present study, we used the value of \((\tilde{x}_1)_{\text{steady}}\) at \(t = 3600\) as the value of \((\tilde{x}_1)_{\text{steady}}\).

Using \(F\) of \(Y\) and the offset obtained above, we can estimate \(e_{\text{max}}\) and \(K_1\) by QE (in (4) of 2.2). Note that 403 sets of \(e_{\text{max}}\) and \(K_1\) are obtained by the corresponding sets of \(E_{\text{init}}\) and \(S_{\text{init}}\). Since the fitting of simulated data strongly depends on the initial values, we further simulate numerically \(E_{\text{init}}\) and \(S_{\text{init}}\) within the above ranges of \(E_{\text{init}}\) and \(S_{\text{init}}\); by a standard technique of the bisection method, \(E_{\text{init}}\) and \(S_{\text{init}}\) for each set of \(e_{\text{max}}\) and \(K_1\) are estimated to minimize the \(SSq\) that is calculated for 300 values of \([P]\) (in (5) of 2.2). Finally, we obtain a set of \(e_{\text{max}}, K_1, E_{\text{init}}\) and \(S_{\text{init}}\) by selecting a minimum \(SSq\) among the 403 \(SSq\)'s.

To judge whether the loop in Figure 1 terminates or not (in (6) of 2.2), the minimum of \(SSq\)'s is compared with the threshold \(\theta\). The threshold is set to 0.04 in the present study.

### 3.2 Observed Data Fitting with the Optimized Parameters

The optimized parameters with the six sets of observed data are listed in Table 1, together with the iteration number, the goodness of fit measured by \(SSq\), the initial values of \(E_{\text{init}}\) and \(S_{\text{init}}\), and the offset. In addition, the fittings of simulated values to the observed data in six cases are described in Figure 3.

One of the remarkable features of the present fitting is that only one point of the observed data are sufficient to fit 300 data points with an \(SSq\) value of less than 0.03. The data point for the optimization is randomly chosen from 300 points of data, and all fittings attain the threshold by one or two iterations of the loop. In one of the six cases, two rounds of iterations were required, but the first fitting in the case agreed well with the observed data. This is partly because QE powerfully restricts the possible ranges of the parameters and the variables, and partly because the present model is simpler than that expected from the complex kinetics of ten parameters and nine variables. These points will be discussed in the following section.

Another feature is that the values of the parameters agree well with those in the previous studies [8, 9]. In particular, the highlighted parameters in this model, the inhibitor binding constant \((k_{52})\) and the deactivation rate constant \((k_6)\), are about 0.10 and 0.097 in the six cases, which are similar values to the constants in one previous study [8]. In contrast, the constants are enormously large in the other previous study [9]. In comparison with both cases, the value in the latter case is unreasonably large for the analysis to be acceptable. Thus, the large dissociation and deactivation rate constants suggest that the potency of the inhibitor is overestimated in terms of the inhibitor reaction.

### 4 DISCUSSION

Two problems in the present optimization remain: the first of them is the choice of the observed data for the optimization, and the second of them is the choice of \((\tilde{x}_1)_{\text{steady}}\) in the offset computation. As for the first problem, the data showing a flat slope in the kinetic curve seem intuitively inadequate for
Figure 3: Fitting to observed data with optimized parameters. The amount of product $[P]$ is multiplied by a coefficient (0.024), according to [9]. The experimental data are denoted by the dots. a and b are in the case of $I = 0$. a is the carve of minimum $SSq (=0.00758)$ and b is the carve of maximum $SSq (=0.00951)$ in the (a) of Table 1. c and d are in the case of $I = 0.0015$. c is $SSq = 0.0321$ and d is $SSq = 0.0320$ in the (b) of Table 1. e and f are in the case of $I = 0.003$. e is the carve of minimum $SSq (=0.00795)$ and f is the carve of maximum $SSq (=0.00835)$ in the (c) of Table 1. The arrow of each figure denotes the time of the observed value used for the symbolic-numeric optimization: a, $t=336$; b, $t=1848$; c, $t=336$; d, $t=984$; e, $t=984$; f, $t=1848$. 
the optimization. Indeed, by using the data of more than $t=2500$ in Figure 3, QE frequently outputs 'false'; this means no parameter and variable spaces for the initial conditions in $F'$. Any data, except for those in the steady states, may possibly output 'true' for the optimization by QE. As for the second problem, fluctuation of data in steady states is the cause of large $SSq$ value. In the case of $I=0$ and $I=0.003$ (see a, b, e, f of Figure 3), there is small fluctuation in the steady states. However, large fluctuation appears in the case of $I=0.0015$ (see c, d of Figure 3) and $(\tilde{z}_1)_{steady}$ of $t=3600$ is a lower value of the steady state. The estimated curve is adjusted to the point is the problem. A rule of data selection is required to attain more good $SSq$ value.

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