数理解析研究所講究録
1536 巻 2007 年 1-18

On the Oseen semigroup with rotating effect

Yoshihiro SHIBATA*
Department of Mathematical Sciences, School of Science and Engineering,
Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan.
e-mail address: yshibata@waseda.jp †

1 Introduction

Consider a rigid body $\mathcal{R}$ moving through an incompressible viscous fluid $\mathcal{L}$ that fills the whole three-dimensional space exterior to $\mathcal{R}$. We assume that with restect to a frame attached to $\mathcal{R}$, the translational velocity u_∞ and the angular velocity ω of $\mathcal{R}$ are both constant vectors. Without loss of generality we may assume that $\omega = {}^T(0,0,a)$ [1]. If the flow is non-slip at the boundry, then the motion is described by the following equation:

$$
(1) \qquad \begin{aligned}
v_t + v\cdot\nabla v - \Delta v + \nabla\pi &= g && \text{in } \Omega(t), && t>0\\
\operatorname{div} v &= 0 && \text{in } \Omega(t), && t>0\\
v(y,t) &= \omega\times y && \text{on } \partial\Omega(t), && t>0\\
v(y,t) &\to u_\infty \neq 0 && \text{as } |y|\to\infty, && t>0\\
v(y,0) &= v_0(y)
\end{aligned}
$$

in the time-dependent exterior domain

$$\Omega(t) = O(at)\Omega$$

where $O(t)$ denotes the orthogonal matrix

$$O(t) = \begin{pmatrix} \cos t & -\sin t & 0\\ \sin t & \cos t & 0\\ 0 & 0 & 1\end{pmatrix}$$

and Ω is a fixed exterior domain in $\mathbb{R}^3$ with $C^{1,1}$ boundary $\partial\Omega$.

*Partly supported by Grants-in-Aid for Scientific Research (B) - 15340204, Japan Society for the Promotion of Science.
†*MOS Subject Classification*: 35Q30, 76D05, 76D07
Keywords: C^0 semigroup, L_p-L_q estimate, rotating body, exterior domain, Oseen equations.
[1] ${}^TM$ denotes the transposed M.

Over the last several years, the study of well-posedness of the initial boundary value problem of (1) and related topics has attracted the attention of several authors; see, e.g.[1]–[36]. Besides the intrinsic mathematical interest, this is probably also due to the fact that problem (1) is at the foundation of several important engineering applications. The interested reader is refered to [11] and to the literature cited therein.

To treat (1) in the time-independent domain Ω, we introduce

$$
\begin{aligned}
x &= O(t)^T y, \\
u(x,t) &= O(t)^T (v(y,t) - u_\infty), \\
p(x,t) &= \pi(y,t)
\end{aligned} \tag{2}
$$

Then, we see that (u,p) satisfies the modified Navier-Stokes equations:

$$
\begin{aligned}
& u_t + u\cdot\nabla u - \Delta u + (O(t)^T u_\infty)\cdot\nabla u \\
&\quad -(\omega\times x)\cdot\nabla u + \omega\times u + \nabla\pi = f && \text{in } \Omega\times(0,\infty) \\
&\quad \operatorname{div} u = 0 && \text{in } \Omega\times(0,\infty) \\
&\quad u(x,t) = \omega\times x - O(t)u_\infty && \text{on } \partial\Omega\times(0,\infty) \\
&\quad u(x,t)\to 0 && \text{as } |x|\to\infty,\ t>0 \\
&\quad u(x,0) = v_0(x)
\end{aligned} \tag{3}
$$

In this paper, we consider only the case where

$$u_\infty = k\mathbf{e}_3$$

with $\mathbf{e}_3 = {}^T(0,0,1)$ so that

$$O(t)^T u_\infty = k\mathbf{e}_3 \quad \text{for all } t>0.$$

Therefore, the equation (1) leads to the system:

$$
\begin{aligned}
& u_t + u\cdot\nabla u - \Delta u + k\partial_3 u \\
&\quad -(\omega\times x)\cdot\nabla u + \omega\times u + \nabla\pi = f && \text{in } \Omega\times(0,\infty), \\
&\quad \operatorname{div} u = 0 && \text{in } \Omega\times(0,\infty), \\
&\quad u(x,t) = \omega\times x - k\mathbf{e}_3 && \text{on } \partial\Omega\times(0,\infty), \\
&\quad u(x,t)\to 0 && \text{as } |x|\to\infty,\ t>0 \\
&\quad u(x,0) = v_0(x)
\end{aligned} \tag{4}
$$

One of the interesting questions is to find the unique existence of physically reasonable stationary solutions to (4) and to show their stability by initial disturbace in the $L_3(\Omega)$ framework when the external force f is independent of time variable, which is an extension of results obtained in [47] to the case where $\omega \neq 0$. But, so far we could not show the existence of stationary solutions of (4) which behave like

$O((|x|s(x)^{\epsilon})^{-1})$ and the gradient of which behave like $O((|x|^{\frac{3}{2}}s(x)^{\frac{1}{2}+\epsilon})^{-1})$ with some small positive constant ϵ as $|x| \to \infty$ (cf. [47], [48] in the case where $a = 0$ and also Theorem 4 below), where $s(x) = |x| + x_3$. Therefore, in this paper we shall only give the existence of solutions and their decay properties of the linearized equations, and assuming the existence of physically reasonable stationary solutions of (4), we shall state their stability theorem.

The crucial step in proving the stability theorem of stationary solutions is to show the L_p-L_q decay estimate of the solutions to the following linearized equations of (4):

$$
\begin{aligned}
u_t + L_{k,a}u + \nabla\pi &= 0 \quad \text{in} \quad \Omega, \ t > 0, \\
\nabla \cdot u &= 0 \quad \text{in} \quad \Omega, \ t > 0, \\
u &= 0 \quad \text{on} \quad \Gamma, \\
u|_{t=0} &= f \quad \text{in} \quad \Omega,
\end{aligned}
\tag{5}
$$

Here,

$$L_{k,a}u = -\Delta u + k\partial_3 u - (\omega \times x) \cdot \nabla u + \omega \times u$$

$\omega = (0, 0, a)$, k and a are real constants, and $u = (u_1, u_2, u_3)$ and π are unknown velocity field and pressure, respectively. The equation in (5) is written componentwise as follows:

$$
\begin{aligned}
&\frac{\partial u_1}{\partial t} - \Delta u_1 + k\frac{\partial u_1}{\partial x_3} - a\Big(x_1\frac{\partial u_1}{\partial x_2} - x_2\frac{\partial u_1}{\partial x_1}\Big) - u_2 + \frac{\partial \pi}{\partial x_1} = 0 \\
&\frac{\partial u_2}{\partial t} - \Delta u_2 + k\frac{\partial u_2}{\partial x_3} - a\Big(x_1\frac{\partial u_2}{\partial x_2} - x_2\frac{\partial u_2}{\partial x_1}\Big) + u_1 + \frac{\partial \pi}{\partial x_2} = 0 \\
&\frac{\partial u_3}{\partial t} - \Delta u_3 + k\frac{\partial u_3}{\partial x_3} - a\Big(x_1\frac{\partial u_3}{\partial x_2} - x_2\frac{\partial u_3}{\partial x_1}\Big) + \frac{\partial \pi}{\partial x_3} = 0
\end{aligned}
$$

To show the existence of solutions to (5), we use the semigroup approach. Since the pressure term π has no time evolution, we have to eliminate π by using the Helmholtz decomposition. Therefore, at this stage we shall introduce the Helmholtz decomposition. Let D be one of $\mathbb{R}^3$, Ω and $\Omega_R = \Omega \cap B_R$ ($B_R = \{x \in \mathbb{R}^3 \mid |x| < R\}$). Let R be a large number such that $B_{R-5} \supset \mathbb{R}^3 \setminus \Omega$ and $1 < q < \infty$. Set

$$
\begin{aligned}
&L_q(D)^3 = \{f = (f_1, f_2, f_3) \mid f_i \in L_q(D) \ (i = 1, 2, 3)\}, \\
&J_q(D) = \overline{C^{\infty}_{0,\sigma}(D)}^{L_q(D)} \\
&G_q(D) = \{\nabla\pi \mid \pi \in \hat{W}^1_q(D)\} \\
&C^{\infty}_{0,\sigma}(D) = \{u \in C^{\infty}_0(D)^3 \mid \operatorname{div} u = 0 \ \text{in} \ D\} \\
&\hat{W}^1_q(D) = \{\pi \in L_{q,\mathrm{loc}}(\overline{D}) \mid \nabla\pi \in L_q(D)^3, \int_{\Omega_R} \pi \, dx = 0\}
\end{aligned}
$$

Then, we have the Helmholtz decomposition:

$$L_q(D)^3 = J_q(D) \oplus G_q(D), \quad \oplus : \text{direct sum}$$

(cf. [39], [41], [46], [49]). In fact, given $f = (f_1, f_2, f_3) \in L_q(D)^3$ let π be a weak solution to the Laplace equation:

$$\Delta\pi = \operatorname{div} f \ \text{ in } D, \quad \partial_\nu \pi = \nu \cdot f \tag{6}$$

Here, $\nu = (\nu_1, \nu_2, \nu_3)$ is the unit outer normal to ∂D and $\partial_\nu = \nu \cdot \nabla$. Setting $g = f - \nabla\pi$, we have the decomposition: $f = g + \nabla\pi$, which is required one. In particular, we know that

$$J_q(D) = \{g \in L_q(D) \mid \operatorname{div} g = 0 \ \text{ in } D, \ \nu \cdot g|_{\partial D} = 0\}$$

When $f = g + \nabla\pi$ with $g \in J_q(D)$ and $\pi \in \hat{W}^1_q(D)$, we set

$$P_D f = g, \quad Q_D f = \pi \tag{7}$$

and therefore $P_D : L_q(D)^3 \to J_q(D)$ and $Q_D : L_q(D)^3 \to \hat{W}^1_q(D)$ are both bounded linear operators.

Now, using the Helmholtz decomposition we shall formulate (5) in the semigoup setting. Let us define the operator $\mathcal{L}_D$ by the formula:

$$\mathcal{L}_D u = P_D L_{k,a} u = P_D(-\Delta u + k\partial_3 u - (\omega \times x) \cdot \nabla u + \omega \times u)$$

with domain:

$$\mathcal{D}_q(D) = \{u \in J_q(D) \cap W^2_q(D) \mid u|_{\partial D} = 0, \ (\omega \times x) \cdot \nabla u \in L_q(D)\}$$

And then, problem (5) is written as follows:

$$\begin{aligned} u_t + \mathcal{L}_\Omega u &= 0 \quad \text{in } J_q(\Omega) \quad \text{for } t > 0 \\ u|_{t=0} &= f \\ u(t) &\in \mathcal{D}_q(\Omega) \qquad\quad \text{for } t > 0 \end{aligned} \tag{8}$$

One of the significant characters of the operator $\mathcal{L}_\Omega$ is that the crucial drift operator $(\omega \times x) \cdot \nabla$ is never subordinate to the viscous term Δ and equation (8) provides both parabolic and hyperbolic features. In fact, the following theorem was proved by Farwig, Nečasová and Neustupa: [9], [10].

Theorem 1. *The essential spectrum of $\mathcal{L}_\Omega$ coincides with*

$$\bigcup_{j=-\infty}^{\infty} \{\sqrt{-1}aj + \{\lambda \in \mathbb{C} \mid k^2 \operatorname{Re} \lambda + (\operatorname{Im} \lambda)^2 > 0\}\}$$

From Theorem 1, we know that the operator $\mathcal{L}_\Omega$ does not generate an analytic semigroup. But, we can show the generation of the continous semigroup and its decay estimate. Namely, the following two theorems are main results which the author would like to report in this paper.

Theorem 2. *Let $1 < q < \infty$. Then, $\mathcal{L}_\Omega$ generates a C^0 semigroup $\{T(t)\}_{t\geqq 0}$ on $J_q(\Omega)$.*

Theorem 3. *There hold the following estimates for $f \in J_q(\Omega)$ and $t > 0$:*

$$\|T(t)f\|_{L_r(\Omega)} \leqq C_{q,r}t^{-\frac{3}{2}\left(\frac{1}{q}-\frac{1}{r}\right)}\|f\|_{L_q(\Omega)} \qquad (1 < q \leqq r \leqq \infty,\ q \neq \infty) \tag{9}$$

$$\|\nabla T(t)f\|_{L_r(\Omega)} \leqq C_{q,r}t^{-\frac{1}{2}-\frac{3}{2}\left(\frac{1}{q}-\frac{1}{r}\right)}\|f\|_{L_q(\Omega)} \qquad (1 < q \leqq r \leqq 3) \tag{10}$$

Here, the constant $C_{q,r}$ depends on a_0 and k_0 whenever $|a| \leqq a_0$ and $|k| \leqq k_0$ but is independent of a, k, t and f.

Remark 1. Theorem 2 was first proved by Hishida [19] when $q = 2$ and $k = 0$. Later on, Geissert-Heck-Hieber [16] proved Theorem 2 when $1 < q < \infty$ and $k = 0$. Our proof is different from [19] and [16] and based on some new consideration on the pressure terms.

Theorem 3 plays an essential role in proving the stability theorem. In fact, the estimates (7) and (10) were proved by Iwashita [42] when $a = k = 0$, by Kobayashi and Shibata [44] when $a = 0$ and $k \neq 0$ and by Hishida and Shibata [24] when $a \neq 0$ and $k = 0$. The restriction: $1 < q \leqq r \leqq 3$ in (10) is unavoidable at least in the case where $a = k = 0$, which was proved by Maremonti-Solonnikov [45].

2 Remark on the stability theorem

Consider the original non-linear problem (4) in the case where $f = f(x)$. And then, the corresponding stationary problem is given as

$$\begin{aligned} (11) \qquad & w \cdot \nabla w - \Delta w + k\partial_3 w \\ & \quad -(\omega \times x)\cdot \nabla w + \omega \times w + \nabla\theta = f && \text{in } \Omega \\ & \operatorname{div} w = 0 && \text{in } \Omega \\ & w = \omega \times x - k\mathbf{e}_3 && \text{on } \partial\Omega \\ & w(x) \to 0 && \text{as } |x| \to \infty \end{aligned}$$

Setting $u(x,t) = w(x)+z(x,t)$ and $\pi(x,t) = \theta(x)+\kappa(x,t)$ in (4), we have the equation for z and κ as follows:

$$
\begin{aligned}
(12)\quad & z_t + z\cdot\nabla z + z\cdot\nabla w + w\cdot\nabla z - \Delta z + k\partial_3 z \\
& \qquad -(\omega\times x)\cdot\nabla z + \omega\times z + \nabla\kappa = 0 && \text{in } \Omega\times(0,\infty) \\
& \qquad \operatorname{div} z = 0 && \text{in } \Omega\times(0,\infty) \\
& \qquad z = \omega\times x - ke_3 && \text{on } \partial\Omega\times(0,\infty) \\
& \qquad z(t,x) \to 0 && \text{as } |x|\to\infty \quad t>0 \\
& \qquad z(0,x) = v_0(x) - w(x)
\end{aligned}
$$

Following Kato [43], instead of (12) we consider the integral equation:

$$(13)\qquad z(t) = T(t)(v_0 - w) - \int_0^t T(t-s)[z(s)\cdot\nabla z(s) + z(s)\cdot\nabla w + w\cdot\nabla z(s)]\,ds$$

Then, using Theorem 2 and employing the same argument as in Shibata [47] we have the following theorem.

Theorem 4 (Stability Theorem). *Assume that problem* (11) *admits solutions* $w(x)$ *and* $\theta(x)$. *Let* σ *be a small positive number and* $3 < q < \infty$. *Then, there exists a small positive number* ϵ *depending on* σ *and* q *such that if* $v_0 - w \in J_3(\Omega)$ *and*

$$(14)\qquad \|w\|_{L_{3-\sigma}(\Omega)\cap L_{3+\sigma}(\Omega)} + \|\nabla w\|_{L_{(3/2)-\sigma}(\Omega)\cap L_{(3/2)+\sigma}(\Omega)} + \|v_0 - w\|_{L_3(\Omega)} \leqq \epsilon$$

then problem (13) *admits a unique solution*

$$z(t) \in C([0,\infty), J_3(\Omega)) \cap C((0,\infty), L_q(\Omega)\cap W_3^1(\Omega))$$

such that

$$
\begin{aligned}
& [z]_{3,0,t} + [z]_{q,\mu(q),t} + [\nabla z]_{3,1/2,t} \leqq \sqrt{\epsilon} \\
& \lim_{t\to 0+} \|z(t) - (v_0 - w)\|_{L_3(\Omega)} = 0 \\
& \lim_{t\to 0+} ([z]_{q,\mu(q),t} + [\nabla z]_{3,1/2,t}) = 0
\end{aligned}
$$

Here, we have set

$$[z]_{p,\rho,t} = \sup_{0<s<t} s^{\rho}\|z(\cdot,s)\|_{L_p(\Omega)}, \quad \mu(q) = \frac{3}{2}\left(\frac{1}{3} - \frac{1}{q}\right) = \frac{1}{2} - \frac{3}{2q}$$

Remark 2. Galdi and Silvestre [14] proved the existence of solutions to (11), but the velocity fields in [14] behave $O(|x|^{-1})$ when $|x|\to\infty$ and they did not show the asymptotic behaviour of the gradient of the velocity fields. We do not know any existence theorem of solutions to (11) which satisfy (14) as far as the author checked. But, the author believes the existence of stationary solutions satisfying (14), because the motion is stabilized by translation.

3 Analysis of the whole space problem

To show theorems 2 and 3, we start with the analysis of the whole space probem:

$$
(15) \qquad \begin{aligned} u_t + L_{k,a}u + \nabla\pi = 0 \quad &\text{in} \quad \mathbb{R}^3, \ t > 0, \\ \nabla \cdot u = 0 \quad &\text{in} \quad \mathbb{R}^3, \ t > 0, \\ u|_{t=0} = f \quad &\text{in} \quad \mathbb{R}^3, \end{aligned}
$$

with initial data $f \in J_q(\mathbb{R}^3)$. We know that the solution u is given by the following formula:

$$
\begin{aligned} u(t) = S_{\mathbf{R}^3}(t)f &= \int_{\mathbf{R}^3} \frac{\exp\left(-\frac{|O(at)x-y-ke_3t|^2}{4t}\right)}{(4\pi t)^{3/2}} O(at)^T P_{\mathbf{R}^3}f(y)\,dy \\ &= \mathcal{F}^{-1}\left[e^{-(|\xi|^2+ik\xi_3)t}O(at)^T\mathcal{F}[P_{\mathbf{R}^3}f](O(at)\xi)\right](x) \end{aligned}
$$

Here, $P_{\mathbf{R}^3}$ and $Q_{\mathbf{R}^3}$ are defined as follows:

$$
P_{\mathbf{R}^3}f = \mathcal{F}^{-1}\left[\left(\delta_{jk} - \frac{\xi_j\xi_k}{|\xi|^2}\right)\hat{f}(\xi)\right](x), \quad Q_{\mathbf{R}^3}f = \mathcal{F}^{-1}\left[\frac{\xi\cdot\hat{f}(\xi)}{i|\xi|^2}\right](x) + c(f)
$$

and $c(f)$ is a constant such that $\int_{\Omega_R} Q_{\mathbf{R}^3}f\,dx = 0$. Set

$$
\begin{aligned} \mathcal{A}_{\mathbf{R}^3,a,k}(\lambda)f = \mathcal{A}(\lambda)f &= \int_0^\infty e^{-\lambda t}S_{\mathbf{R}^3}(t)P_{\mathbf{R}^3}f\,dt \\ &= \mathcal{F}^{-1}\Big[\int_0^\infty e^{-(\lambda+|\xi|^2+i\xi_3k)t}O(at)^T\widehat{P_{\mathbf{R}^3}f}(O(at)\xi)\,dt\Big](x) \end{aligned}
$$

Then, we have the following two theorems:

Theorem 5. *Let* $1 < q < \infty$, $k_0 > 0$ *and* $a_0 > 0$. *Assume that* $|k| \leqq k_0$ *and* $|a| \leqq a_0$.
(1) *Let* $\gamma > 0$, $0 < \epsilon < \pi/2$ *and* $N \in \mathbb{N}$ *with* $N \geqq 4$. *Set*

$$
\begin{aligned} &\mathbb{C}_\gamma = \{\lambda \in \mathbb{C} \mid \mathrm{Re}\,\lambda \geqq \gamma\}, \quad \mathbb{C}_+ = \{\lambda \in \mathbb{C} \mid \mathrm{Re}\,\lambda > 0\}, \\ &\Sigma_\epsilon = \{\lambda \in \mathbb{C}\setminus\{0\} \mid |\arg\lambda| \leqq \pi - \epsilon\}, \\ &L_{q,R+2}(\mathbb{R}^3) = \{f \in L_q(\mathbb{R}^3)^3 \mid f(x) = 0 \ \textit{for}\ x \notin B_{R+2}\}, \\ &\mathcal{L}_R(\mathbb{R}^3) = \mathcal{L}(L_{q,R+2}(\mathbb{R}^3), W_q^2(\mathbb{R}^3)^3) \end{aligned}
$$

Then, $R(\lambda)P_{\mathbf{R}^3} \in \mathrm{Anal}(\mathbb{C}_+, \mathcal{L}_R(\mathbb{R}^3))$ *and there exist three operators:*

$$
\mathcal{A}^N_{1,a}(\lambda),\ \tilde{\mathcal{A}}^N_{1,a}(\lambda) \in \mathrm{Anal}(\mathbb{C}\setminus(-\infty,0], \mathcal{L}_R(\mathbb{R}^3)), \quad \mathcal{A}^N_{2,a}(\lambda) \in \mathrm{Anal}(\mathbb{C}_+, \mathcal{L}_R(\mathbb{R}^3))
$$

such that

$$
\begin{aligned} &\mathcal{A}_{\mathbf{R}^3,a,k}(\lambda)f = \mathcal{A}^N_{1,a}(\lambda) + \mathcal{A}^N_{2,a}(\lambda) \\ &\mathcal{A}^N_{1,a}(\lambda) = (\lambda - \Delta_{\mathbf{R}^3} + k\partial_3)^{-1}P_{\mathbf{R}^3} + \tilde{\mathcal{A}}^N_{1,a}(\lambda) \\ &\|\partial_x^\beta\mathcal{A}^N_{1,a}(\lambda)f\|_{L_q(\mathbf{R}^3)} \leqq C|\lambda|^{-(1-(|\beta|/2))}\|f\|_{L_q(\mathbf{R}^3)} \\ &\|\partial_x^\beta\tilde{\mathcal{A}}^N_{1,a}(\lambda)f\|_{L_q(\mathbf{R}^3)} \leqq C|\lambda|^{-((3/2)-(|\beta|/2))}\|f\|_{L_q(\mathbf{R}^3)} \end{aligned}
$$

for any $\lambda \in \Sigma_\epsilon$ *with* $|\lambda| \geqq c_\epsilon > 0$ *(*c_ϵ *being some constant depending on* ϵ*) and* β *with* $|\beta| \leqq 2$*, and*

$$\|\partial_x^\beta \mathcal{A}^N_{2,a}(\lambda)f\|_{L_q(\mathbb{R}^3)} \leqq C\gamma^{-1}|\lambda|^{-(N/2)+(|\beta|/2)}\|f\|_{L_q(\mathbb{R}^3)}$$

for any $\lambda \in \mathbb{C}_\gamma$ *with* $|\lambda| \geqq 1$ *and* β *with* $|\beta| \leqq 2$ *provided that* $f \in L_{q,R+2}(\mathbb{R}^3)$*. Here, the constant* C *depends on* a_0 *and* k_0 *but is independent of* a *and* k*, and we have set*

$$(\lambda - \Delta_{\mathbb{R}^3} + k\partial_3)^{-1}g = \mathcal{F}_\xi^{-1}[(\lambda + |\xi|^2 + ik\xi_3)^{-1}\hat{g}(\xi)](x)$$

Theorem 6. *Let* $1 < q < \infty$*,* $k_0 > 0$*,* $a_0 > 0$*,* γ_0 *and* $K \geqq 10a_0 + 2$*. Assume that* $|k| \leqq k_0$*,* $|a| \leqq a_0$ *and* $0 \leqq \gamma \leqq \gamma_0$*. Set*

$$\mathcal{L}_{R,\mathrm{comp}}(\mathbb{R}^3) = \mathcal{L}(L_{q,R+2}(\mathbb{R}^3), W_q^2(B_{R+3})^3), \quad [\,\cdot\,]_{\mathbb{R}^3,R} = \|\cdot\|_{\mathcal{L}(L_{q,R+2}(\mathbb{R}^3), W_q^2(B_{R+3})^3)}$$

$$\overline{\mathbb{C}_+} = \{\lambda \in \mathbb{C} \mid \mathrm{Re}\,\lambda \geqq 0\}$$

Then, $\mathcal{A}(\lambda) = \mathcal{A}_{\mathbb{R}^3,a,k}(\lambda) \in C(\overline{\mathbb{C}_+}, \mathcal{L}_{R,\mathrm{comp}}(\mathbb{R}^3))$ *and satisfies the following conditions:*

$$\sup_{|s|\leqq K}[\mathcal{A}(\gamma + is)]_{\mathbb{R}^3,R} \leqq C_{\gamma_0,a_0,K}$$

$$\int_{-K}^{K}[(\partial_\lambda\mathcal{A})(\gamma + is)]^p_{\mathbb{R}^3,R}\,ds \leqq C_{\gamma_0,a_0,K} \quad (1 \leqq p < 2)$$

$$\sup_{0<|h|\leqq 1}|h|^{-1/2}\Big(\int_{-K}^{K}[\mathcal{A}(\gamma + i(s+h)) - \mathcal{A}(\gamma + is)]^p_{\mathbb{R}^3,R}\,ds\Big)^{1/p} \leqq C_{\gamma_0,a_0,K} \quad (1 \leqq p < 4)$$

$$\sup_{0<|h|\leqq 1}|h|^{-1/2}\int_{-K}^{K}[(\partial_\lambda\mathcal{A})(\gamma + i(s+h)) - (\partial_\lambda\mathcal{A})(\gamma + is)]_{\mathbb{R}^3,R}\,ds \leqq C_{\gamma_0,a_0,K}$$

$$\|(\partial_\lambda^m\mathcal{A}_{\mathbb{R}^3,a})(\gamma + is)\|_{\mathcal{L}(L_{q,R+2}(\mathbb{R}^3), W_q^j(B_{R+3})^3)} \leqq C_{\gamma_0,a_0,K}|s|^{-m-(1-(j/2))}$$

for $m = 0, 1, 2, 3$*,* $j = 0, 1, 2$ *and* $s \in \mathbb{R}$ *with* $|s| \geqq K - 2$*, and*

$$\lim_{\gamma\to 0+}\sup_{s\in\mathbb{R}}[\mathcal{A}(\gamma + is) - \mathcal{A}(is)]_{\mathbb{R}^3,R} = 0$$

$$\lim_{\gamma\to 0+}\int_{-\infty}^{\infty}[(\partial_\lambda\mathcal{A})(\gamma + is) - (\partial_\lambda\mathcal{A})(is)]_{\mathbb{R}^3,R} = 0$$

$$\lim_{R\to\infty}R^{-1}\int_{R\leqq|x|\leqq 2R}|[\mathcal{A}(\lambda)f](x)|^2\,dx = 0 \quad (k \neq 0)$$

$$\lim_{R\to\infty}R^{-2}\int_{R\leqq|x|\leqq 2R}|[\mathcal{A}(\lambda)f](x)|^2\,dx = 0 \quad (k = 0)$$

$$\|\mathcal{A}(\lambda_1)f - \mathcal{A}(\lambda_2)f\|_{W_q^2(B_{R+3})} \leqq C_{q,R}|\lambda_1 - \lambda_2|^{1/4}\|f\|_{L_q(\mathbb{R}^3)}$$

for any λ*,* λ_1 *and* $\lambda_2 \in \overline{\mathbb{C}_+}$ *provided that* $f \in L_{q,R+2}(\mathbb{R}^3)$*.*

4 An idea of Theorem 2

Since the whole space problem is solvable, to show the existence of solutions to the equations:

$$
\begin{aligned}
(16) \qquad u_t - \Delta u + M_{k,a}u + \nabla\pi = 0 &\quad \text{in} \quad \Omega, \ t > 0,\\
\nabla\cdot u = 0 &\quad \text{in} \quad \Omega, \ t > 0,\\
u = 0 &\quad \text{on} \quad \Gamma,\\
u|_{t=0} = f &\quad \text{in} \quad \Omega.
\end{aligned}
$$

with $M_{k,a}u = k\partial_3 u - (\omega\times x)\cdot\nabla u + \omega\times u$, the following theorem plays an essential role.

Theorem 7. *Let* $1 < q < \infty$ *and set*

$$L_{q,R-1}(\Omega) = \{f \in L_q(\Omega)^3 \mid f(x) = 0 \ \ (|x| > R-1)\}$$

For every $f \in L_{q,R-1}(\Omega)$, *problem* (16) *admits a unique solution* (u,π) *having the following regularity properties:*

$$
\begin{aligned}
&u \in C^0([0,\infty), J_q(\Omega)) \cap C^1((0,\infty), L_q(\Omega)) \cap C^0((0,\infty), W_q^2(\Omega))\\
&\pi \in C^0((0,\infty), \hat{W}_q^1(\Omega))
\end{aligned}
$$

and satisfying the following estimates:

$$
\begin{aligned}
\|u(t)\|_{L_q(\Omega)} + t^{1/2}\|\nabla u(t)\|_{L_q(\Omega)} + t(\|u_t(t)\|_{L_q(\Omega)} + \|u(t)\|_{W_q^2(\Omega)} + \|\nabla\pi(t)\|_{L_q(\Omega)})&\\
\leqq C_\gamma e^{\gamma t}\|f\|_{L_q(\Omega)}&\\
t^{(1/2)(1+(1/q))}(\|u_t(t)\|_{L_q(\Omega_b)} + \|\pi(t)\|_{L_q(\Omega_b)}) \leqq C_{\gamma,b}e^{\gamma t}\|f\|_{L_q(\Omega)}&
\end{aligned}
$$

for any $t > 0$. *Here,* $\Omega_b = B_b \cap \Omega$ $(b > R)$, $\gamma > 0$ *is any real number, and* C_γ *and* $C_{\gamma,b}$ *are constants depending on* a_0 *and* k_0 *whenever* $|a| \leqq a_0$ *and* $|k| \leqq k_0$ *but are independent of* a, k, t *and* f.

Moreover, if $f \in L_{q,R-1}(\Omega) \cap \mathcal{D}_q(\Omega)$, *then we have*

$$
\begin{aligned}
&u \in C^0([0,\infty), W_q^2(\Omega)) \cap C^1([0,\infty), L_q(\Omega))\\
&\|u(t)\|_{W_q^2(\Omega)} + \|u_t(t)\|_{L_q(\Omega)} \leqq C_\gamma e^{\gamma t}\|f\|_{W_q^2(\Omega)}
\end{aligned}
$$

Now, we shall show Theorem 2 by using Theorem 7.

1 st step

Given $f \in \mathcal{D}_q(\Omega)$, let $\tilde{f} \in \mathcal{D}_q(\mathbb{R}^3)$ be an extension of f to the whole space such that $\tilde{f} = f$ on Ω and $\|\tilde{f}\|_{\mathcal{D}_q(\mathbb{R}^3)} \leqq C_q\|f\|_{\mathcal{D}_q(\Omega)}$ where we have set

$$\|f\|_{\mathcal{D}_q(D)} = \|f\|_{W_q^2(D)} + \|(\omega\times x)\cdot\nabla f\|_{L_q(D)}$$

Let $\varphi \in C_0^\infty(\mathbb{R}^3)$ such that $\varphi(x) = 1$ for $|x| \leqq R-2$ and $\varphi(x) = 0$ for $|x| \geqq R-1$ and set

$$v(t) = (1-\varphi)S_{\mathbf{R}^3}(t)\tilde{f} + \mathbb{B}[(\nabla\varphi)\cdot S_{\mathbf{R}^3}(t)\tilde{f}]$$

where $\mathbb{B}$ denotes the Bogovski operator satisfying the estimates:

$$\|\mathbb{B}[(\nabla\varphi)\cdot v]\|_{W_q^j(\mathbf{R}^3)} \leqq C\|v\|_{W_q^{j-1}(\mathrm{supp}\,(\nabla\varphi))}, \quad j=1,2$$
$$\|\mathbb{B}[(\nabla\varphi)\cdot \nabla v]\|_{W_q^j(\mathbf{R}^3)} \leqq C\|v\|_{W_q^{j}(\mathrm{supp}\,(\nabla\varphi))}, \quad j=0,1,2$$

(cf. [37], [38], [41], [16]).

2nd step

To obtain the solution $u(t)$ of (16), we set $u(t) = v(t) + w(t)$, and then $w(t)$ and $\pi(t)$ satisfy the equations:

$$w_t + L_{k,a}w + \nabla\pi = F, \quad \mathrm{div}\, w = 0 \quad \text{in } \Omega\times(0,\infty)$$
$$w|_{\partial\Omega} = 0, \quad w|_{t=0} = \varphi f - \mathbb{B}[(\nabla\varphi)\cdot f] = g$$

Here

$$F = -2(\nabla\varphi)\cdot\nabla S_{\mathbf{R}^3}(t)\tilde{f} - (\Delta\varphi)S_{\mathbf{R}^3}(t)\tilde{f} + k(\partial_3\varphi)S_{\mathbf{R}^3}(t)\tilde{f} - ((\omega\times x)\cdot\nabla\varphi)S_{\mathbf{R}^3}(t)\tilde{f}$$
$$+ (\partial_t + L_{k,a})\mathbb{B}[(\nabla\varphi)\cdot S_{\mathbf{R}^3}(t)\tilde{f}]$$

Observe that

$$\partial_t\mathbb{B}[(\nabla\varphi)\cdot S_{\mathbf{R}^3}(t)\tilde{f}] = \mathbb{B}[(\nabla\varphi)\cdot\partial_t S_{\mathbf{R}^3}(t)\tilde{f}]$$
$$= \mathbb{B}[(\nabla\varphi)\cdot\Delta S_{\mathbf{R}^3}(t)\tilde{f}] + \mathbb{B}[(\nabla\varphi)\cdot(-k\partial_3 + ((\omega\times x)\cdot\nabla - \omega\times)S_{\mathbf{R}^3}(t)\tilde{f})]$$

Therefore, we have

$$\|F(t)\|_{W_q^2(\Omega)} \leqq C_\gamma t^{-1/2}e^{\gamma t}\|f\|_{D_q(\Omega)}$$
$$\|F(t)\|_{L_q(\Omega)} \leqq C_\gamma t^{-1/2}e^{\gamma t}\|f\|_{L_q(\Omega)}$$

If we write

$$w(t) = S_\Omega(t)g + \int_0^t S_\Omega(t-s)F(s)\,ds$$

then by Theorem 7 we have

$$w(t) \in C^0([0,\infty), W_q^2(\Omega)) \cap C^1([0,\infty), L_q(\Omega))$$
$$\|w(t)\|_{W_q^2(\Omega)} + \|w_t(t)\|_{L_q(\Omega)} \leqq C_\gamma e^{\gamma t}\|f\|_{D_q(\Omega)}$$
$$\|w(t)\|_{L_q(\Omega)} \leqq C_\gamma e^{\gamma t}\|f\|_{L_q(\Omega)}$$

Therefore, we can construct a solution

$$u(t) \in C^0([0,\infty), W_q^2(\Omega)) \cap C^1([0,\infty), L_q(\Omega))$$

which satisfies the estimate:

$$\|u(t)\|_{W_q^2(\Omega)} + \|u_t(t)\|_{L_q(\Omega)} \leqq C_\gamma e^{\gamma t}\|f\|_{\mathcal{D}_q(\Omega)}$$
$$\|u(t)\|_{L_q(\Omega)} \leqq C_\gamma e^{\gamma t}\|f\|_{L_q(\Omega)}$$

If we define $\{T(t)\}_{t\geqq 0}$ by the formula: $T(t)f = u(t)$, then the uniqueness of solutions and the denseness of $\mathcal{D}_q(\Omega)$ in $J_q(\Omega)$ imply that $\{T(t)\}_{t\geqq 0}$ is a C^0 semigroup on $J_q(\Omega)$. Since we know that the resolvent set of $\mathcal{L}_q$ contains the complex plane with positive real part (cf. Theorem 8 below), we see that the generator of $\{T(t)\}_{t\geqq 0}$ is $\mathcal{L}_q$, which completes the proof of Theorem 2.

5 An idea of a proof of Theorem 7

To prove Theorem 7, we consider the corresponding resolvent problem:

$$\lambda u - \Delta u + M_{k,a}u + \nabla\pi = f, \quad \operatorname{div} u = 0 \quad \text{in } \Omega, \quad u|_{\partial\Omega} = 0 \tag{17}$$

We construct the parametrix of the form:

$$\begin{aligned}\Phi(\lambda)f &= (1-\varphi)\mathcal{A}_{\mathbf{R}^3,a,k}(\lambda)f_0 + \varphi R_{\Omega_R}(\lambda)P_{\Omega_R}f|_{\Omega_R}\\ &\quad + \mathbb{B}[(\nabla\varphi)\cdot(\mathcal{A}_{\mathbf{R}^3,a,k}(\lambda)f_0 - R_{\Omega_R}(\lambda)P_{\Omega_R}f|_{\Omega_R})]\\ \Psi(\lambda)f &= (1-\varphi)Q_{\mathbf{R}^3}f_0 + \varphi(Q_{\Omega_R}f|_{\Omega_R} + p_{\Omega_R}(\lambda)P_{\Omega_R}f|_{\Omega_R})\end{aligned}$$

for $\lambda \in \mathbb{C}_+ = \{\lambda \in \mathbb{C} \mid \operatorname{Re}\lambda > 0\}$ and $f \in L_{q,R-1}(\Omega)$. Here, f_0 denotes the zero extension of f to the whole space, $f|_{\Omega_R}$ the restriction of f on Ω_R,

$$\mathcal{A}_{\mathbf{R}^3,a,k}(\lambda)f_0 = \int_0^\infty e^{-\lambda t}S_{\mathbf{R}^3}(t)f_0\,dt$$

and $u = R_{\Omega_R}(\lambda)g$ and $\theta = p_{\Omega_R}(\lambda)g$ are solutions to the equation:

$$\begin{aligned}\lambda u - \Delta u + M_{k,a}u + \nabla\theta &= P_{\Omega_R}g \quad &&\text{in } \Omega_R\\ \operatorname{div} u &= 0 &&\text{in } \Omega_R\\ u|_{\partial\Omega_R} &= 0\end{aligned}$$

We see that

$$\begin{aligned}(\lambda - \Delta + M_{k,a})\Phi(\lambda)f + \nabla\Psi(\lambda)f &= (I+T)f + S(\lambda)f \quad &&\text{in } \Omega\\ \operatorname{div}\Phi(\lambda)f &= 0 &&\text{in } \Omega\\ u|_{\partial\Omega} &= 0\end{aligned}$$

Here T is the operator defined by the formula:

$$Tf = -(\nabla\varphi)(Q_{\mathbf{R}^3}f_0 - Q_{\Omega_R}f|_{\Omega_R}) - \mathbb{B}[(\nabla\varphi)\cdot\nabla(Q_{\mathbf{R}^3}f_0 - Q_{\Omega_R}f|_{\Omega_R})] \tag{18}$$

and $S(\lambda)$ is a linear operator is defined as follows:

$$\begin{aligned}
S(\lambda)f &= 2(\nabla\varphi) : \nabla(C_a(\lambda)f) + (\Delta\varphi)C_a(\lambda)f - k(\partial_3\varphi)C_a(\lambda)f \\
&\quad + [(\omega \times x)\cdot\nabla\varphi]C_a(\lambda)f + \mathbb{B}[(\nabla\varphi)\cdot\Delta C_a(\lambda)f] \\
&\quad - \mathbb{B}[(\nabla\varphi)\cdot M_{k,a}C_a(\lambda)f] + \mathbb{B}[(\nabla\varphi)\cdot\nabla\mathcal{B}_{\Omega_R,a}(\lambda)\gamma_{\Omega_R}f] \\
&\quad - \Delta\mathbb{B}[(\nabla\varphi)\cdot C_a(\lambda)f] + M_{k,a}\mathbb{B}[(\nabla\varphi)\cdot C_a(\lambda)f] + (\nabla\varphi)\mathcal{B}_{\Omega_R,a}(\lambda)\gamma_{\Omega_R}f \\
C_a(\lambda)f &= \mathcal{A}_{\mathbf{R}^3,a}(\lambda)f_0 - \mathcal{A}_{\Omega_R,a}(\lambda)\gamma_{\Omega_R}f
\end{aligned}$$

We can show the following lemma which is one of the points in our argument.

Lemma 1. *Let $1 < q < \infty$. Then, there exists the inverse operator $(I+T)^{-1} \in \mathcal{L}(L_{q,R-1}(\Omega))$.*

Moreover, we know that

$$\|S(\lambda)f\|_{L_q(\Omega)} \leqq C_q|\lambda|^{-\frac{1}{2}(1-\frac{1}{q})}, \quad |\lambda| \geqq 1, f \in L_{q,R-1}(\Omega) \tag{19}$$

We can define the operators:

$$T_1(\lambda) \in Anal(\tilde{\Sigma}_\epsilon, \mathcal{L}(L_{q,R-1}(\Omega))), \quad T_2(\lambda) \in Anal(\mathbb{C}_+, \mathcal{L}(L_{q,R-1}(\Omega)))$$

such that

$$\begin{aligned}
(I + T + S(\lambda))^{-1} &= (I + \sum_{j=1}^{\infty}((I+T)^{-1}S(\lambda))^j)(I+T)^{-1} \\
&= I + T_1(\lambda) + T_2(\lambda)
\end{aligned}$$

for $\lambda \in \mathbb{C}_+$ with $|\lambda| \geqq c$ and

$$\begin{aligned}
\|T_1(\lambda)f\|_{L_q(\Omega)} &\leqq C|\lambda|^{-(1/2)(1-(1/q))}\|f\|_{L_q(\Omega)} \quad (\lambda \in \tilde{\Sigma}_\epsilon, |\lambda| \geqq 1) \\
\|T_2(\lambda)f\|_{L_q(\Omega)} &\leqq C_\gamma|\lambda|^{-3}\|f\|_{L_q(\Omega)} \quad (\lambda \in \mathbb{C}_+ : \ \mathrm{Re}\,\lambda \geqq \gamma > 0, |\lambda| \geqq 1)
\end{aligned}$$

Here, c is some large positive number, $0 < \epsilon < \pi/2$,

$$\tilde{\Sigma}_\epsilon = \mathbb{C}_+ \cup \{\lambda \in \mathbb{C} \setminus \{0\} \mid |\arg\lambda| \leqq \pi - \epsilon, \ |\lambda| \geqq c(\epsilon)\}$$

and $c(\epsilon)$ is some lalrge positive number depending on ϵ, a_0 and k_0 whenever $|a| \leqq a_0$ and $|k| \leqq k_0$.

We can also show that $(I+T+S(\lambda))^{-1}$ exists for λ with $\mathrm{Re}\lambda > 0$. And therefore, we set $(R(\lambda), \Xi(\lambda)) = (\Phi(\lambda), \Psi(\lambda))(I+T+S(\lambda))^{-1}$. Then, $(R(\lambda), \Xi(\lambda))$ is the solution operator to the equation:

$$(\lambda - \Delta + M_{k,a})u = f, \quad \mathrm{div}\, u = 0 \quad \text{in } \Omega \quad u|_{\partial\Omega} = 0$$

Moreover, using Theorem 5, we have

$$
\begin{aligned}
(20)\qquad & R(\lambda) = R_1(\lambda) + R_2(\lambda), \quad \Xi(\lambda) = \Xi_0 + \Xi_1(\lambda) + \Xi_2(\lambda)\\
& R_1(\lambda) \in Anal(\tilde{\Sigma}_\epsilon, \mathcal{L}(L_{q,R-1}(\Omega), W_q^2(\Omega)))\\
& R_2(\lambda) \in Anal(\mathbb{C}_+, \mathcal{L}(L_{q,R-1}(\Omega), W_q^2(\Omega)))\\
& \Xi_0 \in \mathcal{L}(L_{q,R-1}(\Omega), \hat{W}_q^1(\Omega))\\
& \Xi_1(\lambda) \in Anal(\tilde{\Sigma}_\epsilon, \mathcal{L}(L_{q,R-1}(\Omega), \hat{W}_q^1(\Omega)))\\
& \Xi_2(\lambda) \in Anal(\mathbb{C}_+, \mathcal{L}(L_{q,R-1}(\Omega), \hat{W}_q^1(\Omega)))\\
& \|\nabla^j R_1(\lambda) f\|_{L_q(\Omega)} \leqq C|\lambda|^{-1+(j/2)}\|f\|_{L_q(\Omega)}\\
& |\lambda|^{(1/2)(1-(1/q))}\|\Xi_1(\lambda)f\|_{L_q(\Omega_b)} + \|\nabla\Xi_1(\lambda)f\|_{L_q(\Omega)} \leqq C_b\|f\|_{L_q(\Omega)}
\end{aligned}
$$

for $\lambda \in \tilde{\Sigma}_\epsilon$ with $|\lambda| \geqq 1$

$$
\begin{aligned}
& \|R_2(\lambda)f\|_{W_q^2(\Omega)} \leqq C_\gamma|\lambda|^{-3}\|f\|_{L_q(\Omega)}\\
& \|\Xi_2(\lambda)f\|_{L_q(\Omega_b)} + \|\nabla\Xi_2(\lambda)f\|_{L_q(\Omega)} \leqq C_b|\lambda|^{-3}\|f\|_{L_q(\Omega)}
\end{aligned}
$$

for $\lambda \in \mathbb{C}_+$ with $|\lambda| \geqq 1$ and $\mathrm{Re}\,\lambda \geqq \gamma > 0$.

Using the above solution formula of (17), we have

Theorem 8. *Let $1 < q < \infty$. Then, the resolvent set of $\mathcal{L}_q$ contains $\mathbb{C}_+$.*

Moreover, if we define $u(t)$ and $\pi(t)$ by the formula:

$$
\begin{aligned}
(21)\qquad u(t) &= \frac{1}{2\pi i}\int_\Gamma e^{\lambda t}R_1(\lambda)f\,d\lambda + \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\lambda t}R_2(\lambda)f\,d\lambda\\
\theta(t) &= \frac{1}{2\pi i}\int_\Gamma e^{\lambda t}(\Xi_0 f + \Xi_1(\lambda)f)\,d\lambda + \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\lambda t}\Xi_2(\lambda)f\,d\lambda
\end{aligned}
$$

for $f \in L_{q,R-1}(\Omega)$ where

$$
\Gamma = \bigcup_{\pm}\{\sigma + se^{\pm i\kappa} \mid s \geqq 0\}
$$

with large $\sigma > 0$ and $\pi/2 < \kappa < \pi$, then we can show Theorem 7 by (20).

6 An idea of Proof of Theorem 3

To show Theorem 3, the main step is to show the following theorem.

Theorem 9 (Local Energy Decay). *Let $1 < q < \infty$. Then, we have*

$$
\|\partial_t^j T(t)P_\Omega f\|_{W_q^2(\Omega_R)} \leqq Ct^{-\frac{3}{2}}\|f\|_{L_q(\Omega)}, \quad t > 1
$$

for any $f \in L_{q,R-1}(\Omega)$ and $j = 0, 1$.

We can show Theorem 9 by shifting the contour in the definition of $u(t)$ in (21) to the imaginary axis, applying Theorem 6 and using the following lemma:

Lemma 2. *Let* $0 < \kappa < 1$. *Let* X *and* $\|\cdot\|_X$ *be a Banach space and its norm, respectively. Let* $f(s)$ *be a function in* $L_1(\mathbb{R}, X)$, *which satisfies the condition:*

$$\sup_{0<|h|\leqq 1} |h|^{-\kappa} \int_{\mathbb{R}} \|f(s+h) - f(s)\|_X \, ds \leqq C_\kappa M \tag{22}$$

for some $M > 0$. *Set* $g(t) = \int_{\mathbb{R}} e^{its} f(s)\, ds$. *Then, we have*

$$\|g(t)\|_X \leqq |e^{-i} - 1|^{-1} M t^{-\kappa}$$

for any $t \geqq 1$.

By Young's inequality, we see easily that

$$\|\nabla^j S_{\mathbb{R}^3}(t) f\|_{L_r(\mathbb{R}^3)} \leqq C_{q,r} t^{-\frac{j}{2}-\frac{3}{2}\left(\frac{1}{q}-\frac{1}{r}\right)} \|f\|_{L_q(\mathbb{R}^3)} \tag{23}$$

for $1 < q \leqq r \leqq \infty$ with $q \neq \infty$ and $t > 0$. Combining (23) with Theorem 9 by cut-off technique, we have Theorem 3.

References

[1] W. Borchers, *Zur Stabilität und Faktorisierungsmethode für die Navier-Stokes Gleichungen inkompressibler viskoser Flüssigkeiten*, Habilitationsschrift, Universität Paderborn, 1992.

[2] Z. Chen and T. Miyakawa, *Decay properties of weak solutions to a perturbed Navier-Stokes system in* $\mathbb{R}^n$, Adv. Math. Sci. Appl., 7 (1997), 741–770

[3] P. Cumsille and M. Tucsnak, Strong solutions for the Navier-Stokes flow in the exterior of a rotating obstacle, preprint.

[4] E. Dintelmann, M. Geissert and M. Hieber, *Strong* L^p *solutions to the Navier-Stokes flow past moving obstacles: the case of several obstacles and time dependent velocity*, TU Darmstadt, Fachbereich Mathematik, preprint 2006

[5] R. Farwig, An L^q-analysis of viscous fluid flow past a rotating obstacle, Tôhoku Math. J. **58** (2006), 129–147.

[6] R. Farwig, M. Krbec and Š. Nečasová, *A weighted* L^q*-approach to Stokes flow around a rotating body*, TU Darmstadt, Fachbereich Mathematik, Preprint No. 2422

[7] R. Farwig and T. Hishida, *Stationary Navier-Stokes flow around a rotating obstacle*, TU Darmstadt, Fachbereich Mathematik, Preprint No. 2445 (2006).

[8] R. Farwig, T. Hishida and D. Müller, L^q-theory of a singular "winding" integral operator arising from fluid dynamics, Pacific. J. Math. **215** (2004), 297–312.

[9] R. Farwig, Š. Nečasová and J. Neustupa, *On the essential spectrum of a Stokes-type operator arising from flow around a rotationg body in the L^q-framework*, preprint in 2006.

[10] R. Farwig and J. Neustupa, On the spectrum of a Stokes-type operator arising from flow around a rotating body, TU Darmstadt, Fachbereich Mathematik, Preprint No. 2423.

[11] G. P. Galdi, On the motion of a rigid body in a viscous liquid: a mathematical analysis with applications, *Handbook of Mathematical Fluid Dynamics*, Volume I, 653–791, Ed. by S. Friedlander and D. Serre, North-Holland, Amsterdam, 2002.

[12] G. P. Galdi, Steady flow of a Navier-Stokes fluid around a rotating obstacle, J. Elasticity **71** (2003), 1–31.

[13] G. P. Galdi and A. L. Silvestre, Strong solutions to the Navier-Stokes equations around a rotating obstacle, Arch. Rational Mech. Anal., **176** (2005), 331–350.

[14] G. P. Galdi and A. L. Silvestre, *On the steady motion of a Navier-Stokes liquid around a rigid body*, preprint.

[15] G. P. Galdi and A. L. Silvestre, *Existence of time-periodic solutions to the Navier-Stokes equations around a moving body*, preprint.

[16] M. Geissert, H. Heck and M. Hieber, *L^p-theory of the Navier-Stokes flow in the exterior of a moving or rotating obstacle*, J. Reine Angew. Math., to appear.

[17] R. B. Gunther, R. T. Hudspeth and E. A. Thomann, *Hydrodynamic flows on submerged rigid bodies- steady flow*, J. Math. Fluid Mech. **4** (2002), 187–202.

[18] M. Hieber and O. Sawada *The Navier-Stokes equations in $\mathbb{R}^n$ with linearly growing initial data,* Arch. Ration. Mech. Anal., **175** (2005), pp. 269-285.

[19] T. Hishida, *An existence theorem for the Navier-Stokes flow in the exterior of a rotating obstacle*, Arch. Rational Mech. Anal., **150** (1999), 307–348.

[20] T. Hishida, *The Stokes operator with rotation effect in exterior domains*, Analysis, **19** (1999), 51–67.

[21] T. Hishida, *L^2 theory for the operator $\Delta + (k \times x) \cdot \nabla$ in exterior domains*, Nihonkai Math. J., **11** (2000), 103–135.

[22] T. Hishida, *The nonstationary Stokes and Navier-Stokes flows through an aperture*, Contributions to Current Challenges in Mathematical Fluid Mechanics, 79–123, Adv. Math. Fluid Mech. 3, Birkhäuser, Basel, 2004.

[23] T. Hishida, *L^q estimates of weak solutions to the stationary Stokes equations around a rotating body*, J. Math. Soc. Japan, **58** (2006), 743–767.

[24] T. Hishida and Y. Shibata, *L_p-L_q estimate of the Stokes operator and Navier-Stokes flows in the exterior of a rotating obstacle*, preprint in 2006.

[25] G. Kirchhoff, Über die Bewegung eines Rotationskörpers in einer Flüssigkeit, Crelle J. **71** (1869), 237–281

[26] S. Kračmar, Š. Nečasová and P. Penel, *Estimates of weak solutions in anisotropically weighted Sobolev spaces to the stationary rotating Oseen equations*, IASME Transactions **6** N.2 (2005), 854–861.

[27] S. Kračmar, Š. Nečasová and P. Penel, *On the weak solution to the Oseen-type problem arising from flow aroung a rotating rigid body in the whole space*, WSEAS Transactions of Mathematics **3**, No.5 (2006), 243–251.

[28] Š. Nečasová, *On the problem of the Stokes flow and Oseen flow in $\mathbb{R}^3$ with Coriolis force arising from fluid dynamics*, IASME Transactions **2** (2005), 1262–1270.

[29] Š. Nečasová, *Asymptotic properties of the steady fall of a body in viscous fluids*, Math. Meth. Appl. Sci., **27** (2004), 1969–1995.

[30] J. A. San Martín, V. Starovoitov and M. Tucsnak, *Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid*, Arch. Rational Mech.Anal. **161** (2002), 113–147.

[31] D. Serre, *Chute libre d'un solide dans un fluide visqueux incompressible. Existence*, Japan J. Appl. Math. **4** (1987), 99–110.

[32] A. L. Silvestre, On the existence of steady flows of a Navier-Stokes liquid around a moving rigid body, *Math. Meth. Appl. Sci.* **27** (2004), 1399–1409.

[33] T. Takahashi and M. Tucsnak, *Global strong solutions for the two-dimensional motion of an infinite cylinder in a viscous fluid*, J. Math. Fluid Mech., **6** (2004), 53–77.

[34] E. A. Thomann and R. B. Guenther, *The fundamental solution of the linearized Navier-Stokes equations for spining bodies in three spatial dimensions-time dependent case*, J. Math. Fluid Mech., (2005), in press

[35] W. Thomson (Lord Kelvin), *Mathematical and Physical Papers*, **Vol 4**, Cambridge University Press 1982.

[36] H. F. Weinberger, *On the steady fall of a body in a Navier-Stokes fluid*, Proc. Symp. Pure Mathematics **23** (1973), 421–440.

[37] H. F. Weinberger, *Variational properties of steady fall in Stokes flow*, J. Fluid Mech. **52** (1972), 321–344.

[38] M. E. Bogovskiǐ, *Solution of the First Boundary Value Problem for the Equation of Continuity of an Incompressible Medium*, Soviet Math. Dokl., **20** (1979), 1094–1098.

[39] M. E. Bogovskiǐ, *Solution of Some Vector Analysis Problems connected with Operators div and grad*, (Russian), Trudy Seminar S.L.Sobolve, No. 1, **80**, Akademia Nauk SSR, Sibirskoe Otdelenie Matematiki, Nowosibirsk, 1980, 5–40.

[40] D. Fujiwara and H. Morimoto *An L_r-theory of the Helmholtz decomposition of vector fields*, J. Fac. Sc. Univ. Tokyo, Sect. Math. **24**, 685–700 (1977).

[41] M. Geissert, H. Heck and M. Hieber, *On the equation* $\operatorname{div} u = g$ *and Bogovskiǐ's operator in Sobolev spaces of negative order*, preprint 2005, April.

[42] G. P. Galdi, *An Introduction to the Mathematical Theory of the Navier-Stokes equations, Vol. I: Linear Steady Problems, Vol. II: Nonlinear Steady Problems*. Springer Tracts in Nat. Ph. **38**, **39**, Springer Verlag New York (1994) 2nd edition: 1998

[43] H. Iwashita, *L_q-L_r estimates for solutions of the nonstationary Stokes equations in an exterior domain and the Navier-Stokes initial value problem in L_q spaces*, Math. Ann., **285** (1989), 265–288.

[44] T. Kato, *Strong L^p- solutions of the Navier-Stokes equation in $\mathbb{R}^m$ with applications to weak solutions*, Math. Z. **187** (1984), 471–480.

[45] T. Kobayashi and Y. Shibata, *On the Oseen equation in the three dimensional exterior domains*, Math. Ann., **310** (1998), 1–45.

[46] P. Maremonti and V. A. Solonnikov, *On nonstationary Stokes problem in exterior domains*, Ann. Sc. Norm. Sup. Pisa, **24** (1997), 395–449.

[47] T. Miyakawa, *On non-stationary solutions of the Navier-Stokes equations in an exterior domain*, Hiroshima Math. J., **12**, (1982) 115-140.

[48] Y. Shibata, *On an exterior initial boundary value problem for Navier-Stokes equations*, Quartely Appl. Math. **LVII** No.1, (1999), 117–155.

[49] Y. Shibata, *Time-global solutions of nonlinear evolution equations and their stability*, Amer. Math. Soc. Transl. (2) Vol **211**, 2003, 87–105.

[50] C. G. Simader and H. Sohr, *The Helmholtz decomposition in L^q and related topics* Mathematical Problems Related to the Navier-Stokes Equation, G.P. Galdi ed., Advances in Maths. Appl. Sci., **11**, World Scientific, 1-35