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Uniform Boundedness of the Solutions for a One-Dimensional Isentropic Model System of Compressible Viscous Gas

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1 External Force Problem

In this section we consider the one-dimensional motion of a general viscous isentropic gas in a bounded region, with an external force. In Lagrangian mass coordinate, such a model system is well formulated by the system of equations

$$v_t - u_x = 0, \tag{1.1}$$

$$u_t + \left(\frac{a}{v^\gamma}\right)_x = \mu\left(\frac{u_x}{v}\right)_x + f\left(\int_0^x v dx, t\right), \tag{1.2}$$

where v denotes the specific volume, u the velocity, μ the viscosity coefficient, f the the external force and $a > 0, \gamma \geq 1$ are the constants appearing in the equation of state. In what follows, assuming that the viscosity coefficient is a positive constant, we consider these equations in a fixed domain Q

$$Q = \{\ (x,t) \mid 0 < x < 1,\ t > 0\}. \tag{1.3}$$

together with the initial conditions

$$v(x,0) = v_0(x), \quad u(x,0) = u_0(x) \quad \text{on} \ \ 0 < x < 1, \tag{1.4}$$

and with the boundary conditions

$$u(0,t) = u(1,t) = 0 \quad \text{on} \ \ t > 0. \tag{1.5}$$

For the above data, it is natural to impose

$$v_0 \in H^1(0,1), \quad u_0 \in H_0^1(0,1), \tag{1.6}$$

$$C_0^{-1} \leq v_0(x) \leq C_0 \quad \textit{for some constant} \ \ C_0 > 1, \tag{1.7}$$

and

$$\int_0^1 v_0(x)dx = 1. \tag{1.8}$$

Furthermore, for the external force $f = f(\xi, t)$, $\xi = \int_0^x v dx$, we suppose that

$$f, \;\; f_\xi \;\text{ and }\; f_t \in L^\infty\left((0,1)\times(0,\infty)\right). \tag{1.9}$$

We are interested in the existence of uniformly bounded global solution with respect to time t. Here and throughout this paper, the term "uniformly bounded global in time solution" means the time-global solution which is uniformly bounded and its density also being uniformly positive with respect to t.

In the case $f \equiv 0$, the existence and uniqueness of the uniformly bounded global in time solution have been obtained by a number of authors including Kanel'[5], Itaya [3], Kazhikhov [6], Kazhikhov & Shelukhin [9], Kazhikhov & Nikolaev [7, 8], etc, under various conditions on the initial data, the equation of state p, and so on. Among them, Kazhikhov's result [6] shows that for arbitrary large initial data, our problem with $f \equiv 0$ has a unique uniformly bounded global in time solution. If the external force vanishes sufficiently fast as time tends to infinity, we can extend their results to obtain uniform estimates or the asymptotic behavior of the solution. However this assumption is too restrictive to cover physically meaningfull cases, such as time periodic external forces or time independent ones. In this points of view, Beirão da Veiga [1] proved the following result. For suitably small f, if some norm of the initial data is bounded by some constant which is determined by the L^∞-norm of f, then uniformly bounded global in time solution uniquely exists. Since the constant mentioned above tends to infinity as the L^∞-norm of f tends to 0, there is no gap between his result and Kazhikhov's one. His result also shows that for any fixed initial data, if the external force is sufficiently small, then the uniformly bounded global in time solution uniquely exists. However it does not cover Matsumura & Nishida's result [10] : when the gas is assumed to be isothermal, namely the equation of state is given by $p = a/v$, then there exists a unique uniformly bounded global in time solution for arbitrary large external force and large initial data. From this point, our interest in the present work is to make up for the difference between them. To do so, regarding γ as a parameter, we shall get the sufficient condition on the external force f so as to have uniform estimates on the solution, and study precisely how this condition depends on γ. Of course we expect that when γ tends to 1, our goal will be achieved.

In what follows, we denote the norm in L^∞, L^2 and H^1 by $|\cdot|_\infty, \|\cdot\|$ and $\|\cdot\|_1$, respectively. The following is our main theorem.

Theorem 1. 1 *Assume (1.6) - (1.9), and $1 < \gamma \leq 2$. Then there exists a constant $C(\gamma)$, which tends to ∞ as γ tends to 1, such that if $E_1(0) < \frac{\mu}{4}\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}}$ and $|f|_\infty \leq C(\gamma)$, then the initial and the boundary value problem (1.1), (1.2) with (1.5), (1.6) has a unique uniformly bounded global in time solution (v, u) satisfying*

$$C^{-1} \leq v(x,t) \leq C \qquad \forall \;\; (x,t) \in \; Q, \tag{1.10}$$

and

$$\sup_{t\geq 0} \| (v,u)(t) \|_1 \leq C, \tag{1.11}$$

where $E_1(0)$ shall be defined in (1.19), and C is a positive constant depending only on a, μ, γ, C_0, $\| (v_0, u_0) \|_1$, and $|f|_\infty$.

Remark 1.1
The above constant $C(\gamma)$ can be chosen to satisfy $C(\gamma) \geq C \left(\log (\gamma-1)^{-1}\right)^\beta$ as $\gamma \to 1$ for any β satisfying $0 < \beta < 1$.

Remark 1.2
Theorem 1.1 shows that for arbitrary large initial data and large external force, there exists a unique uniformly bounded global in time solution, provided the adiabatic constant γ is suitably close to 1.

Proof of Theorem 1.1.

Let us begin with the following easy result. Integrating (1.1) over $[0,1]$ gives

$$\int_0^1 v(x,t)dx = 1, \qquad \forall \ t \geq 0. \tag{1.12}$$

Multiplying (1.2) by u and integrating it over $[0,1]$ yield

$$\frac{d}{dt}\int_0^1 \left\{\frac{1}{2}u^2 + \Phi(v)\right\} dx + \mu \int_0^1 \frac{u_x^2}{v} dx = \int_0^1 uf dx, \tag{1.13}$$

where Φ is defined by $\Phi(v) = \dfrac{a}{\gamma-1}(v^{-\gamma+1} - 1) + a(v-1) \ \ (\geq 0)$.

Using the relation $u = \int_0^x u_x dx$, we have the estimate $|u|_\infty \leq \left(\int_0^1 \frac{u_x^2}{v} dx\right)^{\frac{1}{2}}$, then the right hand side of (1.13) is estimated as

$$\int_0^1 uf dx \leq |u|_\infty |f|_\infty \leq \frac{\mu}{2}\int_0^1 \frac{u_x^2}{v}dx + \frac{1}{2\mu}|f|_\infty^2, \tag{1.14}$$

from which one gets

$$\frac{d}{dt}\int_0^1 \left\{\frac{1}{2}u^2 + \Phi(v)\right\} dx + \frac{\mu}{2}\int_0^1 \frac{u_x^2}{v}dx \leq \frac{1}{2\mu}|f|_\infty^2. \tag{1.15}$$

Multiplying (1.2) by $\frac{v_x}{v}$ and integrating it over $[0,1]$ give

$$\frac{d}{dt}\int_0^1 \left\{\frac{\mu}{2}\left(\frac{v_x}{v}\right)^2 - \frac{uv_x}{v}\right\} dx + a\gamma\int_0^1 \frac{v_x^2}{v^{\gamma+2}}dx = \int_0^1 \frac{u_x^2}{v}dx - \int_0^1 \frac{v_x}{v} f dx. \tag{1.16}$$

As the last term in the right hand side of (1.16) is bounded by $|f|_\infty \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}}$, we have

$$(1.17)\qquad \frac{d}{dt}\int_0^1 \left\{\frac{\mu}{2}\left(\frac{v_x}{v}\right)^2 - \frac{uv_x}{v}\right\}dx + a\gamma\int_0^1 \frac{v_x^2}{v^{\gamma+2}}dx \le \int_0^1 \frac{u_x^2}{v}dx + |f|_\infty \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}}.$$

Multiplying (1.17) by $\frac{\mu}{4}$, adding it with (1.15), one shows that

$$(1.18)\qquad \frac{d}{dt}E_1^2(t) + E_2^2(t) \le \frac{1}{2\mu}|f|_\infty^2 + \frac{\mu}{4}|f|_\infty \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}},$$

where $E_1^2(t)$ and $E_2^2(t)$ are defined by

$$(1.19)\qquad E_1^2(t) = \int_0^1 \left\{\frac{1}{2}u^2 + \frac{\mu^2}{8}\left(\frac{v_x}{v}\right)^2 - \frac{\mu u v_x}{4v} + \Phi(v)\right\}dx,$$

$$(1.20)\qquad E_2^2(t) = \frac{\mu}{4}\int_0^1 \frac{u_x^2}{v}dx + \frac{a\mu\gamma}{4}\int_0^1 \frac{v_x^2}{v^{\gamma+2}}dx.$$

Since the absolute value of the term $\frac{\mu u v_x}{4v}$ is bounded by $\frac{1}{4}u^2 + \frac{\mu^2}{16}\left(\frac{v_x}{v}\right)^2$, $E_1^2(t)$ can be estimated as

$$(1.21)\qquad \begin{aligned} &\frac{1}{2}\int_0^1 \left\{\frac{1}{2}u^2 + \frac{\mu^2}{8}\left(\frac{v_x}{v}\right)^2\right\}dx + \int_0^1 \Phi(v)dx \le \\ \le\ & E_1^2(t) \le \frac{3}{2}\int_0^1 \left\{\frac{1}{2}u^2 + \frac{\mu^2}{8}\left(\frac{v_x}{v}\right)^2\right\}dx + \int_0^1 \Phi(v)dx. \end{aligned}$$

Now we would like to estimate $E_2^2(t)$ from below and $|f|_\infty \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}}$ from above. To do so, we shall use some methods found in [10]. Let X and Y be defined by

$$(1.22)\qquad X = \int_0^1 \frac{v_x^2}{v^2}dx, \quad Y = \int_0^1 \frac{v_x^2}{v^{\gamma+2}}dx.$$

Using Hölder's inequality, one has

$$(1.23)\qquad \int_0^1 \frac{v_x^2}{v^3}dx \le X^{\frac{\gamma-1}{\gamma}}Y^{\frac{1}{\gamma}}.$$

Then it follows from (1.20) - (1.23) that

$$(1.24)\qquad \begin{aligned} |f|_\infty \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}} &\le |f|_\infty X^{\frac{\gamma-1}{2\gamma}}Y^{\frac{1}{2\gamma}} \\ &\le \frac{\epsilon}{2\gamma}Y + \frac{2\gamma-1}{2\gamma}\epsilon^{-\frac{1}{2\gamma-1}}|f|_\infty^{\frac{2\gamma}{2\gamma-1}}X^{\frac{\gamma-1}{2\gamma-1}} \\ &\le \frac{2\epsilon}{a\mu\gamma^2}E_2^2(t) + \frac{2\gamma-1}{2\gamma}\left(\frac{16}{\mu^2}\right)^{\frac{\gamma-1}{2\gamma-1}}\epsilon^{-\frac{1}{2\gamma-1}}|f|_\infty^{\frac{2\gamma}{2\gamma-1}}E_1^{\frac{2(\gamma-1)}{2\gamma-1}}, \end{aligned}$$

for any $\epsilon > 0$. If we determine ϵ that satisfies $\frac{\mu}{4}\frac{2\epsilon}{a\mu\gamma^2} = \frac{1}{2}$; i.e., $\epsilon = a\gamma^2$, then (1.18) is reduced to

$$\frac{d}{dt}E_1^2(t) + \frac{1}{2}E_2^2(t) \le \frac{1}{2\mu}|f|_\infty^2 + C_1|f|_\infty^{\frac{2\gamma}{2\gamma-1}} E_1^{\frac{2(\gamma-1)}{2\gamma-1}}, \tag{1.25}$$

where $C_1 = \frac{\mu}{4}\frac{2\gamma-1}{2\gamma}\left(\frac{16}{\mu^2}\right)^{\frac{\gamma-1}{2\gamma-1}}\left(a\gamma^2\right)^{-\frac{1}{2\gamma-1}}$.

Next, it easily follows from (1.12) that there exists a point $x_0(t) \in [0,1]$ such that $v(x_0(t),t) = 1$. Therefore we have

$$|\log v| \le \left|\int_{x_0}^x \frac{v_x}{v}dx\right| \le \left(\int_0^1 \frac{v_x^2}{v^3}dx\right)^{\frac{1}{2}}, \tag{1.26}$$

from which one obtains the following relation between X and Y

$$X \le |v|_\infty^\gamma Y \le Y\exp\left(\gamma X^{\frac{\gamma-1}{2\gamma}} Y^{\frac{1}{2\gamma}}\right). \tag{1.27}$$

In order to proceed this relation, we use the following lemma, without proof.

Lemma 1. 1 *Let $g(x)$ be a function in $C([0,\infty))$ satisfying $g(0) = 0$, and is monotone increasing on some interval $[0, A_0]$. Let A be an arbitrary number satisfying $0 < A \le A_0$. Then the following inequality is vallid for all $B \ge 0$*

$$AB \le \begin{cases} \int_0^A g(x)dx + \int_0^B g^{-1}(x)dx & \text{for } 0 \le B \le g(A_0), \\ \int_0^A g(x)dx + A_0 B - \int_0^{A_0} g(x)dx & \text{for } B \ge g(A_0). \end{cases} \tag{1.28}$$

Putting $A = X^{\frac{\gamma-1}{2\gamma}}, A_0 = \left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma-1}{2\gamma}}, B = Y^{\frac{1}{2\gamma}}$ and $g(x) = \frac{1}{\gamma-1}\frac{x^{\frac{\gamma+1}{\gamma-1}}}{x^{\frac{2\gamma}{\gamma-1}}+1}$ into (1.28), one shows from (1.27) that

$$\frac{X}{\sqrt{X+1}} \le \begin{cases} Y\exp\left(\gamma\int_0^{Y^{\frac{1}{2\gamma}}} g^{-1}(\xi)d\xi\right) & \text{for } 0 \le Y \le \alpha(\gamma), \\ Y\left(\frac{\gamma-1}{2\gamma}\right)^{\frac{1}{2}}\exp\left(\gamma\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma-1}{2\gamma}} Y^{\frac{1}{2\gamma}}\right) & \text{for } Y \ge \alpha(\gamma), \end{cases} \tag{1.29}$$

provided that $X \le \frac{\gamma+1}{\gamma-1}$, where $\alpha(\gamma)$ is defined by $\alpha(\gamma) = \left(\frac{1}{2\gamma}\right)^{2\gamma}\left(\frac{\gamma+1}{\gamma-1}\right)^{\gamma+1}$.

Let us consider the function

$$G(y) = \begin{cases} y\exp\left(\gamma\int_0^{y^{\frac{1}{2\gamma}}} g^{-1}(\xi)d\xi\right) & \text{for } 0 \le y \le \alpha(\gamma), \\ y\left(\frac{\gamma-1}{2\gamma}\right)^{\frac{1}{2}}\exp\left(\gamma\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma-1}{2\gamma}} y^{\frac{1}{2\gamma}}\right) & \text{for } y \ge \alpha(\gamma). \end{cases} \tag{1.30}$$

Since the function $G(y)$ is a monotone increasing one with respect to y, there exists the inverse function $y = G^{-1}(x)$, which has a following property

$$
\begin{aligned}
(1.31)\qquad H(x) &\equiv \frac{G^{-1}(x)}{x} \\
&= \begin{cases} \exp\left(-\gamma \displaystyle\int_0^{G^{-1}(x)^{\frac{1}{2\gamma}}} g^{-1}(\xi)d\xi\right) & \text{for } 0 \le x \le G(\alpha(\gamma)) \\ \left(\dfrac{2\gamma}{\gamma-1}\right)^{\frac{1}{2}} \exp\left(-\gamma\left(\dfrac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma-1}{2\gamma}} G^{-1}(x)^{\frac{1}{2\gamma}}\right) & \text{for } x \ge G(\alpha(\gamma)) \end{cases} \\
&\le 1,
\end{aligned}
$$

and $H(x)$ is a decreasing function of x.

We are now in a position to estimate $E_2^2(t)$ from below. Assuming $X \le \dfrac{\gamma+1}{\gamma-1}$, we have from (1.20) and (1.29)

$$
(1.32)\qquad G^{-1}\left(\frac{X}{\sqrt{X+1}}\right) \le Y \le \frac{4}{a\mu\gamma}E_2^2,
$$

or equivalently

$$
(1.33)\qquad H\left(\frac{X}{\sqrt{X+1}}\right)\frac{X}{\sqrt{X+1}} \le \frac{4}{a\mu\gamma}E_2^2.
$$

Using the monotonicity of the functions $H(x)$ and $\dfrac{x}{\sqrt{x+1}}$, and the relation $X \le C_2E_1^2$ $(C_2 = 16/\mu^2)$ shown in (1.21), one obtains from (1.33)

$$
(1.34)\qquad H\left(\frac{C_2E_1^2}{\sqrt{C_2E_1^2+1}}\right)\frac{X}{\sqrt{C_2E_1^2+1}} \le \frac{4}{a\mu\gamma}E_2^2,
$$

namely

$$
(1.35)\qquad G^{-1}\left(\frac{C_2E_1^2}{\sqrt{C_2E_1^2+1}}\right)\frac{X}{C_2E_1^2} \le \frac{4}{a\mu\gamma}E_2^2.
$$

Next we shall estimate $\Phi(v)$. For the sake of the point $x_0(t)$, it follows that

$$
\begin{aligned}
(1.36)\qquad \frac{a}{\gamma-1}\left(v^{-\gamma+1}-1\right) &= \frac{a}{\gamma-1}\int_{x_0}^{x}\frac{\partial}{\partial x}\left(v^{-\gamma+1}-1\right)dx \\
&= -a\int_{x_0}^{x}\frac{v_x}{v^\gamma}dx \le a\int_0^1\frac{|v_x|}{v^\gamma}dx.
\end{aligned}
$$

Thus

$$
(1.37)\qquad \int_0^1 \Phi(v)dx \le a\int_0^1\frac{|v_x|}{v^\gamma}dx \le a\left(\int_0^1\frac{v_x^2}{v^{\gamma+2}}dx\right)^{\frac{1}{2}}\left(\int_0^1 v^{2-\gamma}dx\right)^{\frac{1}{2}}.
$$

As we are interested in γ which is near 1, we may assume $1 < \gamma \leq 2$, so it is easily verified that the integration $\int_0^1 v^{2-\gamma} dx$ is less or equal to 1. Using (1.20), one gets

$$\int_0^1 \Phi(v)dx \leq a \left(\int_0^1 \frac{v_x^2}{v^{\gamma+2}} dx \right)^{\frac{1}{2}} \leq 2\sqrt{\frac{a}{\mu\gamma}} E_2. \tag{1.38}$$

Therefore it follows from (1.38) and the property of $H(x)$ that

$$H\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right) \int_0^1 \Phi(v)dx \leq 2\sqrt{\frac{a}{\mu\gamma}} E_2, \tag{1.39}$$

namely

$$G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right) \frac{1}{C_2 E_1^2} \int_0^1 \Phi(v)dx \leq 2\sqrt{\frac{a}{\mu\gamma}} E_2. \tag{1.40}$$

Similar consideration as above yields

$$G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right) \frac{1}{C_2 E_1^2} \int_0^1 u^2 dx \leq \frac{4}{\mu} E_2^2. \tag{1.41}$$

Multiplying (1.35) by $\frac{3\mu^2}{16}$, (1.41) by $\frac{3}{4}$ and adding the results together with (1.40) imply

$$G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right) \leq \frac{12}{\mu^2}\left(\frac{\mu}{a\gamma} + \frac{4}{\mu} \right) E_2^2 + \frac{32}{\mu^2}\sqrt{\frac{a}{\mu\gamma}} E_2, \tag{1.42}$$

where we have used (1.21). As the last term in the right hand side of (1.42) is bounded by $\epsilon + \frac{256a}{\epsilon\mu^5\gamma} E_2^2$, easy calculation shows that

$$\frac{\epsilon}{1+\epsilon}\left(G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right) - \epsilon \right) \leq C_3 E_2^2, \tag{1.43}$$

for any $\epsilon > 0$ and $C_3 = \max\left(\frac{12}{\mu^2}\left(\frac{\mu}{a\gamma} + \frac{4}{\mu} \right), \frac{256a}{\mu^5\gamma} \right)$. Putting $\epsilon = \frac{1}{2} G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right)$ into (1.43), and substituting it into (1.25), we derive

$$\frac{d}{dt} E_1^2(t) + \frac{1}{4C_3} \frac{G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right)^2}{2 + G^{-1}\left(\frac{C_2 E_1^2}{\sqrt{C_2 E_1^2 + 1}} \right)} \leq \frac{1}{2\mu} |f|_\infty^2 + C_1 |f|_\infty^{\frac{2\gamma}{2\gamma-1}} E_1^{\frac{2(\gamma-1)}{2\gamma-1}}. \tag{1.44}$$

If $E_1(0)$ and $|f|_\infty$ are sufficiently small so as to satisfy

$$E_1(0) < \left(\frac{1}{C_2} \frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}}, \tag{1.45}$$

and

$$(1.46)\qquad \frac{1}{2\mu}|f|_\infty^2 + C_1 C_2^{-\frac{\gamma-1}{2\gamma-1}}|f|_\infty^{\frac{2\gamma}{2\gamma-1}}\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma-1}{2\gamma-1}} < \frac{1}{4C_3}\frac{G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)^2}{2+G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)},$$

then (1.44) shows that $E_1(t) < \left(\frac{1}{C_2}\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}}$ for all $t>0$, therefore $X < \frac{\gamma+1}{\gamma-1}$. One of the sufficient condition on f that satisfies (1.46) is

$$(1.47)\qquad |f|_\infty \le \min\left\{\begin{array}{l}\left(\frac{\mu}{4C_3}\frac{G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)^2}{2+G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)}\right)^{\frac{1}{2}}, \\ C_2^{\frac{\gamma-1}{2\gamma}}\left(\frac{1}{8C_1C_3}\frac{G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)^2}{2+G^{-1}\left(\frac{\gamma+1}{\sqrt{2\gamma(\gamma-1)}}\right)}\right)^{\frac{2\gamma-1}{2\gamma}}\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{\gamma-1}{2\gamma}}.\end{array}\right.$$

We have already got the following result.

Proposition 1. 1 *Let the asuumptions in Theorem 1.1 be satisfied. If the initial conditions and the external force satisfy (1.45) and (1.46), then the following estimates are vallid*

$$(1.48)\qquad C^{-1} \le v(x,t) \le C \qquad \forall\ (x,t) \in\ Q,$$

and

$$(1.49)\qquad \sup_{t\ge 0}(\| v(t) \|_1 + \| u(t) \|) \le C,$$

where C is a positive constant depending only on a, μ, γ, C_0, $\| (v_0, u_0) \|_1$, and $|f|_\infty$.

Let $C(\gamma)$ be defined by the right hand side of (1.47). Then the proof of Theorem 1.1 shall be completed if we estimate $\| u_x(t) \|$. Multiplying (1.2) by $-u_{xx}$ and integrating it over $[0,1]$ yield

$$(1.50)\qquad \frac{1}{2}\frac{d}{dt}\int_0^1 u_x^2 dx + \mu\int_0^1 \frac{u_{xx}^2}{v}dx = -a\gamma\int_0^1 \frac{v_x u_{xx}}{v^{\gamma+1}}dx + \mu\int_0^1 \frac{u_x v_x u_{xx}}{v^2}dx - \int_0^1 u_{xx} f dx.$$

Using Proposition 1.1, we can esimate each term in the right hand side of (1.50) as

$$(1.51)\qquad \left|a\gamma\int_0^1 \frac{v_x u_{xx}}{v^{\gamma+1}}dx\right| \le \epsilon\int_0^1 u_{xx}^2 dx + \frac{C}{\epsilon},$$

$$(1.52)\qquad \left|\mu\int_0^1 \frac{u_x v_x u_{xx}}{v^2}dx\right| \le \epsilon\int_0^1 u_{xx}^2 dx + \frac{C}{\epsilon}\int_0^1 v_x^2 u_x^2 dx,$$

$$\left|\int_0^1 u_{xx} f\,dx\right| \le \epsilon \int_0^1 u_{xx}^2 dx + \frac{C}{\epsilon}, \tag{1.53}$$

for any $\epsilon > 0$. Since u satisfies the boundary conditions (1.5), there exists a point $x_1(t) \in (0,1)$ such that $u_x(x_1(t), t) = 0$. Using this point, we have the relation $u_x^2 = \int_{x_1}^{x} \frac{\partial}{\partial x}\left(u_x^2\right) dx = 2\int_{x_1}^{x} u_x u_{xx} dx$, which gives

$$|u_x|_\infty^2 \le \epsilon^2 \int_0^1 u_{xx}^2 dx + \frac{1}{\epsilon^2}\int_0^1 u_x^2 dx, \tag{1.54}$$

for any $\epsilon > 0$. Substituting (1.54) into the last term in the right hand side of (1.52) imply

$$\left|\mu \int_0^1 \frac{u_x v_x u_{xx}}{v^2} dx\right| \le C\left(\epsilon \int_0^1 u_{xx}^2 dx + \frac{1}{\epsilon^3}\int_0^1 u_x^2 dx\right). \tag{1.55}$$

By choosing ϵ sufficiently small, we obtain from (1.50), (1.51), (1.53) and (1.55)

$$\frac{d}{dt}\int_0^1 u_x^2 dx + \int_0^1 u_{xx}^2 dx \le C\left(1 + \int_0^1 u_x^2 dx\right). \tag{1.56}$$

It easily follows from (1.18) and (1.56) that

$$\frac{d}{dt}\left(E_1^2(t) + \int_0^1 u_x^2 dx\right) + \left(E_1^2(t) + \int_0^1 u_x^2 dx\right) \le C, \tag{1.57}$$

from which we conclude

$$E_1^2(t) + \int_0^1 u_x^2 dx \le C. \tag{1.58}$$

This completes the proof of Theorem 1.1.

It is to be noted that for the proof of Remark 1.1, we can refer to [11].

2 Piston Problem

In this section we consider the piston problem for a one-dimensional isentropic model system of compressible viscous gas, represented by (1.1) and (1.2) with $f \equiv 0$. We also assume that the viscosity coefficient is a positive constant, and consider these equations in a fixed domain Q defined by (1.3). The piston problem consists of finding a solution to (1.1), (1.2) subject to the initial and boundary conditions

$$v(x,0) = v_0(x), \quad u(x,0) = u_0(x) \quad \text{on } 0 < x < 1, \tag{2.1}$$

$$u(0,t) = 0, \quad u(1,t) = u_1(t) \quad \text{on } t > 0, \tag{2.2}$$

where $u_1(t)$ denotes a given velocity of the piston. For the above data, it is natural to assume

$$(v_0, u_0) \in H^1(0,1) \times H^1(0,1), \tag{2.3}$$

$$\int_0^1 v_0(x)dx = 1, \tag{2.4}$$

$$C_0^{-1} \leq v_0(x) \leq C_0 \quad \textit{for some constant} \quad C_0 > 1, \tag{2.5}$$

$$u_1, \ u_1' = \frac{du_1}{dt} \in L^\infty(0,\infty), \tag{2.6}$$

and the compatibility condition

$$u_0(0) = 0, \quad u_0(1) = u_1(0). \tag{2.7}$$

Let $X(t)$ be the path of the piston in Eulerian coordinate, which is expressed by

$$\begin{aligned} X(t) \ &\equiv \ \int_0^1 v(x,t)dx = \int_0^1 v_0(x)dx + \int_0^t u_1(s)ds \\ &= \ 1 + \int_0^t u_1(s)ds. \end{aligned} \tag{2.8}$$

For this, we further assume

$$X_0^{-1} \leq X(t) \leq X_0 \quad \textit{for some constant} \quad X_0 > 1, \tag{2.9}$$

whose physical meaning is that the piston remains bounded and away from the fixed boundary $x = 0$.

For this piston problem, Itaya [4] proved the existence of global in time solution: For any fixed $T > 0$, there exists a unique solution $(v,u) \in B^{1+\theta}(Q_T) \times H^{2+\theta}(Q_T)$ to (1.1), (1.2) with (2.1), (2.2), where $Q_T = \{\ (x,t) \mid 0 < x < 1,\ 0 < t < T\}$ and $0 < \theta < 1$. The bound of the solution that he has constructed, however, depends on T. In the case of isothermal gas $\gamma = 1$, Matsumura & Nishida [10] established the unique existence of uniformly bounded global in time solution to the problem for arbitrary initial data and the velocity of the piston satisfying (2.3) - (2.9).

The aim of this section is to establish the similar results to [10] for $\gamma > 1$, and to elucidate the relation between $\gamma = 1$ and $\gamma > 1$.

In what follows, we denote the norms in L^∞, L^2 and H^1 by $|\cdot|_\infty, \|\cdot\|$ and $\|\cdot\|_1$, respectively. The following is our main theorem.

Theorem 2. 1 *Assume (2.3) - (2.9) and* $1 < \gamma \leq 2$. *Then there exists a constant* $C(\gamma)$, *which has the same property as in section 1, such that if* $E_1(0) < \frac{\sqrt{2}}{8}\mu X_0^{-1}\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}}$ *and* $|u_1|_\infty, |u_1'|_\infty \leq C(\gamma)$, *then the piston problem (1.1), (1.2) with (2.1), (2.2) has a unique uniformly bounded global in time solution* (v,u) *satisfying*

$$C^{-1} \leq v(x,t) \leq C \qquad \textit{for any} \ (x,t) \in Q, \tag{2.10}$$

and

$$\sup_{t\geq 0} \| (v,u)(t) \|_1 \leq C, \tag{2.11}$$

where $E_1(0)$ *will be defined in (2.37), and* C (>1) *is a constant depending only on* a, μ, γ, C_0, $\| (v_0, u_0) \|_1$, $|u_1|_\infty$ *and* $|u_1'|_\infty$.

Proof of Theorem 2.1.

It is sufficient to prove (2.10) and (2.11). Because from these inequalities, the unique existence of uniformly bounded global in time solution can be established, according to the same arguments as in [10].

In what follows, the letters $C_1, C_2, \cdots$ denote the positive constants which depend only on the data.

Let the function $U(x,t)$ be

$$U(x,t) = \frac{u_1(t)}{X(t)} \int_0^x v(x,t)dx. \tag{2.12}$$

Changing the unknown functions $w = (u-U)X$, $m = \dfrac{v}{X}$ in (1.1), (1.2), (2.1) and (2.2) leads to

$$m_t - \frac{w_x}{X^2} = 0, \tag{2.13}$$

$$w_t + X\left(\frac{a}{(mX)^\gamma}\right)_x = \mu\left(\frac{w_x}{mX}\right)_x - u_1' X \int_0^x m dx, \tag{2.14}$$

$$m(x,0) = m_0(x), \quad w(x,0) = w_0(x) \qquad (0 < x < 1), \tag{2.15}$$

$$w(0,t) = w(1,t) = 0 \qquad (t > 0), \tag{2.16}$$

$$\int_0^1 m_0(x)dx = 1. \tag{2.17}$$

Here $m_0(x) = v_0(x)$, $w_0(x) = u_0(x) - u_1(0)\displaystyle\int_0^x v_0(x)dx$, $(m_0, w_0) \in H^1 \times H_0^1$.

First we note that integration of (2.13) over $[0,1]$ gives

$$\int_0^1 m(x,t)dx = 1 \quad \mathit{for\ all}\ t \geq 0. \tag{2.18}$$

Multiplying (2.14) by w and integrating the result over $[0,1]$ imply

$$\frac{1}{2}\frac{d}{dt}\int_0^1 w^2 dx - \int_0^1 X\frac{a}{(mX)^\gamma} w_x dx + \mu\int_0^1 \frac{w_x^2}{mX}dx = -u_1' X \int_0^1 w dx \int_0^x m dx'. \tag{2.19}$$

By using (2.13), the second term in the left hand side of (2.19) becomes

$$
\begin{aligned}
(2.20)\qquad -\int_0^1 X\frac{a}{(mX)^\gamma}w_x dx &= -\int_0^1 X^3\frac{a}{(mX)^\gamma}m_t dx \\
&= \frac{d}{dt}\int_0^1 X^2\Phi(mX)dx + \int_0^1 X^2\frac{a}{(mX)^\gamma}mu_1 dx - aX^{2-\gamma}u_1 - \\
&\quad -2\int_0^1 Xu_1\Phi(mX)dx \\
&= \frac{d}{dt}\int_0^1 X^2\Phi(mX)dx - a\frac{3-\gamma}{\gamma-1}u_1X^{2-\gamma}\int_0^1\left(m^{1-\gamma}-1\right)dx,
\end{aligned}
$$

where $\Phi(v) = \dfrac{a}{\gamma-1}\left(v^{-\gamma+1} - X^{-\gamma+1}\right) + a(v-X)\ (\geq 0)$. Thus, (2.19) can be written in the form

$$
\begin{aligned}
(2.21)\qquad &\frac{d}{dt}\int_0^1\left\{\frac{1}{2}w^2 + X^2\Phi(mX)\right\}dx + \mu\int_0^1\frac{w_x^2}{mX}dx \\
&= -u_1'X\int_0^1 wdx\int_0^x mdx' + a\frac{3-\gamma}{\gamma-1}u_1X^{2-\gamma}\int_0^1\left(m^{1-\gamma}-1\right)dx.
\end{aligned}
$$

Let us evaluate each term in the right hand side of (2.21) as follows. Using the estimate

$$
(2.22)\qquad |w|_\infty \leq \left(\int_0^1\frac{w_x^2}{mX}dx\right)^{\frac{1}{2}}\left(\int_0^1 mXdx\right)^{\frac{1}{2}} = \left(X\int_0^1\frac{w_x^2}{mX}dx\right)^{\frac{1}{2}},
$$

which comes from $w = \displaystyle\int_0^x w_x dx$, we have for the first term in the right hand side of (2.21)

$$
\begin{aligned}
(2.23)\qquad \left|u_1'X\int_0^1 wdx\int_0^x mdx'\right| &\leq |u_1'|_\infty X^{\frac{3}{2}}\left(\int_0^1\frac{w_x^2}{mX}dx\right)^{\frac{1}{2}} \\
&\leq \frac{\mu}{2}\int_0^1\frac{w_x^2}{mX}dx + \frac{1}{2\mu}|u_1'|_\infty^2 X^3.
\end{aligned}
$$

Since (2.18) implies that there exists a point $x_0(t)\in[0,1]$ satisfying $m(x_0(t),t)=1$, we have

$$
\begin{aligned}
(2.24)\qquad \left|m^{1-\gamma}-1\right| &= \left|\int_{x_0}^x\left(m^{1-\gamma}-1\right)_x dx\right| \\
&\leq (\gamma-1)\int_0^1\frac{|m_x|}{m^\gamma}dx \leq (\gamma-1)\left(\int_0^1\frac{m_x^2}{m^{\gamma+2}}dx\right)^{\frac{1}{2}}\left(\int_0^1 m^{2-\gamma}dx\right)^{\frac{1}{2}} \\
&\leq (\gamma-1)\left(\int_0^1\frac{m_x^2}{m^{\gamma+2}}dx\right)^{\frac{1}{2}},
\end{aligned}
$$

provided that $1<\gamma\leq 2$. Hence the last term in the right hand side of (2.21) is estimated as

$$
(2.25)\qquad \left|a\frac{3-\gamma}{\gamma-1}u_1X^{2-\gamma}\int_0^1\left(m^{1-\gamma}-1\right)dx\right| \leq a(3-\gamma)|u_1|_\infty X^{2-\gamma}\left(\int_0^1\frac{m_x^2}{m^{\gamma+2}}dx\right)^{\frac{1}{2}}.
$$

Therefore we conclude from (2.21), (2.23) and (2.25) that

$$\frac{d}{dt}\int_0^1 \left\{\frac{1}{2}w^2 + X^2\Phi(mX)\right\} dx + \frac{\mu}{2}\int_0^1 \frac{w_x^2}{mX}dx \tag{2.26}$$
$$\leq \frac{1}{2\mu}|u_1'|_\infty^2 X^3 + a(3-\gamma)|u_1|_\infty X^{2-\gamma}\left(\int_0^1 \frac{m_x^2}{m^{\gamma+2}}dx\right)^{\frac{1}{2}}$$
$$\leq \frac{1}{2\mu}|u_1'|_\infty^2 X^3 + \epsilon X^{-\gamma}\int_0^1 \frac{m_x^2}{m^{\gamma+2}}dx + \frac{1}{4\epsilon}a^2(3-\gamma)^2 X^{4-\gamma}|u_1|_\infty^2$$

holds for any $\epsilon > 0$.

Multiplying (2.14) by $\dfrac{m_x}{mX}$ and integrating the result over $[0,1]$ yield

$$\frac{d}{dt}\int_0^1 \left\{\frac{\mu}{2}\frac{m_x^2}{m^2} - \frac{wm_x}{mX} + u_1 m\log m\right\} dx + a\gamma X^{-\gamma}\int_0^1 \frac{m_x^2}{m^{\gamma+2}}dx \tag{2.27}$$
$$= \frac{1}{X^2}\int_0^1 \frac{w_x^2}{mX}dx + u_1'\int_0^1 \frac{m_x}{m}dx\int_0^x mdx' + u_1'\int_0^1 m\log m dx,$$

where we have used (2.18) and the equation

$$\frac{u_1}{X^2}\int_0^1 \frac{wm_x}{m}dx = -u_1\int_0^1 m_t\log m dx \tag{2.28}$$

followed from (2.13). The second and the last terms in the right hand side of (2.27) are estimated as

$$\left|u_1'\int_0^1 \frac{m_x}{m}dx\int_0^x mdx'\right| \leq |u_1'|_\infty\left(\int_0^1 \frac{m_x^2}{m^3}dx\right)^{\frac{1}{2}}, \tag{2.29}$$

$$\left|u_1'\int_0^1 m\log m dx\right| = \left|u_1'\int_0^1 mdx\int_{x_0}^x \frac{m_x}{m}dx'\right| \tag{2.30}$$
$$\leq |u_1'|_\infty\int_0^1 \frac{|m_x|}{m}dx \leq |u_1'|_\infty\left(\int_0^1 \frac{m_x^2}{m^3}dx\right)^{\frac{1}{2}}.$$

Whence we derive from (2.27), (2.29) and (2.30)

$$\frac{d}{dt}\int_0^1 \left\{\frac{\mu}{2}\frac{m_x^2}{m^2} - \frac{wm_x}{mX} + u_1 m\log m\right\} dx + a\gamma X^{-\gamma}\int_0^1 \frac{m_x^2}{m^{\gamma+2}}dx \tag{2.31}$$
$$\leq \frac{1}{X^2}\int_0^1 \frac{w_x^2}{mX}dx + 2|u_1'|_\infty\left(\int_0^1 \frac{m_x^2}{m^3}dx\right)^{\frac{1}{2}}.$$

Multiplying (2.31) by $\frac{\mu}{4}X_0^{-2}$, and adding the result to (2.26) with $\epsilon = \frac{1}{2}a\gamma\frac{\mu}{4}X_0^{-2} = \frac{a\gamma\mu}{8}X_0^{-2}$, we have

$$\frac{d}{dt}\tilde{E}_1(t) + \tilde{E}_2(t) \tag{2.32}$$
$$\leq \frac{1}{2\mu}|u_1'|_\infty^2 X_0^3 + \frac{2a}{\mu}\frac{(3-\gamma)^2}{\gamma}X_0^{6-\gamma}|u_1|_\infty^2 + \frac{\mu}{2}X_0^{-2}|u_1'|_\infty\left(\int_0^1 \frac{m_x^2}{m^3}dx\right)^{\frac{1}{2}},$$

where

$$\tilde{E}_1(t) = \int_0^1 \left\{ \frac{1}{2} w^2 + X^2 \Phi(mX) + \frac{\mu^2}{8} X_0^{-2} \frac{m_x^2}{m^2} - \frac{\mu}{4} X_0^{-2} \frac{w m_x}{mX} + \right.$$
$$\left. + \frac{\mu}{4} X_0^{-2} u_1 m \log m \right\} dx, \tag{2.33}$$

$$\tilde{E}_2(t) = \frac{\mu}{4} \int_0^1 \frac{w_x^2}{mX} dx + \frac{a\gamma\mu}{8} X_0^{-\gamma-2} \int_0^1 \frac{m_x^2}{m^{\gamma+2}} dx. \tag{2.34}$$

Taking into account the inequalities

$$\left| \frac{\mu}{4} X_0^{-2} \frac{w m_x}{mX} \right| \le \frac{\mu^2}{16} X_0^{-2} \frac{m_x^2}{m^2} + \frac{1}{4} X_0^{-2} \frac{w^2}{X^2} \le \frac{\mu^2}{16} X_0^{-2} \frac{m_x^2}{m^2} + \frac{1}{4} w^2, \tag{2.35}$$

$$\begin{aligned} \int_0^1 \left| \frac{\mu}{4} X_0^{-2} u_1 m \log m \right| dx &\le \frac{\mu}{4} X_0^{-2} |u_1|_\infty \left(\int_0^1 \frac{m_x^2}{m^2} dx \right)^{\frac{1}{2}} \\ &\le \frac{\mu^2}{32} X_0^{-2} \int_0^1 \frac{m_x^2}{m^2} dx + \frac{1}{2} X_0^{-2} |u_1|_\infty^2, \end{aligned} \tag{2.36}$$

we introduce the functions $E_1^2(t)$ and $E_2^2(t)$ as

$$E_1^2(t) = \tilde{E}_1(t) + \frac{1}{2} X_0^{-2} |u_1|_\infty^2, \tag{2.37}$$

$$E_2^2(t) = \tilde{E}_2(t) + \frac{1}{2} X_0^{-2} |u_1|_\infty^2. \tag{2.38}$$

Then it is easily seen that from (2.35) and (2.36)

$$\begin{aligned} &\int_0^1 \left\{ \frac{1}{4} w^2 + X^2 \Phi(mX) + \frac{\mu^2}{32} X_0^{-2} \frac{m_x^2}{m^2} \right\} dx \le E_1^2(t) \le \\ &\le \int_0^1 \left\{ \frac{3}{4} w^2 + X^2 \Phi(mX) + \frac{7\mu^2}{32} X_0^{-2} \frac{m_x^2}{m^2} \right\} dx + X_0^{-2} |u_1|_\infty^2 \end{aligned} \tag{2.39}$$

follows and from the differential inequality (2.32)

$$\begin{aligned} &\frac{d}{dt} E_1^2(t) + E_2^2(t) \\ &\le \frac{1}{2\mu} |u_1'|_\infty^2 X_0^3 + \left(\frac{2a}{\mu} \frac{(3-\gamma)^2}{\gamma} X_0^{6-\gamma} + \frac{1}{2} X_0^{-2} \right) |u_1|_\infty^2 + \\ &+ \frac{\mu}{2} X_0^{-2} |u_1'|_\infty \left(\int_0^1 \frac{m_x^2}{m^3} dx \right)^{\frac{1}{2}}. \end{aligned} \tag{2.40}$$

In what follows, repeating the same argument in [11] leads to Theorem 2.1; we omit the detail (see [14]).

3 Existence of Periodic Solutions

In this section we only introduce a result on the existence of periodic solutions for one-dimensional motion of a general viscous isentropic gas in a fixed domain, with periodic external forces.

Theorem 3. 1 *Let the external force f be periodic in time with period $\omega > 0$, i.e.*

$$f(\cdot, t+\omega) = f(\cdot, t), \quad \forall t \geq 0. \tag{3.1}$$

Furthermore, suppose f and its first derivatives being bounded. Assume $1 < \gamma \leq 2$. Then there exits a constant $C(\gamma) > 0$, which has the same property as in section 1, such that if $\| f \|_\infty \leq C(\gamma)$, then the system (1.1), (1.2), (1.5) with the normalized condition

$$\int_0^1 v dx = 1. \tag{3.2}$$

has at least one ω- periodic solution, belonging to the class

$$v \in C^0\left(0, \omega; H^1\right), \quad v_t \in C^0\left(0, \omega; L^2\right) \cap L^2\left(0, \omega; H^1\right), \tag{3.3}$$

$$u \in C^0\left(0, \omega; H_0^1\right) \cap L^2\left(0, \omega; H^2\right), \quad u_t \in L^2\left(0, \omega; L^2\right), \tag{3.4}$$

satisfying

$$\max_{0 \leq t \leq \omega} \| (v, u)(t) \|_1^2 + \int_0^\omega \{\| v(t) \|_1^2 + \| u(t) \|_2^2\} dt \leq C, \tag{3.5}$$

where $\| \cdot \|_k$ $(k = 1, 2)$ denotes the norm in H^k, and C is a positive constant depending only on a, μ, γ, ω and $\| f \|_\infty$.

For the proof of this theorem, and for further detail, we can refer to [13].

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