数理解析研究所講究録
923 巻 1995 年 68-86

An Inverse Problem for the Rotating Wave Approximation on a Partial $*$-Algebra

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I. INTRODUCTION.

A long-time behavior of the canonical correlation function as an infinite volume limit interests us. In this paper, we would like to apply Arai's results [5] concerning long-time behavior of two-point functions to a class of canonical correlation functions of position operators as infinite volume limit. In [5], Arai argued long-time behavior of two-point functions of position operators for some models of a quantum harmonic oscillator interacting with bosons.

We consider a quantum harmonic oscillator in thermal equilibrium with any system in certain classes of bosons with infinitely many degrees of freedom in a finite volume $V > 0$. Our models include photons in a laser interacting with oscillation caused by a heat bath, which can be observed when the laser passes in the heat bath, and are photons in a laser interacting with oscillation caused by phonons on the surface of a material, which can be observed when we irradiate the weak laser on the surface.

When a two-point function (or canonical correlation function) $R^V(t_1, t_2)$ of the position or momentum operator of the harmonic oscillator is given by an observation, we take an infinite volume limit, $V \to \infty$, for $R^V(t_1, t_2)$, and get $R_\beta^\infty(t_1, t_2) \equiv \lim_{V \to \infty} R^V(t_1, t_2)$ under suitable conditions. And we argue long-time behavior of $R_\beta^\infty(t_1, t_2)$.

Let O_{bs} be the position operator $q \overset{\text{def}}{=} (a + a^+)/\sqrt{2\omega}$, momentum operator

$p \stackrel{\text{def}}{=} i\sqrt{\omega}\,(a^+ - a)\,/\sqrt{2}$, or their smeared operators O^{sm}_{bs}, where a and a^+ are the annihilation and creation operators of the quantum harmonic oscillator, respectively, and $\omega > 0$ denotes the original frequency of the quantum harmonic oscillator. In this paper, we let that ω is equal to 1, i.e., $\omega = 1$, for the sake of simplicity. $R^V(t_1, t_2)$ is the observed two-point function of O_{bs}, given by the Bogoliubov scalar product. In our system to be considered, for $R^V(t_1, t_2)$ there exists a Hamiltonian $H^V_{a,b}$ which governs our system and is described by the annihilation operator a, the creation operator a^+ of the quantum harmonic oscillator; and annihilation operators b_k, creation operators b^+_k of bosons, i.e., $H^V_{a,b} \stackrel{\text{def}}{=} H^V\left(a, a^+; b_k, b^+_k; k \in \boldsymbol{N}\right)$, such that $e^{-\beta H^V_{a,b}}$ is a trace class operator (where β denotes the inverse temperature and we set the Planck constant $\hbar = 1$), furthermore, $R^V(t_1, t_2)$ is defined by

$$R^V(t_1,t_2) \stackrel{\text{def}}{=} \frac{1}{\beta \operatorname{tr}\left(e^{-\beta H^V_{a,b}}\right)} \int_0^\beta d\lambda \operatorname{tr}\left(e^{-(\beta-\lambda)H^V_{a,b}} e^{iH^V_{a,b}t_1} O_{bs} e^{-iH^V_{a,b}t_1} e^{-\lambda H^V_{a,b}} e^{iH^V_{a,b}t_2} O_{bs} e^{-iH^V_{a,b}t_2}\right).$$

Of course, the operator form of $H^V_{a,b}$ is unknown, so there is a possibility that $H^V_{a,b}$ is non-quadratic.

For applying Arai's result [5, Theorem 1.3] to our case, we first solve the following inverse problem: In terms of $R^V(t_1, t_2)$ only, determine positive frequencies x_0 and x_k $(k \in \boldsymbol{N})$ of the quantum harmonic oscillator and scalar bosons, respectively; and coupling constants $y_k \in \boldsymbol{C}$ $(k \in \boldsymbol{N})$; appearing in the Hamiltonian of the rotating wave approximation (RWA),

$$H^V_{\text{RWA}}(x,y) \stackrel{\text{def}}{=} x_0 a^+ a + \sum_{k=1}^{\infty} x_k b^+_k b_k + \sum_{k=1}^{\infty} \left(y_k a^+ b_k + \overline{y_k} b^+_k a\right),$$
$$x \stackrel{\text{def}}{=} (x_0, x_1, x_2, \cdots), \qquad y \stackrel{\text{def}}{=} (y_1, y_2, \cdots),$$

(where $\overline{c}$ means the complex conjugate of $c \in \boldsymbol{C}$), and determine constants $c_1, c_2 \in \boldsymbol{C}$ in terms of $R^V(t_1, t_2)$ only such that the Hamiltonian $H^V_{\text{RWA}}(x, y)$ recovers the original $R^V(t_1, t_2)$ in the following sense:

$$\begin{aligned} &\left\{\text{energy levels of } H^V_{\text{RWA}}(x,y)\right\} = \left\{\text{positive poles of } \int_0^\infty dt e^{itz} R^V(t)\right\}, \\ &R^V(t_1,t_2) = \text{a representation using } W^V(t_1,t_2), \end{aligned} \tag{1.1}$$

where

$$W^{V}(t_1,t_2) \stackrel{\text{def}}{=} (\Omega_0\,,\, e^{iH^{V}_{\text{RWA}}(x,y)t_1}O_{bs}e^{-iH^{V}_{\text{RWA}}(x,y)t_1}e^{iH^{V}_{\text{RWA}}(x,y)t_2}O_{bs}e^{-iH^{V}_{\text{RWA}}(x,y)t_2}\Omega_0)_{a,b},$$

which is the vacuum expectation of $e^{iH^{V}_{\text{RWA}}(x,y)t_1}O_{bs}e^{-iH^{V}_{\text{RWA}}(x,y)t_1}e^{iH^{V}_{\text{RWA}}(x,y)t_2}O_{bs}e^{-iH^{V}_{\text{RWA}}(x,y)t_2}$, and $(\cdot,\cdot)_{a,b}$ is a natural inner product of the Fock space $\mathcal{F}_{a,b}$. Moreover Ω_0 is the Fock vacuum and the ground state of $H^{V}_{\text{RWA}}(x,y)$.

Indeed there are some negative criticisms against RWA [14, §V.D] and there exists the independent-oscillator model which is more useful in physics than RWA [14,26], but we venture to use RWA in so far as our purpose of investigating the long-time behavior. Why do we represent $R^{V}(t_1,t_2)$ by using RWA? Because it is nothing but easy to argue an infinite volume limit for the Hamiltonian of RWA in mathematics, and RWA is established in mathematics by [4,5,23]. So there is a possibility that we can investigate the long-time behavior of infinite volume limit of $R^{V}(t_1,t_2)$ through infinite volume limit of $W^{V}(t_1,t_2)$ in representation (1.1) of $R^{V}(t_1,t_2)$ using $W^{V}(t_1,t_2)$. Actually, what is better, the long-time behavior of the infinite volume limit $W(t_1,t_2)$ of $W^{V}(t_1,t_2)$ of the position operator is investigated exactly by Arai in [5].

An answer for this inverse problem for RWA is given by Theorem 2.1 in this paper. By representation (1.1), we can consider an infinite volume $R^{\infty}_{\beta}(t_1,t_2)$ of $R^{V}(t_1,t_2)$ for the position operator q, through the right side of (1.1). Then, we have a representation of $R^{\infty}_{\beta}(t_1,t_2)$ by using $W(t_1,t_2)$. And, applying Arai's results in [5] to the representation, we consider the long-time behavior of $R^{\infty}_{\beta}(t) \equiv R^{\infty}_{\beta}(0,t)$ for the position operator q in Theorem 2.3 of this paper.

II. STATEMENT OF MAIN RESULTS.

In this section, in order to introduce canonical correlation functions defined by the Bogoliubov scalar product, and explain our main results, we first set up a general framework. For a while, we fix a finite volume $V > 0$.

We give a complex Hilbert space $l^2(\boldsymbol{N})$ by $l^2(\boldsymbol{N}) \stackrel{\mathrm{def}}{=} \left\{(c_1, c_2, \cdots) \,\middle|\, c_k \in \boldsymbol{C},\ k \in \boldsymbol{N},\ \sum_{k=1}^{\infty} |c_k|^2 < \infty\right\}$. For each $f \in \boldsymbol{C} \oplus l^2(\boldsymbol{N})$, we denote f by $(f_0, f_1, f_2, \cdots)$, i.e., $f = (f_0, f_1, f_2, \cdots)$, where $f_0 \in \boldsymbol{C}$ and $(f_1, f_2, \cdots) \in l^2(\boldsymbol{N})$. An inner product $(\ \ ,\ \ )_{l^2}$ of $\boldsymbol{C} \oplus l^2(\boldsymbol{N})$ is given by $(f, g)_{l^2} \stackrel{\mathrm{def}}{=} \sum_{k=0}^{\infty} \overline{f_k} g_k$ $(f, g \in \boldsymbol{C} \oplus l^2(\boldsymbol{N}))$, where $\overline{c}$ denotes the complex conjugate of $c \in \boldsymbol{C}$. We denote the symmetric Fock space over $\boldsymbol{C} \oplus l^2(\boldsymbol{N})$ by $\mathcal{F}_S(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))$, which is defined by $\mathcal{F}_S(\boldsymbol{C} \oplus l^2(\boldsymbol{N})) \stackrel{\mathrm{def}}{=} \bigoplus_{n=0}^{\infty} S_n(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))^n$, where $S_n(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))^n$ is the n-fold symmetric tensor product of $\boldsymbol{C} \oplus l^2(\boldsymbol{N})$ for each $n \in \boldsymbol{N}$ and $S_0(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))^0 \stackrel{\mathrm{def}}{=} \boldsymbol{C}$ (see [32, p.53, Example 2]), and S_n denotes an orthogonal projection onto $S_n(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))^n$ for each $n \in \boldsymbol{N}^* \stackrel{\mathrm{def}}{=} \{0, 1, \cdots\}$ (see again [32, p.53, Example 2]).

The operators a and a^+ physically denote the annihilation and creation operators of the quantum harmonic oscillator, respectively, and likewise, operators b_k and b_k^+ $(k \in \boldsymbol{N})$ are the annihilation and creation operators of the bosons with infinitely many degrees of freedom.

We consider a quantum harmonic oscillator in thermal equilibrium with a system of bosons with infinitely many degrees of freedom in the finite volume. So, we give a state space for our system by a symmetric Fock space, $\mathcal{F}_S(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))$, which is denoted by simply $\mathcal{F}_{a,b}$ for convenience, i.e., $\mathcal{F}_{a,b} \stackrel{\mathrm{def}}{=} \mathcal{F}_S(\boldsymbol{C} \oplus l^2(\boldsymbol{N}))$. And we denote the inner product of $\mathcal{F}_{a,b}$ by $(\ \ ,\ \ )_{a,b}$.

For our system, there exists a Hamiltonian $H_{a,b}^V = H^V(a, a^+; b_k, b_k^+, k \in \boldsymbol{N})$ whose form is unknown. So $H_{a,b}^V$ may be non-quadratic, but must be realized as a self-adjoint operator acting in the Fock space $\mathcal{F}_{a,b}$. Since we are now considering the thermal equilibrium quantum system, $H_{a,b}^V$ is a self-adjoint operator acting in $\mathcal{F}_{a,b}$, and

(H) $\quad e^{-\tau H_{a,b}^V}$ is a trace class operator on $\mathcal{F}_{a,b}$ for every $\tau \in (0, \beta]$,

where β is the inverse temperature. This condition implies that the spectra of $H_{a,b}^V$ are purely discrete and the eigenvectors $\{\varphi_n \mid n \in \boldsymbol{N}^*\}$ of $H_{a,b}^V$ form a complete orthonormal system of $\mathcal{F}_{a,b}$, where $\boldsymbol{N}^* \stackrel{\text{def}}{=} \{0,1,\cdots\}$. We count the eigenvalues λ_n $(n \in \boldsymbol{N}^*)$ of $H^V_{a,b}$ in such a way that $H^V_{a,b}\varphi_n = \lambda_n\varphi_n$ and $0 < \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_n \leq \lambda_{n+1} \leq \cdots \nearrow \infty$.

For the Hamiltonian $H^V_{a,b}$, we can construct a Liouville space $\mathbf{X}_c(H^V_{a,b})$, which is a set of adequate quantum operators acting in $\mathcal{F}_{a,b}$ [22–24]. We denote the linear hull of $\{\varphi_n \,|n \in \boldsymbol{N}^*\}$ by $\mathbf{D}_{a,b}$, i.e., $\mathbf{D}_{a,b} \stackrel{\text{def}}{=} \mathbf{L.h.}\,[\{\varphi_n \,|\, n \in \boldsymbol{N}^*\}]$. From here on, we denote the linear hull of a set S by $\mathbf{L.h.}[S]$. Obviously $\mathbf{D}_{a,b}$ is dense in $\mathcal{F}_{a,b}$. Further, we denote by $\mathbf{B}(\mathbf{D}_{a,b}, \mathcal{F}_{a,b})$ the space of bounded linear operators from $\mathbf{D}_{a,b}$ to $\mathcal{F}_{a,b}$. Every element A in $\mathbf{B}(\mathbf{D}_{a,b}, \mathcal{F}_{a,b})$ has a unique extension to an element in $\mathbf{B}(\mathcal{F}_{a,b})$, the space of bounded linear operators on $\mathcal{F}_{a,b}$. We denote the extension of A by A^-, and $A^*\lceil\mathbf{D}_{a,b}$ by A^+, which means that the domain of operator A^* is restricted to $\mathbf{D}_{a,b}$.

In this paper, we consider the restricted position and momentum operators as $q \stackrel{\text{def}}{=} (a + a^+)\,/\sqrt{2}\lceil\mathbf{D}_{a,b}$ and $p \stackrel{\text{def}}{=} i\,(a^+ - a)\,/\sqrt{2}\lceil\mathbf{D}_{a,b}$ respectively.

We first define a class $\mathbf{T}(H^V_{a,b})$ of quantum operators, which is a set of quantum operators A satisfying the following conditions:

(**T.1**) the domain of each operator is equal to $\mathbf{D}_{a,b}$, and the domain of the adjoint operator of each operator includes $\mathbf{D}_{a,b}$ (i.e., $\mathrm{D}(A) = \mathbf{D}_{a,b}$ and $\mathrm{D}(A^*) \supset \mathbf{D}_{a,b}$, where $\mathrm{D}(B)$ denotes the domain of each operator B);

(**T.2**) for all τ in $(0,\beta]$ operators $e^{-\tau H^V_{a,b}}A$ and $Ae^{-\tau H^V_{a,b}}$ are in $\mathbf{B}(\mathbf{D}_{a,b}, \mathcal{F}_{a,b})$, furthermore, $(e^{-\tau H^V_{a,b}}A)^-$ and $(Ae^{-\tau H^V_{a,b}})^-$ are Hilbert-Schmidt operators on $\mathcal{F}_{a,b}$ with the Hilbert-Schmidt norm $\|\quad\|_2$.

We must now turn our attention to the unboundedness of operators because it is known that limits on the precision of the measurement of observables for bounded operators (e.g., fermion) and unbounded operators (e.g., boson) are different [7,12,35]. For unbounded operators, the problem of their domains is delicate, so we provide condition (**T.1**). Condition (**T.2**) addresses convergency with respect to the Bogoliubov scalar product [23,24], [8, p. 96]. We note here that $\mathbf{T}(H^V_{a,b})$ is a linear space. We can then introduce the Bogoliubov (Kubo-Mori) scalar product $< \ ; \ >_{H^V_{a,b}}$ as

$$< A; B >_{H^V_{a,b}} \stackrel{\text{def}}{=} \frac{1}{\beta Z(\beta)} \int_0^\beta d\lambda \mathrm{tr}((e^{-(\beta-\lambda)H^V_{a,b}}A^*)^-(e^{-\lambda H^V_{a,b}}B)^-), \quad A, B \in \mathbf{T}(H^V_{a,b}),$$

where $Z(\beta) \stackrel{\text{def}}{=} \mathrm{tr}(e^{-\beta H^V_{a,b}})$. It can be easily proven that $< \ ; \ >_{H^V_{a,b}}$ is an inner product of $\mathbf{T}(H^V_{a,b})$ (see, [23]). The inner product introduces a norm: $\|A\|_{H^V_{a,b}} \stackrel{\text{def}}{=} < A; A >^{1/2}_{H^V_{a,b}}$. We can therefore obtain a partial $*$-algebra $\mathbf{X}_c(H^V_{a,b})$ defined by a Hilbert space which is the completion of $\mathbf{T}(H^V_{a,b})$ with respect to the norm $\| \ \ \|_{H^V_{a,b}}$. The definition of the partial $*$-algebra with a unit appears in [1–3,11,27]. We also note here that an element in $\mathbf{X}_c(H^V_{a,b})$ is not always an operator acting in $\mathcal{F}_{a,b}$. It is noteworthy that Naudts et al. attempted to argue in general about linear response theory on the Hilbert space which is constructed by a completion of a von Neumann algebra with KMS-state [30]. Similarly, we deal with Mori's theory in statistical physics on $\mathbf{X}_c(H^V_{a,b})$, which was constructed by the completion concerning the Bogoliubov scalar product.

Because we consider a system governed by the Hamiltonian $H^V_{a,b}$ constructed by a, a^+, b_k, and b_k^+ $(k \in \boldsymbol{N}^*)$, on condition that $e^{-\tau H^V_{a,b}}$ is a trace class operator for all $\tau \in (0, \beta]$, the condition that $O_{bs} \in \mathbf{T}(H^V_{a,b})$ is natural assumption. Thus, in this paper we assume that the position operator O_{bs} belongs to a dense subspace $\mathbf{T}(H^V_{a,b})$ in the Liouville space.

(O) For every $V > 0$, $O_{bs} \in \mathbf{T}(H^V_{a,b})$.

REMARK 2.1. There are several examples satisfying condition **(O)**. For instance, there is an example that $q \in \mathbf{T}(H^V_{a,b})$, even if $H^V_{a,b}$ is non-quadratic Hamiltonian.

In order to introduce the Heisenberg operator $O_{bs}(t)$ of the observable, we define here the Liouville operator $\mathcal{L}^V_{a,b}$ determined by the Hamiltonian $H^V_{a,b}$.

We can define, for adequate operators A, the Liouville operator $\mathcal{L}^{V}_{a,b}$ by $\mathcal{L}^{V}_{a,b}A \stackrel{\text{def}}{=} [H^{V}_{a,b}, A] = H^{V}_{a,b}A - AH^{V}_{a,b}$ [23, Lemma 3.8]. The domain $\mathrm{D}(\mathcal{L}^{V}_{a,b})$ of the Liouville operator $\mathcal{L}^{V}_{a,b}$ then contains a dense subspace $\mathcal{D}_{a,b}$ of all elements $A \in \mathbf{T}(H^{V}_{a,b})$ satisfying that $H^{V}_{a,b}A$ and $AH^{V}_{a,b}\lceil\mathbf{D}_{a,b}$ are in $\mathbf{T}(H^{V}_{a,b})$; furthermore, $Ax, A^{+}x, H^{V}_{a,b}Ax, H^{V}_{a,b}A^{+}x, AH^{V}_{a,b}x$, and $A^{+}H^{V}_{a,b}x$ are in $\mathbf{D}_{a,b}$ for all x in $\mathbf{D}_{a,b}$. Actually, the subspace $\mathcal{D}_{a,b}$ is a core for $\mathcal{L}^{V}_{a,b}$ [23, Lemmas 3.7 and 3.8]. More exact and easier definition of $\mathcal{L}^{V}_{a,b}$ is as follows: We first define linear operators $\Phi_{m,n} : \mathbf{D}_{a,b} \to \mathbf{D}_{a,b}, \quad m, n \in \boldsymbol{N}^{*}$ defined by

$$
\begin{cases} \mathrm{D}(\Phi_{m,n}) \stackrel{\text{def}}{=} \mathbf{D}_{a,b}, \\ \Phi_{m,n}x \stackrel{\text{def}}{=} \beta^{1/2}Z(\beta)^{1/2}W^{1/2}_{m,n}(\varphi_n, x)_{a,b}\varphi_m, \qquad x \in \mathbf{D}_{a,b};\ m, n \in \boldsymbol{N}^{*}, \end{cases} \tag{2.1}
$$

where

$$
W_{m,n} \stackrel{\text{def}}{=} \begin{cases} \dfrac{\lambda_n - \lambda_m}{e^{-\beta\lambda_m} - e^{-\beta\lambda_n}} & \text{if } \lambda_m \neq \lambda_n, \\ \beta^{-1}e^{\beta\lambda_m} & \text{if } \lambda_m = \lambda_n. \end{cases}
$$

It must be noted that

$$
\begin{cases} W_{m,n} > 0, & m, n \in \boldsymbol{N}^{*}, \\ W_{m,n} = W_{n,m}, & m, n \in \boldsymbol{N}^{*}. \end{cases} \tag{2.2}
$$

Then $\{\Phi_{m,n}\}_{m,n=0,1,\dots}$ is a complete orthonormal basis of $\mathbf{X}_c(H^{V}_{a,b})$, and

$$
\begin{cases} \mathcal{L}^{V}_{a,b}\Phi_{m,n} = (\lambda_m - \lambda_n)\Phi_{m,n}, & m, n \in \boldsymbol{N}^{*}, \\ \Phi^{+}_{m,n} = \Phi_{n,m}, & m, n \in \boldsymbol{N}^{*}. \end{cases} \tag{2.3}
$$

So, since it is clear that $\mathcal{L}^{V}_{a,b}$ is symmetric in $\mathbf{X}_c(H^{V}_{a,b})$, $\mathcal{L}^{V}_{a,b}$ can be extended to a self-adjoint operator acting in $\mathbf{X}_c(H^{V}_{a,b})$, which is denoted by the same symbol. And it is easy to show that the linear hull $\mathbf{L.h.}\,[\{\Phi_{m,n}\}_{m,n=0,1,\dots}]$ is included in $\mathcal{D}_{a,b}$. It is clear that $\mathbf{L.h.}\left[\{\Phi_{m,n}\}_{m,n=0,1,\dots}\right] \subset \mathcal{D}_{a,b} \subset \mathrm{D}(\mathcal{L}^{V}_{a,b})$, so $\mathcal{D}_{a,b}$ is a core for $\mathcal{L}^{V}_{a,b}$.

For every $A \in \mathbf{X}_c(H^{V}_{a,b})$, we set

$$
A(t) \stackrel{\text{def}}{=} e^{i\mathcal{L}^{V}_{a,b}t}A,
$$

$$
R^{V}(t) \stackrel{\text{def}}{=} R^{V}(0,t) \equiv \langle O_{bs}(0); O_{bs}(t) \rangle_{H^{V}_{a,b}}.
$$

REMARK 2.2. The time evolution $A(t)$ coincides with the Heisenberg picture $e^{iH^V_{a,b}t}Ae^{-iH^V_{a,b}t}$ for every operator A in $\mathcal{D}_{a,b}$ and $t \in \boldsymbol{R}$ [23, Proposition 3.13].

We can prove that

$$\sigma(\mathcal{L}^V_{a,b}) = \overline{\{\lambda_m - \lambda_n \,|\, m,n \in \boldsymbol{N}^*\}}^{\text{closure}}. \tag{2.4}$$

In order to obtain suitable data of $R^V(t)$ for reconstruction, we note the following fact: There exist non-negative constants $A_{m,n}$ $(m,n \in \boldsymbol{N}^*)$ such that

$$R^V(t) = \sum_{m,n=0}^{\infty} A_{m,n}e^{it(\lambda_m - \lambda_n)}, \tag{2.5}$$

$$0 \leq \sum_{m,n=0}^{\infty} A_{m,n} < \infty. \tag{2.6}$$

We define here a function $[R^V](z)$ $(z \in \boldsymbol{C}$ with $\mathrm{Im}z > 0)$ by the Fourier-Laplace transform as

$$\left[R^V\right](z) \stackrel{\text{def}}{=} \int_0^\infty dte^{itz}R^V(t).$$

We denote the set of all positive poles of $[R^V](z)$ by $\mathbf{P}^R_+$, and the set of all negative poles of $[R^V](z)$ by $\mathbf{P}^R_-$.

We here assume that

(A.0) $\quad \mathbf{P}^R_+ = \{\varepsilon_p \,|\, p = 0,1,\cdots\}$ with $\inf_{p=0,1,\cdots}(\varepsilon_{p+1} - \varepsilon_p) > 0$.

Moreover, $\mathbf{P}^R_- = \{\eta_p \,|\, p = 0,1,\cdots\}$ with $\inf_{p=0,1,\cdots}(\eta_p - \eta_{p+1}) > 0$.

When if condition (**A.0**) does not hold, we consider smeared observables given in Definition 2.1 below.

By (2.5) and (**A.0**), it is clear that, for any ε_p and η_p there exist $m^+(p), n^+(p) \in \boldsymbol{N}^*$; and $m^-(p), n^-(p) \in \boldsymbol{N}^*$ such that

$$\varepsilon_p = -\left(\lambda_{m^+(p)} - \lambda_{n^+(p)}\right), \tag{2.7}$$

$$\eta_p = -\left(\lambda_{m^-(p)} - \lambda_{n^-(p)}\right), \tag{2.8}$$

and for the zero 0 there exist $m^0(p), n^0(p) \in \boldsymbol{N}^*$ such that

$$0 = -\left(\lambda_{m^0(p)} - \lambda_{n^0(p)}\right). \tag{2.9}$$

For every $z \in \boldsymbol{C}$ with $z \neq 0$ and $z \notin \mathbf{P}^R_{\pm}$, there exists a point $c \in \{0\} \cup \mathbf{P}^R_+ \cup \mathbf{P}^R_-$ such that $|z-c| \leq |z-c'|$ for all $c' \in \{0\} \cup \mathbf{P}^R_+ \cup \mathbf{P}^R_-$ by assumption (**A.0**). So, we have $|\int_0^\infty dte^{-itc'}e^{itz}| \leq |z-c|^{-1}$ if $\mathrm{Im}z > 0$. Thus, by applying Lebesgue's dominated convergence theorem to (2.5) and (2.6), we note that

$$\begin{aligned}
\left[R^V\right](z) = i\sum_{p=0}^{\infty} &\left(\sum_{\substack{m^+(p), n^+(p);\\ \lambda_{m^+(p)} - \lambda_{n^+(p)} = -\varepsilon_p}} A_{m^+(p), n^+(p)} \right) \frac{1}{z - \varepsilon_p} \\
&+ i\sum_{p=0}^{\infty} \left(\sum_{\substack{m^-(p), n^-(p);\\ \lambda_{m^-(p)} - \lambda_{n^-(p)} = -\eta_p}} A_{m^-(p), n^-(p)} \right) \frac{1}{z - \eta_p} \\
&+ i\sum_{p=0}^{\infty} \left(\sum_{\substack{m^0(p), n^0(p);\\ \lambda_{m^0(p)} - \lambda_{n^0(p)} = 0}} A_{m^0(p), n^0(p)} \right) \frac{1}{z}.
\end{aligned} \tag{2.10}$$

And, for $z \notin \{0\} \cup \mathbf{P}^R_+ \cup \mathbf{P}^R_-$, $|\frac{d}{dz}\left(\frac{1}{z-c'}\right)| \leq |z-c|^{-2}$. Thus, by applying Weierstrass' M-test to (2.10), it is evident that $[R](z)$ can be extended into a meromorphic function on the complex plain with singularities only at points in $\{0\} \cup \mathbf{P}^R_+ \cup \mathbf{P}^R_-$ by (**A.0**) and (2.10).

When condition (**A.0**) does not hold, we consider the following smeared observable. We can expand O_{bs} as

$$O_{bs} = \sum_{m,n} < \Phi_{m,n}\,;\, O_{bs} >_{H^V_{a,b}} \Phi_{m,n}$$

in $\mathbf{X}_c(H^V_{a,b})$.

DEFINITION 2.1. By an observation, select ε_p and η_p with (**A.0**) for every $V > 0$. Then, for O_{bs}, we define *smeared observable* O^{sm}_{bs} by

$$
\begin{aligned}
O_{bs}^{sm} &\equiv O_{bs}^{sm}\left(\varepsilon_p, \eta_p; p \in \boldsymbol{N}^*\right) \\
&\stackrel{\text{def}}{=} \sum_{p=0}^{\infty} \left(\sum_{\substack{m^+(p), n^+(p); \\ \lambda_{m^+(p)} - \lambda_{n^+(p)} = -\varepsilon_p}} < \Phi_{m^+(p), n^+(p)} \,;\, O_{bs} >_{H_{a,b}^V} \right) \Phi_{m^+(p), n^+(p)} \\
&\quad + \sum_{p=0}^{\infty} \left(\sum_{\substack{m^-(p), n^-(p); \\ \lambda_{m^-(p)} - \lambda_{n^-(p)} = -\eta_p}} < \Phi_{m^-(p), n^-(p)} \,;\, O_{bs} >_{H_{a,b}^V} \right) \Phi_{m^-(p), n^-(p)}.
\end{aligned}
$$

We call O_{bs}^{sm} *smeared position operator*, which denotes q^{sm}, if $O_{bs} = q$. And we call O_{bs}^{sm} *smeared momentum operator*, which denotes p^{sm}, if $O_{bs} = p$.

Of course, any smeared observable O_{bs}^{sm} satisfies condition (**A.0**).

The following fact is derived from (2.10) by Weierstrass' M-test, which tells us that proper summations of $A_{m,n}$ are determined in terms of $R^V(t)$ only: For each $p \in \boldsymbol{N}^*$,

(2.11)
$$\lim_{z \to \varepsilon_p} \frac{1}{i}(z - \varepsilon_p)\left[R^V\right](z) = \sum_{\substack{m^+(p), n^+(p); \\ \lambda_{m^+(p)} - \lambda_{n^+(p)} = -\varepsilon_p}} A_{m^+(p), n^+(p)},$$

(2.12)
$$\lim_{z \to \eta_p} \frac{1}{i}(z - \eta_p)\left[R^V\right](z) = \sum_{\substack{m^-(p), n^-(p); \\ \lambda_{m^-(p)} - \lambda_{n^-(p)} = -\eta_p}} A_{m^-(p), n^-(p)},$$

(2.13)
$$\lim_{z \to 0} \frac{1}{i} z \left[R^V\right](z) = \sum_{\substack{m^0(p), n^0(p); \\ \lambda_{m^0(p)} - \lambda_{n^0(p)} = 0}} A_{m^0(p), n^0(p)}.$$

DEFINITION 2.2. For each $n \in \boldsymbol{N}^*$, we say that $d^n R^V(t)/dt^n$ is *computable* if

$$\sum_{p=0}^{\infty} \left(\lim_{z \to \varepsilon_p} \frac{1}{i}(z - \varepsilon_p)\left[R^V\right](z) \right) \varepsilon_p^n < \infty, \quad \text{and} \quad \sum_{p=0}^{\infty} \left(\lim_{z \to \eta_p} \frac{1}{i}(z - \eta_p)\left[R^V\right](z) \right) (-\eta_p)^n < \infty.$$

REMARK 2.3.

(1) If $q \in \mathrm{D}\left(\left(\mathcal{L}_{a,b}^V\right)^{n/2}\right)$ for some $n \in \boldsymbol{N}$, then $d^n R^V(t)/dt^n$ is computable.

(2) If $d^n R^V(t)/dt^n$ is computable, then we have for $n \in \boldsymbol{N}$

$$\frac{d^n R^V(t)}{dt^n}$$
$$= (-i)^n \sum_{p=0}^{\infty} \left\{ \left(\lim_{z\to\varepsilon_p} \frac{1}{i}(z-\varepsilon_p)\left[R^V\right](z) \right) \varepsilon_p^n e^{-it\varepsilon_p} + \left(\lim_{z\to\eta_p} \frac{1}{i}(z-\eta_p)\left[R^V\right](z) \right) \eta_p^n e^{-it\eta_p} \right\},$$

which is the meaning of "*computable.*"

(3) If $d^2 R^V(t)/dt^2$ is computable, then we have $O_{bs} \in \mathrm{D}(\mathcal{L}^V_{a,b})$ since we assumed (**A.0**).

(4) There is an example of non quasi-free Hamiltonian satisfying $q \in \mathrm{D}(\mathcal{L}^V_{a,b})$.

Here we remember (2.6), (2.11)-(2.13), and note if $d^2R(t)/dt^2$ is computable, $\sum_{p=0}^{\infty} \varepsilon_p^2 \sum_{\substack{m^+(p),n^+(p);\\ \lambda_{m^+(p)}-\lambda_{n^+(p)}=-\varepsilon_p}} A_{m^+(p),n^+(p)}$ converges by (2.11). So, we can define a constant ω_0 by

$$\omega_0 \stackrel{\text{def}}{=} \frac{\sum_{p=0}^{\infty} \varepsilon_p \left(\lim_{z\to\varepsilon_p} \frac{1}{i}(z-\varepsilon_p)\left[R^V\right](z) \right)}{\sum_{p=0}^{\infty} \left(\lim_{z\to\varepsilon_p} \frac{1}{i}(z-\varepsilon_p)\left[R^V\right](z) \right)}. \tag{2.14}$$

We furthermore define a function $D^V(z)$ by

$$D^V(z) \stackrel{\text{def}}{=} \left(\frac{2}{\omega_0\left(R^V(0)-R_0^V\right)} \sum_{p=0}^{\infty} \left(\lim_{z\to\varepsilon_p} \frac{1}{i}(z-\varepsilon_p)\left[R^V\right](z) \right) \frac{\varepsilon_p}{z-\varepsilon_p} \right)^{-1}, \tag{2.15}$$

$$R_0^V \stackrel{\text{def}}{=} -i \lim_{z\to 0;\, \mathrm{Im}z>0} z\left[R^V\right](z).$$

REMARK 2.4. We here note that the constants ω_0, R_0^V, and the function $D^V(z)$ also determined by $\left[R^V\right](z)$ only.

Now, we can state one of our main theorems:

THEOREM 2.1. *Under* (**H**) *and* (**O**), *suppose that we get a two-point function* $R^V(t_1, t_2)$, *defined by the Bogoliubov scalar product, with* (**A.0**) *from an experimental observation satisfying the conditions that* $d^2 R^V(t)/dt^2$ *is computable. Then the function* $D^V(z)$ *can be extended to a meromorphic function on the complex plane, and the set* $\{\omega_k \mid k \in \boldsymbol{N}\}$ *of all zero points of* $D^V(z) - D^V(0)$ *except* $z = 0$ *is counted in such a way that*

$$\omega_k \in (\varepsilon_{k-1}, \varepsilon_k), \qquad k \in \boldsymbol{N}.$$

And the total Hamiltonian of RWA is given by

$$H^V_{\mathrm{RWA}} \stackrel{\mathrm{def}}{=} \omega_0 a^+ a + \sum_{k=1}^{\infty} \omega_k b_k^+ b_k + \sum_{k=1}^{\infty} \rho_k \left(a^+ b_k + b_k^+ a\right),$$

where $\rho_k \stackrel{\mathrm{def}}{=} \left(\omega_0 \omega_k / (D^V)'(\omega_k)\right)^{1/2}$, $k \in \boldsymbol{N}$, $(D^V)'(z) \equiv dD^V(z)/dz$. H^V_{RWA} *is realized as a positive (i.e.,* $H^V_{\mathrm{RWA}} \geq 0$*) self-adjoint operator acting in the Fock space* $\mathcal{F}_{a,b}$ *such that*

$$\sigma(H^V_{\mathrm{RWA}}) = \{\varepsilon_0 n_0 + \cdots + \varepsilon_N n_N \mid n_0, \cdots, n_N \in \boldsymbol{N}^*,\, N \in \boldsymbol{N}^*\},$$

where $\{\varepsilon_p \mid p \in \boldsymbol{N}^*\}$ *is equal to the set of all positive poles of* $\left[R^V\right](z)$, *which is the set of all zero points of* $D^V(z)$. *Furthermore,* $O_{bs}(t)$ *is reconstructed as* $O_{\mathrm{RWA}}(t) \stackrel{\mathrm{def}}{=} e^{iH^V_{\mathrm{RWA}} t} O_{bs} e^{-iH^V_{\mathrm{RWA}} t}$ *such that*

$$R^V(t_1, t_2) = 2\left(R^V(0) - R^V_0\right) \mathrm{Re} W^V(t_1, t_2) + R^V_0,$$

for every $t_1, t_2 \in \boldsymbol{R}$.

From now on, we consider the case that the observable is given by the position operator, i.e., $O_{bs} = q$. We define a set $\boldsymbol{\Gamma}_V$ of lattice points by

$$\boldsymbol{\Gamma}_V \stackrel{\mathrm{def}}{=} \left\{ k \,\middle|\, k = \frac{2\pi n}{V}, n = 0, \pm 1, \pm 2, \cdots \right\}. \tag{2.16}$$

Here we assume the following technical conditions for existence of an infinite volume limit:

(**A.1**) $\omega_0 \longrightarrow \omega_{\beta,0}^{\infty} > 0$ as $V \to \infty$.

(**A.2**) There exist a non-negative, continuously differentiable function $\omega_\beta(k)$, and real-valued continuous function $\rho_\beta(k)$ in $L^2(\boldsymbol{R})$, which satisfy the following conditions;

$$\omega_\beta(k') < \omega_\beta(k) \quad \text{for } 0 \le k' < k, \qquad \text{and} \qquad \omega_\beta(-k) = \omega_\beta(k) \quad \text{for } k \in \boldsymbol{R},$$

there are one-to-one maps, δ_1 and δ_2: $\boldsymbol{N} \equiv \{1, 2, \cdots\} \longrightarrow \boldsymbol{Z} \equiv \{0, \pm 1, \pm 2, \cdots\}$ such that

$$\omega_n = \omega_\beta\left(\frac{2\pi\delta_1(n)}{V}\right), \qquad \rho_n = \rho_\beta\left(\frac{2\pi\delta_2(n)}{V}\right) / \sqrt{V}, \tag{2.17}$$

and

$$m \equiv \inf_{-\infty<k<\infty} \omega_\beta(k) > 0, \qquad \int_{-\infty}^{\infty} \frac{\rho_\beta(k)^2}{\omega_\beta(k)} dk < \infty. \tag{2.18}$$

We define for every $V > 0$ and $t \in \boldsymbol{R}$ a function $R_1^{V}(t)$ by

$$R_1^{V}(t) \stackrel{\text{def}}{=} \sum_{p=0}^{\infty} \left(\lim_{z \to \varepsilon_p} \frac{1}{i}(z - \varepsilon_p) \left[R^{V}\right](z) \right) e^{-it\varepsilon_p},$$

where $\{\varepsilon_p \,|\, p = 0, 1, \cdots\}$ is the set of all positive poles of $\left[R^{V}\right](z)$, which was appeared in condition (**A.0**). And we set

$$\left[R_1^{V}\right](z) \stackrel{\text{def}}{=} \int_0^{\infty} dt e^{itz} R_1^{V}(t), \qquad z \in \boldsymbol{C}^+ \equiv \{\zeta \in \boldsymbol{C} \,|\, \mathrm{Im}\zeta > 0\}.$$

Then, we have

LEMMA 2.2. *If* $R_\beta^{\infty}(0) \equiv \lim_{V\to\infty} R^{V}(0)$ *and* $R_{\beta,0}^{\infty} \equiv \lim_{V\to\infty} R_0^{V}$ *exist, then* $\left[R_{\beta,1}^{\infty}\right](z) \equiv \lim_{V\to\infty} \left[R_1^{V}\right](z)$ *also exists.*

Here, for using Theorem 2.1 and Lemma 2.3, we assume that

(**A.3**) For every $V > 0$, $d^2R^V(t)/dt^2$ is computable. And $R^\infty_\beta(0) \equiv \lim_{V\to\infty} R^V(0)$ and $R^\infty_{\beta,0} \equiv \lim_{V\to\infty} R^V_0$ exist.

We define a function $D^\beta_{\mathrm{RWA}}(z)$ by

$$D^\beta_{\mathrm{RWA}}(z) \stackrel{\mathrm{def}}{=} \left(\frac{R^\infty_\beta(0) - R^\infty_{\beta,0}}{2}\right) \times \frac{1}{i\left[R^\infty_{\beta,1}\right](z)}.$$

It is clear that there exists the inverse function $\varphi_\beta(x)$ such that $\varphi_\beta(x)$ is differentiable and monotone increasing in (m, ∞) with

$$\lim_{x\downarrow m} \varphi_\beta(x) = 0, \qquad \varphi'_\beta(x) = \left(\omega'_\beta\left(\varphi_\beta(x)\right)\right)^{-1}, \qquad x > m.$$

For using Arai's results in [5], we assume a little more assumptions:

$$(\mathbf{A.4}) \qquad \sup_{\epsilon>0, x\geq m}\left|\int_{-\infty}^{\infty} \frac{\rho_\beta(k)^2}{(x - i\epsilon) - \omega_\beta(k)}\right| < \infty, \qquad \inf_{\epsilon>0, x\geq m}\left|D^\beta_{\mathrm{RWA}}(x - i\epsilon)\right| < \infty.$$

(**A.5**) There exists a constant $\theta(\beta) \in (0, 2\pi)$ such that the function $\varphi'_\beta(x)\rho_\beta(\varphi_\beta(x))^2$ has an analytic continuation $I^{(0)}_\beta(z)$ onto the domain $\mathbf{D}^\beta_{m,\theta} \stackrel{\mathrm{def}}{=} \{z \in \boldsymbol{C} \,|\, \mathrm{Re}z > m,\ -\theta(\beta) < \arg z < 0\}$ with the following properties:

$$\lim_{\epsilon\downarrow 0} I^{(0)}_\beta(x - i\epsilon) = I^{(0)}_\beta(x), \quad x \geq m, \qquad |I^{(0)}_\beta(z)| \leq \mathrm{const}|z|^{-\mathsf{q}_0(\beta)} \tag{2.19}$$

for all sufficiently large $|z|$ ($z \in \mathbf{D}^\beta_{m,\theta}$) with a constant $\mathsf{q}_0(\beta) \geq 0$,

$$\lim_{z\to 0; z\in\mathbf{D}^\beta_{m,\theta}} \frac{I^{(0)}_\beta(m + z)}{z^{\mathsf{p}_0(\beta;m)}} = A^{(0)}_m(\beta), \tag{2.20}$$

with constant $A^{(0)}_m(\beta) \neq 0$ and $\mathsf{p}_0(\beta; m) \geq 0$,

$$\inf_{0<\epsilon<\epsilon_0; x\geq m} |D^\beta_{\mathrm{RWA}}(x - i\epsilon) - 2i\pi I^{(0)}_\beta(x - i\epsilon)| > 0 \tag{2.21}$$

for all sufficiently small $\epsilon_0 > 0$.

So by using Arai's result [5, Theorem 1.3], we obtain the following theorem:

THEOREM 2.3. *Let* $O_{bs} = q$. *There exists* $R^{\infty}_{\beta}(t_1, t_2) \equiv \lim_{V \to \infty} R^{V}(t_1, t_2)$. *Let* $B^{(0)}_m(\beta) \stackrel{\text{def}}{=} (D^{\beta}_{\text{RWA}}(m) - 2i\pi\delta_{0,\mathsf{p}_0(m)}A^{(0)}_m)D^{\beta}_{\text{RWA}}(m)$, *and* $R^{\infty}_{\beta}(t) \equiv R^{\infty}_{\beta}(0,t)$.

(a) *If* $R^{\infty}_{\beta,0} \neq 0$, *then* $\lim_{t\to\infty} R^{\infty}_{\beta}(t) = R^{\infty}_{\beta,0}$.

(b) *If* $R^{\infty}_{\beta,0} = 0$, *then*

$$\begin{aligned} R^{\infty}_{\beta}(t) &= R^{\infty}_{\beta,1}(t) + R^{\infty}_{\beta,1}(-t), \\ R^{\infty}_{\beta,1}(t) \underset{t\to\infty}{\sim} & \quad \omega^{\infty}_{\beta,0}\left(R^{\infty}_{\beta}(0) - R^{\infty}_{\beta,0}\right) \frac{A^{(0)}_m(\beta)e^{-i\pi(\mathsf{p}_0(\beta;m)+1)/2}\Gamma(\mathsf{p}_0(\beta;m)+1)}{B^{(0)}_m(\beta)} \\ & \quad \times e^{-itm}t^{-(\mathsf{p}_0(\beta;m)+1)} \end{aligned}$$

REMARK 2.5. Concerning part (a), if the condition that $R^{\infty}_{\beta,0} \neq 0$ occurs, maybe it will be the case when there are infinitely many elements in the thermal states for every $V > 0$ such that the elements are not orthogonal to q just like the superfluidity at $T = 0$. Here the thermal states is a physical notion given by $\overline{\mathbf{L.h.}\,[\{\Phi_{n,n}\}_{n=0,1,\cdots}]}$ (i.e., $\mathcal{L}^{V}_{a,b}(\text{thethermalstates}) = \{0\}$) in thermo field dynamics (e.g. [22]).

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