FIXED POINT THEOREMS IN COMPLETE METRIC SPACES

東工大・大学院理工学研究科 鈴木智成 (TOMONARI SUZUKI)

1. INTRODUCTION

In 1990, Takahashi proved the following nonconvex minimization theorem, which was used to obtain Caristi's fixed point theorem [1], Ekeland's ε-variational principle [3] and Nadler's fixed point theorem [6].

Theorem 1 (Takahashi [8]). Let $X$ be a complete metric space with metric $d$ and let $f : X \to (-\infty, \infty]$ be a proper lower semicontinuous function, bounded from below. Suppose that, for each $u \in X$ with $f(u) > \inf_{x \in X} f(x)$, there exists $v \in X$ such that $v \neq u$ and $f(v) + d(u, v) \leq f(u)$. Then there exists $x_0 \in X$ such that $f(x_0) = \inf_{x \in X} f(x)$.

This theorem was improved by several authors; see [5], [9] and [10]. On the other hand, Ćirić [2] proved an interesting fixed point theorem for a quasi-contraction which generalizes some fixed point theorems in a complete metric space. Recently Kada, Suzuki and Takahashi introduced the following concept.

Definition ([4]). Let $X$ be a metric space with metric $d$. Then a function $p : X \times X \to [0, \infty)$ is called a $w$-distance on $X$ if the following are satisfied:

1. $p(x, z) \leq p(x, y) + p(y, z)$ for any $x, y, z \in X$;
2. for any $x \in X$, $p(x, \cdot) : X \to [0, \infty)$ is lower semicontinuous;
3. for any $\varepsilon > 0$, there exists $\delta > 0$ such that $p(x, x) \leq \delta$ and $p(z, y) \leq \delta$ imply $d(x, y) \leq \varepsilon$.

The metric $d$ is a $w$-distance on $X$. Other examples of $w$-distance are stated in [4] and [7]. Using it, Kada, Suzuki and Takahashi [4] generalized Caristi's fixed point theorem, Ekeland’s $\varepsilon$-variational principle, Takahashi’s nonconvex minimization theorem and Ćirić’s fixed point theorem. One of them is the following fixed point theorem.

Theorem 2 ([4]). Let $X$ be a complete metric space, let $p$ be a $w$-distance on $X$ and let $T$ be a mapping from $X$ into itself. Suppose that there exists $r \in [0, 1)$ such that

$$p(Tx, T^2x) \leq rp(x, Tx).$$
for every $x \in X$ and
\[
\inf\{p(x, y) + p(x, Tx) : x \in X\} > 0
\]
for every $y \in X$ with $y \neq Ty$. Then there exists $x_0 \in X$ such that $x_0 = Tx_0$. Moreover, if $z = Tz$, then $p(z, z) = 0$.

In this paper, we first give some Examples and Lemmas connected with $w$-distance. Next we give another proof of a generalization of Theorem 1. Further we prove two fixed point theorems which generalize Ćirić’s fixed point theorem. Finally, using them, we give another proof of a characterization of metric completeness.

2. Preliminaries

In this Section, we state, without the proofs, Examples and Lemmas connected with $w$-distance.

**Example 1.** Let $X = \mathbb{R}$ be a metric space with the usual metric and let $f, g : X \to [0, \infty)$ be continuous functions such that
\[
\inf_{x \in X} \int_x^{x+r} f(u)du > 0 \quad \text{and} \quad \inf_{x \in X} \int_x^{x+r} g(u)du > 0
\]
for any $r > 0$. Then a function $p : X \times X \to [0, \infty)$ defined by
\[
p(x, y) = \begin{cases} \int_x^y f(u)du, & \text{if } x \leq y, \\ \int_y^x g(u)du, & \text{if } y \leq x \end{cases}
\]
for every $x, y \in X$ is a $w$-distance on $X$.

**Example 2 ([4]).** Let $X$ be a metric space and let $T$ be a continuous mapping from $X$ into itself. Then a function $p : X \times X \to [0, \infty)$ defined by
\[
p(x, y) = \max\{d(Tx, y), d(Tx, Ty)\} \quad \text{for every } x, y \in X
\]
is a $w$-distance on $X$.

**Example 3.** Let $X$ be a metric space with metric $d$, let $T$ be a mapping from $X$ into itself such that, for every $x \in X$, the orbit $\{x, Tx, T^2x, \cdots\}$ is bounded. Then a function $p : X \times X \to [0, \infty)$ given by
\[
p(x, y) = \sup\{d(T^kx, y) : k \in \mathbb{N} \cup \{0\}\} \quad \text{for every } x, y \in X
\]
is a $w$-distance on $X$.

**Example 4.** Let $X$ be a metric space with metric $d$ and let $\{x_n\}$ be a sequence in $X$ such that
(i) $\{x_n\}$ is Cauchy;
(ii) $\{x_n\}$ does not converge;
(iii) \( x_i \neq x_j \) if \( i \neq j \).

Then a function \( p : X \times X \to [0, \infty) \) defined by

\[
p(x, y) = \begin{cases} 
2^{-i} + 2^{-j}, & \text{if } x = x_i \text{ and } y = x_j, \\
2^{-i} + 1, & \text{if } x = x_i \text{ and } y \notin \{x_n\}, \\
1 + 2^{-j}, & \text{if } x \notin \{x_n\} \text{ and } y = x_j
\end{cases}
\]

is a w-distance on \( X \).

**Lemma 1.** Let \( X \) be a metric space, let \( p \) be a w-distance on \( X \) and let \( f \) be a bounded lower semicontinuous function from \( X \) into \( \mathbb{R} \). Assume that \( c \) is a positive real number with \( c \geq \sup f(X) - \inf f(X) \). Then a function \( q : X \times X \to [0, \infty) \) defined by

\[
q(x, y) = \begin{cases} 
f(x) - \inf f(Mx), & \text{if } y \in Mx, \\
c, & \text{if } y \notin Mx
\end{cases}
\]

is a w-distance on \( X \), where \( Mx = \{ y \in X : f(y) + p(x, y) \leq f(x) \} \).

**Lemma 2.** Let \( X \) be a metric space with metric \( d \), let \( p \) be a w-distance on \( X \) and let \( \alpha \) be a function from \( X \) into \( [0, \infty) \). Then a function \( q : X \times X \to [0, \infty) \) given by

\[
q(x, y) = \max\{\alpha(x), p(x, y)\}
\]

for every \( x, y \in X \)

is also a w-distance.

**Lemma 3.** Let \( X \) be a metric space, let \( p \) be a w-distance on \( X \), let \( \{x_n\}, \{y_n\} \) and \( \{z_n\} \) be sequences in \( X \) and let \( x, y, z \in X \). Then the following hold:

(i) If \( p(x_n, y) \to 0 \) and \( p(x_n, z) \to 0 \), then \( y = z \). In particular, if \( p(x, y) = 0 \) and \( p(x, z) = 0 \), then \( y = z \), see [4];

(ii) If \( p(x_n, y_n) \to 0 \) and \( p(x_n, z_n) \to 0 \), then \( \{y_n\} \) converges to \( z \), see [4];

(iii) If \( p(x_n, y_n) \to 0 \) and \( p(x_n, z_n) \to 0 \), then \( \{d(y_n, z_n)\} \) converges to \( 0 \).

**Lemma 4.** Let \( X \) be a metric space with metric \( d \), let \( p \) be a w-distance on \( X \) and let \( \{x_n\} \) be a sequence in \( X \). Suppose that

\[
\lim \sup_{n \to \infty} \min_{m \geq n} \{ p(x_n, x_m), p(x_m, x_n) \} = 0.
\]

Then \( \{x_n\} \) is Cauchy. In particular, the following hold:

(i) If \( \lim_{n \to \infty} \sup_{m \geq n} p(x_n, x_m) = 0 \), then \( \{x_n\} \) is Cauchy, see [4];

(ii) If \( \lim_{n \to \infty} \sup_{m \geq n} p(x_m, x_n) = 0 \), then \( \{x_n\} \) is Cauchy.
3. Minimization Theorem

In this Section, using Theorem 2, we prove a nonconvex minimization theorem which improves Theorem 1.

**Theorem 3.** Let $X$ be a complete metric space, and let $f : X \to (-\infty, \infty]$ be a proper lower semicontinuous function, bounded from below. Assume that there exists a $w$-distance $p$ on $X$ such that for any $u \in X$ with $f(u) > \inf_{x \in X} f(x)$, there exists $v \in X$ with $v \neq u$ and

$$f(v) + p(u, v) \leq f(u).$$

Then there exists $x_0 \in X$ such that $f(x_0) = \inf_{x \in X} f(x)$.

**Proof.** Assume $f(x) > \inf f(X)$ for every $x \in X$. Put

$$Y = \{x \in X : f(x) \leq \inf f(X) + 1\}$$

and

$$Mx = \{y \in Y : f(y) + p(x, y) \leq f(x)\}$$

for every $x \in Y$ and define $q : Y \times Y \to [0, \infty)$ by

$$q(x, y) = \begin{cases} f(x) - \inf f(Mx), & \text{if } y \in Mx, \\ 1, & \text{if } y \notin Mx \end{cases}$$

for every $x, y \in Y$. Then, since $f$ is lower semicontinuous, $Y$ is closed and hence $Y$ is complete. From Lemma 1, we have that $q$ is a $w$-distance on $Y$. And it is clear that $y \in Mx$ and $z \in My$ imply $z \in Mz$. Let $x \in Y$ be fixed. By assumption, there exists $v \in X$ with $v \neq x$ and $f(v) + p(x, v) \leq f(x)$. Then since

$$f(v) \leq f(v) + p(x, v) \leq f(x) \leq \inf f(X) + 1,$$

we have $v \in Y$ and hence $Mx \setminus \{x\} \neq \emptyset$. So, we can choose $Tx$ such that

$$f(Tx) \leq \frac{1}{2}\{f(x) + \inf f(Mx)\} \quad \text{and} \quad Tx \in Mx \setminus \{x\}.$$

Then, since $MTx \subseteq Mx$, we have

$$q(Tx, T^2x) = f(Tx) - \inf f(MTx) \leq f(Tx) - \inf f(Mx) \leq \frac{1}{2}\{f(x) + \inf f(Mx)\} - \inf f(Mx) = \frac{1}{2}\{f(x) - \inf f(Mx)\} = \frac{1}{2}q(x, Tx).$$
Let \( \{x_n\} \subseteq Y, y \in Y \) with \( q(x_n, y) \to 0 \). By the definition of \( q \), we may assume \( y \in Mx_n \) for every \( n \in \mathbb{N} \). Since \( Ty \in My \subseteq Mx_n \), we have

\[
q(x_n, Ty) = q(x_n, y) \to 0
\]

and hence \( y = Ty \) by Lemma 3. Therefore we have

\[
\inf\{q(x, y) + q(x, Tx) : x \in Y\} > 0
\]

for every \( y \in Y \) with \( y \neq Ty \). So, by Theorem 2, there exists \( x_0 \in Y \) such that \( x_0 = Tx_0 \). This is a contradiction and this completes the proof. \( \square \)

**Remark.** Theorem 1 is not applied to the function \( f(x) = x^2 \). But, putting \( p(x, y) = \left| \int_x^y 2|t|dt \right| \), Theorem 3 is applied to such \( f \).

Using Theorem 3 and Example 2, we have the following corollary which generalizes the results of [5] and [10].

**Corollary 1 (Takahashi [9]).** Let \( X \) be a complete metric space with metric \( d \), let \( T \) be a continuous mapping from \( X \) into itself and let \( f : X \to (-\infty, \infty] \) be a proper lower semicontinuous function, bounded from below. Assume that for any \( u \in X \) with \( f(u) > \inf_{x \in X} f(x) \), there is \( v \in X \) with \( v \neq u \) and

\[
f(v) + \max\{d(Tu, v), d(Tu, TV)\} \leq f(u).
\]

Then there exists \( x_0 \in X \) such that \( f(x_0) = \inf_{x \in X} f(x) \).

### 4. Fixed Point Theorems

In this Section, we first prove the following theorem, which is more useful than Theorem 2.

**Theorem 4.** Let \( X \) be a complete metric space, let \( p \) be a \( w \)-distance on \( X \). Let \( T \) be a mapping from \( X \) into itself and \( r \in [0, 1) \) with

\[
p(Tx, Tx') \leq rp(x, Tx)
\]

for every \( x \in X \). Suppose either of the following holds:

(i) \( \inf\{p(x, Tx) + p(x, y) : x \in X\} > 0 \) for every \( y \in X \) with \( y \neq Ty \);

(ii) it implies \( y = Ty \) that there exists a sequence \( \{x_n\} \subseteq X \) such that \( \{x_n\} \) and \( \{Tx_n\} \) converge to \( y \);

(iii) \( T \) is continuous; see [4].

Then there exists \( x_0 \in X \) such that \( x_0 = Tx_0 \). Moreover, if \( v = Tv \), then \( p(v, v) = 0 \).
Proof. In the case of (i), it is already proved. Let us prove that (ii) implies (i). Let $y \in X$ with $\inf\{p(x, Tx) + p(x, y) : x \in X\} = 0$. Then there exists $\{z_n\}$ such that $p(z_n, Tz_n) \to 0$ and $p(z_n, y) \to 0$. By Lemma 3, we have $Tz_n \to y$. Since

$$p(z_n, T^2z_n) \leq p(z_n, Tz_n) + p(Tz_n, T^2z_n) \leq (1 + r)p(z_n, Tz_n) \to 0,$$

we have $T^2z_n \to y$ by Lemma 3. Put $x_n = Tz_n$. Then both $\{x_n\}$ and $\{Tx_n\}$ converge to $y$. This implies $y = Ty$ by (ii). Hence (i) is satisfied. To complete the proof, we show that (iii) implies (ii). Let $T$ be a continuous mapping of $X$. Assume that $\{x_n\}$ and $\{Tx_n\}$ converge to $y$. Then we have

$$Ty = T(\lim_{n \to \infty} x_n) = \lim_{n \to \infty} Tx_n = y.$$

Therefore (ii) holds. □

In general, a w-distance $p$ on $X$ does not satisfy that $p(x, y) = p(y, x)$ for every $x, y \in X$. So, the condition $p(T^2x, Tx) \leq rp(Tx, x)$ for every $x \in X$, differs from the condition $p(Tx, T^2x) \leq rp(x, Tx)$. Theorem 4 is a fixed point theorem for the latter condition. We can also prove a fixed point theorem for the former condition.

**Theorem 5.** Let $X$ be a complete metric space, let $p$ be a w-distance on $X$. Let $T$ be a mapping from $X$ into itself and $r \in [0, 1)$ such that

$$p(T^2x, Tx) \leq rp(Tx, x)$$

for every $x \in X$. Suppose either of the following holds:

(i) It implies $p(Ty, y) = 0$ (or equivalently $Ty = y$) that there exists a sequence $\{x_n\} \subseteq X$ such that $\{x_n\} \to y$ and $p(Tx_n, x_n) \to 0$;

(ii) it implies $y = Ty$ that there exists a sequence $\{x_n\} \subseteq X$ such that $\{x_n\}$ and $\{Tx_n\}$ converge to $y$;

(iii) $T$ is continuous.

Then there exists $x_0 \in X$ such that $x_0 = Tx_0$. Moreover, if $v = Tv$, then $p(v, v) = 0$.

**Proof.** First, we shall show $p(Ty, y) = 0$ is equivalent to $Ty = y$ for every $y \in X$. If $p(Ty, y) = 0$, we have

$$p(T^2y, Ty) \leq rp(Ty, y) = 0$$

and

$$p(T^2y, y) \leq p(T^2y, Ty) + p(Ty, y) = 0.$$

So, we obtain $Ty = y$ by Lemma 3. If $Ty = y$, we have

$$p(y, y) = p(T^2y, Ty) \leq rp(Ty, y) = rp(y, y).$$
and hence \( p(y, y) = 0 \). Next, we shall show (ii) implies (i). Let \( \{x_n\} \) be a sequence in \( X \), which converges to some point \( y \) in \( X \) and satisfies \( \lim_{n \to \infty} p(Tx_n, x_n) = 0 \). Then we have

\[
p(T^2x_n, Tx_n) \leq rp(Tx_n, x_n) \to 0 \quad (n \to \infty)
\]

and

\[
p(T^2x_n, x_n) \leq p(T^2x_n, Tx_n) + p(Tx_n, x_n)
\leq rp(Tx_n, x_n) + p(Tx_n, x_n)
= (1 + r)p(Tx_n, x_n) \to 0 \quad (n \to \infty).
\]

By Lemma 3 and \( \{x_n\} \) converges to \( y \), we have \( \{Tx_n\} \) also converges to \( y \). So, from (ii), \( y \) is a fixed point of \( T \) and hence (i) holds. It is from the proof of Theorem 4 that (iii) implies (ii). So, to complete the proof, we prove \( T \) has a fixed point in the case of (i). Let \( u \in X \) and define

\[
u_n = T^n u \quad \text{for any} \quad n \in \mathbb{N}.
\]

Then we have, for any \( n \in \mathbb{N} \),

\[
p(u_{n+1}, u_n) \leq rp(u_n, u_{n-1}) \leq \cdots \leq r^n p(u_1, u).
\]

So, if \( m > n \),

\[
p(u_m, u_n) \leq p(u_m, u_{m-1}) + \cdots + p(u_{n+1}, u_n)
\leq r^{m-1} p(u_1, u) + \cdots + r^n p(u_1, u)
\leq \frac{r^n}{1 - r} p(u_1, u).
\]

By Lemma 4, \( \{u_n\} \) is a Cauchy sequence. Since \( X \) is complete, \( \{u_n\} \) converges to some point \( x_0 \in X \). And we have

\[
p(Tu_n, u_n) \leq r^n p(u_1, u) \to 0.
\]

So, by assumption, we have \( p(Tx_0, x_0) = 0 \). Therefore \( x_0 \) is a fixed point of \( T \). This completes the proof. \( \square \)

Now, we prove Čirić’s fixed point theorem by two methods.

**Corollary 2 (Čirić [2]).** Let \( X \) be a complete metric space with metric \( d \), and let \( T \) be a mapping from \( X \) into itself. Suppose \( T \) is quasi-contraction, i.e., there exists \( r \in (0, 1) \) such that

\[
d(Tx, Ty) \leq r \cdot \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}
\]

for every \( x, y \in X \). Then \( T \) has a unique fixed point.
Proof by Theorem 4. By lemma 2 in [2], \( \{x, Tx, T^2x, \cdots \} \) is bounded for every \( x \in X \). Hence we can define a function \( p : X \times X \to [0, \infty) \) by

\[
p(x, y) = \max\{ \text{diam}\{x, Tx, T^2x, \cdots \}, d(x, y) \}
\]

for every \( x, y \in X \). By Lemma 2, \( p \) is a w-distance on \( X \). Let \( x \in X \). Then we have, using lemma 1 in [2],

\[
p(Tx, T^2x) = \text{diam}\{Tx, T^2x, T^3x, \cdots \} = \sup_{n \in \mathbb{N}} \text{diam}\{Tx, T^2x, T^3x, \cdots T^nx\} \\
\leq \sup_{n \in \mathbb{N}} r \cdot \text{diam}\{x, Tx, T^2x, \cdots T^nx\} \\
= r \cdot \text{diam}\{x, Tx, T^2x, \cdots \} \\
= r \cdot p(x, Tx).
\]

Assume \( \{x_n\} \) and \( \{Tx_n\} \) converge to \( y \). Since \( T \) is quasi-contraction,

\[
d(Tx_n, Ty) \leq r \max\{d(x_n, y), d(x_n, Tx_n), d(y, Ty), d(Tx_n, Ty), d(y, Tx_n)\}
\]

for any \( n \in \mathbb{N} \). So,

\[
d(y, Ty) \leq r \max\{d(y, y), d(y, y), d(y, Ty), d(y, Ty), d(y, y)\} \\
= rd(y, Ty)
\]

and hence \( y = Ty \). By Theorem 4, there exists a fixed point \( z \) of \( T \). Clearly, a fixed point is unique. This completes the proof. \( \square \)

Proof by Theorem 5. We can define a function \( p : X \times X \to [0, \infty) \) by

\[
p(x, y) = \sup\{d(T^kx, y) : k \in \mathbb{N} \cup \{0\}\}
\]

for every \( x, y \in X \). By Example 3, \( p \) is a w-distance on \( X \). Let \( x \in X \). Then we have, using lemma 1 in [2],

\[
p(T^2x, Tx) = \sup\{d(T^kx, Tx) : k = 2, 3, 4, \cdots \} \\
\leq r \cdot \sup\{d(T^kx, x) : k = 1, 2, 3, \cdots \} \\
= r \cdot p(x, Tx).
\]

So, by Theorem 5, there exists a fixed point \( z \) of \( T \). This completes the proof. \( \square \)
5. Metric Completeness

In this Section, we discuss a characterization of metric completeness. First, we give a definition. A mapping $T : X \to X$ is called weakly contractive if there exist a w-distance $p$ on $X$ and $r \in [0, 1)$ such that $p(Tx, Ty) \leq rp(x, y)$ for every $x, y \in X$. The following Theorem was proved in [7]. We give another proof of "if" part and two proofs of "only if" part.

**Theorem 6** ([7]). Let $X$ be a metric space. Then $X$ is complete if and only if every weakly contractive mapping from $X$ into itself has a fixed point in $X$.

**Proof of "if" part.** Assume that $X$ is not complete. Then there exists a sequence $\{x_n\}$ in $X$ satisfying the following conditions:

(i) $\{x_n\}$ is Cauchy;

(ii) $\{x_n\}$ does not converge;

(iii) $x_i \neq x_j$ if $i \neq j$.

A function $p : X \times X \to [0, \infty)$ defined by

$$p(x, y) = \begin{cases} 2^{-i} + 2^{-j}, & \text{if } x = x_i \text{ and } y = x_j, \\ 2^{-i} + 1, & \text{if } x = x_i \text{ and } y \notin \{x_n\}, \\ 1 + 2^{-j}, & \text{if } x \notin \{x_n\} \text{ and } y = x_j \end{cases}$$

is a w-distance on $X$, by Example 4. Define a mapping $T$ from $X$ into itself as follows:

$$Tx = \begin{cases} x_{i+1}, & \text{if } x = x_i, \\ x_1, & \text{otherwise}. \end{cases}$$

Then we have $p(Tx, Ty) \leq \frac{1}{2}p(x, y)$ for every $x, y \in X$. But, $T$ has not a fixed point in $X$. This completes the proof. □

**Proof of "only if" part by Theorem 4.** Clearly,

$$p(Tx, T^2x) \leq rp(x, Tx)$$

for every $x \in X$. Let $y \in X$ with $y \neq Ty$ be fixed. Assume that there exists $\{x_n\}$ such that

$$\lim_{n \to \infty} \{p(x_n, y) + p(x_n, Tx_n)\} = 0.$$ 

Then we have

$$p(x_n, Ty) \leq p(x_n, Tx_n) + p(Tx_n, Ty) \leq p(x_n, Tx_n) + rp(x_n, y) \to 0.$$ 

Then, by Lemma 3, we have $Ty = y$. This is a contradiction. Hence, we have

$$\inf\{p(x, y) + p(x, Tx) : x \in X\} > 0.$$ 

By Theorem 4, $T$ has a fixed point. □
Proof of “only if” part by Theorem 5. Clearly, 

\[ p(T^2x, Tx) \leq rp(Tx, x) \]

for every \( x \in X \). Let \( \{x_n\} \) be a sequence in \( X \) which converges to some point \( y \) in \( X \) and satisfies \( \lim_{n \to \infty} p(Tx_n, x_n) = 0 \). Let \( k \in \mathbb{N} \) be fixed. Then we have

\[
p(T^ky, x_n) \leq p(T^ky, T^kx_n) + \sum_{i=1}^{k-1} p(T^{i+1}x_n, T^ix_n) + p(Tx_n, x_n) \\
\leq r^k p(y, x_n) + \sum_{i=0}^{k-1} r^i p(Tx_n, x_n) \\
= r^k p(y, x_n) + \frac{1 - r^k}{1 - r} p(Tx_n, x_n)
\]

and hence \( p(T^ky, y) \leq r^k p(y, y) \). So, we obtain

\[ p(T^ky, Ty) \leq rp(T^{k-1}y, y) \leq r^k p(y, y). \]

By Lemma 3, we have \( Ty = y \). Therefore, by Theorem 5, \( T \) has a fixed point. \( \square \)

Acknowledgment. The author wishes to express his hearty thanks to his supervisor Professor W. Takahashi for many valuable suggestions and constant advice.

REFERENCES


DEPARTMENT OF INFORMATION SCIENCES, TOKYO INSTITUTE OF TECHNOLOGY, OHOKAYAMA, MEGURO-KU, TOKYO 152, JAPAN

E-mail address: tomonari@is.titech.ac.jp