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Numerical Analysis of a Supersonic Rarefied Gas Flow past a Flat Plate at an Angle of Attack

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ABSTRACT: A steady supersonic flow of a rarefied gas past a flat plate at an angle of attack is analyzed numerically on the basis of the Boltzmann-Krook-Welander equation (the so-called BGK model) with the diffuse reflection boundary condition. An accurate finite-difference method giving the correct description of the discontinuity of the velocity distribution function, which was developed in the authors' recent study for the case of a flat plate at zero angle of attack, is employed. The behavior of the velocity distribution function as well as that of the macroscopic quantities is clarified, and the overall quantities such as the drag and lift on the plate are obtained accurately.

1 INTRODUCTION

A steady supersonic rarefied gas flow past a flat plate is one of the basic problems in rarefied gas dynamics, to which a great deal of literature has been devoted so far (see, e.g., Refs. [1]–[7] and their references). In our recent paper,[7] we investigated the case in which the plate is parallel to the uniform flow (i.e., at zero angle of attack), aiming to establish the precise description of the flow field around the plate on the level of the velocity distribution function of the gas molecules. Using the Boltzmann-Krook-Welander (BKW) equation (or the so-called BGK model),[8,9] we developed, on the basis of the method in Ref. [10], an accurate finite-difference scheme that gives the correct description of the discontinuity inherent in the velocity distribution function in the gas. Then, by a careful numerical analysis, we clarified the features of the gas flow, in particular, those around the leading and trailing edges, for a wide range of the Knudsen number. In the present study, we extend the analysis to the case where the plate has an angle of attack and obtain the accurate behavior of the macroscopic quantities as well as that of the velocity distribution function around the plate.

2 PROBLEM AND BASIC EQUATION

Let us consider a uniform supersonic flow of a rarefied gas (with density ρ_∞, temperature T_∞, and flow speed U_∞) past a flat plate (with chord length L and temperature T_w; with infinitely long span and without thickness) at angle of attack α. Let us assume that the plate is placed at the position $-L/2 < X_1 < L/2$, $X_2 = 0$, where X_i is the Cartesian coordinate system, and that the uniform flow is perpendicular to the spanwise direction of the plate [i.e., the velocity of the uniform flow is $(U_\infty \cos\alpha, U_\infty \sin\alpha, 0)$]. We investigate the steady behavior of the gas on the basis of kinetic theory under the assumptions that the behavior of the gas is described by the BKW equation and that the interaction of the gas molecules with the plate is described by the diffuse-reflection boundary condition.

The BKW equation in the present spatially two-dimensional case is written as[8,9]

$$\xi_1 \frac{\partial f}{\partial X_1} + \xi_2 \frac{\partial f}{\partial X_2} = A_c \rho (f_e - f), \tag{1}$$

$$f_e(\rho, T, v_1, v_2, v_3; \xi_i) = \frac{\rho}{(2\pi RT)^{3/2}} \exp\left(-\frac{(\xi_i - v_i)^2}{2RT}\right), \tag{2}$$

$$\rho = \int f d\boldsymbol{\xi}, \qquad v_i = \frac{1}{\rho}\int \xi_i f d\boldsymbol{\xi}, \qquad T = \frac{1}{3R\rho}\int (\xi_i - v_i)^2 f d\boldsymbol{\xi}, \tag{3}$$

where ξ_i is the molecular velocity; $d\boldsymbol{\xi} = d\xi_1 d\xi_2 d\xi_3$; f is the velocity distribution function of the gas molecules; ρ is the density, v_i is the flow velocity ($v_3 = 0$), and T is the temperature of the gas; R is the specific gas constant; and A_c is a constant ($A_c\rho$ is the collision frequency of the gas molecules). The boundary condition on the plate is[8,9]

$$f = f_e(\rho_w, T_w, 0, 0, 0; \xi_i), \qquad \text{for } \xi_i n_i > 0, \tag{4}$$

$$\rho_w = -\left(\frac{2\pi}{RT_w}\right)^{1/2} \int_{\xi_i n_i < 0} \xi_i n_i f d\boldsymbol{\xi}, \tag{5}$$

where $n_i = (0, 1, 0)$ on the upper surface, and $n_i = (0, -1, 0)$ on the lower surface. The condition at infinity is

$$f \to f_e(\rho_\infty, T_\infty, U_\infty \cos\alpha, U_\infty \sin\alpha, 0; \xi_i), \qquad \text{as } (X_1^2 + X_2^2)^{1/2} \to \infty. \tag{6}$$

By introducing appropriate nondimensional variables, we find that our boundary-value problem is characterized by the four nondimensional parameters: T_w/T_∞, M_∞, Kn, and α, where $M_\infty = (5RT_\infty/3)^{-1/2} U_\infty$ is the Mach number at infinity, and $Kn = \ell_\infty / L$ is the Knudsen number, ℓ_∞ being the mean free path of the gas molecules in the equilibrium state at rest with density ρ_∞ and temperature T_∞ [$\ell_\infty = (8RT_\infty/\pi)^{1/2}(A_c\rho_\infty)^{-1}$].

3 NUMERICAL ANALYSIS

We first eliminate ξ_3 from Eqs. (1), (4), and (6) by the standard procedure and then solve the resulting boundary-value problem of two simultaneous integro-differential equations numerically by exploiting the accurate finite-difference method developed in Ref. [7].

As pointed out in Refs. [10] and [11], the velocity distribution function of the gas molecules is, in general, discontinuous in the gas around a convex boundary (see also Ref. [12]). In the case of a flat plate, the convex nature of the boundary is concentrated at the leading and trailing edges. As a consequence, at a point in the gas, the discontinuity (of the velocity distribution function) appears for the molecular velocities in the two directions: one is from the leading edge to the point under consideration, and the other is from the trailing edge to it.[7] The discontinuity decays with the distance from the edges owing to molecular collisions. Thus, it is appreciable at the points whose distance from the leading or trailing edge is less than a few mean free paths. Such behavior of the discontinuity is described precisely by the method mentioned above, in which a method of characteristics is incorporated in a standard finite-difference scheme. The reader is referred to Ref. [7] for the details.

4 RESULTS

Some of the results of the numerical analysis are presented in this section. Here we restrict ourselves to the case of $T_w/T_\infty = 1$ and $M_\infty = 3$.

In Fig. 1, we show the streamlines of the flow and the isolines of the density, the temperature, and the local Mach number $M = (v_1^2 + v_2^2)^{1/2}(5RT/3)^{-1/2}$ around the plate for $\alpha = 30^\circ$ and $Kn = 0.05$ and 0.5. In Fig. 1(a) are shown the contours of $\rho/\rho_\infty = 0.2m$ ($m = 1, \cdots, 4, 6, \cdots, 15$), 1.01, and $4 + m$ ($m = 0, \cdots, 5$); $T/T_\infty = 1.01$ and $1.2 + 0.2m$ ($m = 0, \cdots, 11$); $M = 0.2m$ ($m = 1, \cdots, 14$) and 2.99, and in Fig. 1(b) those of $\rho/\rho_\infty = 0.2m$ ($m = 1, \cdots, 4, 6, \cdots, 15$), 1.01, and $4 + m$ ($m = 0, 1, 2$); $T/T_\infty = 1.2 + 0.2m$ ($m = 0, \cdots, 9$); $M = 0.2m$ ($m = 1, \cdots, 14$). For $Kn = 0.05$ [Fig. 1(a)], a curved shock, originating from the leading edge, develops below the plate. A weak compression layer is also formed upward from the leading edge, but it decays rapidly with the distance from the edge. There is a significant density variation in the gas, i.e., the density increases to $9.22\rho_\infty$ on the lower surface and decreases to $0.198\rho_\infty$ on the upper surface. For $Kn = 0.5$ [Fig. 1(b)], the variations of the flow properties are milder, and the effect of the plate tends to extend upstream.

In Fig. 2 are shown the marginal velocity distribution function $g = \int f(X_1, X_2, \xi_i) d\xi_3$ at eight points on the line $X_1/L = -0.248$, as a function of ξ_1 and ξ_2, for $\alpha = 30^\circ$ and $Kn = 0.5$. Figures 2(a)–2(d) show the g below the plate, and Figs. 2(e)–2(h) that above it [Fig. 2(d) and 2(e) are the g on the lower and the upper surface, respectively]. The height of g significantly increases on the lower surface and decreases on the upper surface because of the large density variation. [Note that the scale for g is different in Fig. 2(d).] The discontinuity of g for the molecular velocities in the two directions mentioned in Sec. 3 [i.e., (ξ_1, ξ_2) with $\xi_1/\xi_2 = (X_1 + L/2)/X_2$ and $\xi_1/\xi_2 = (X_1 - L/2)/X_2$ ($\xi_2 > 0$ for

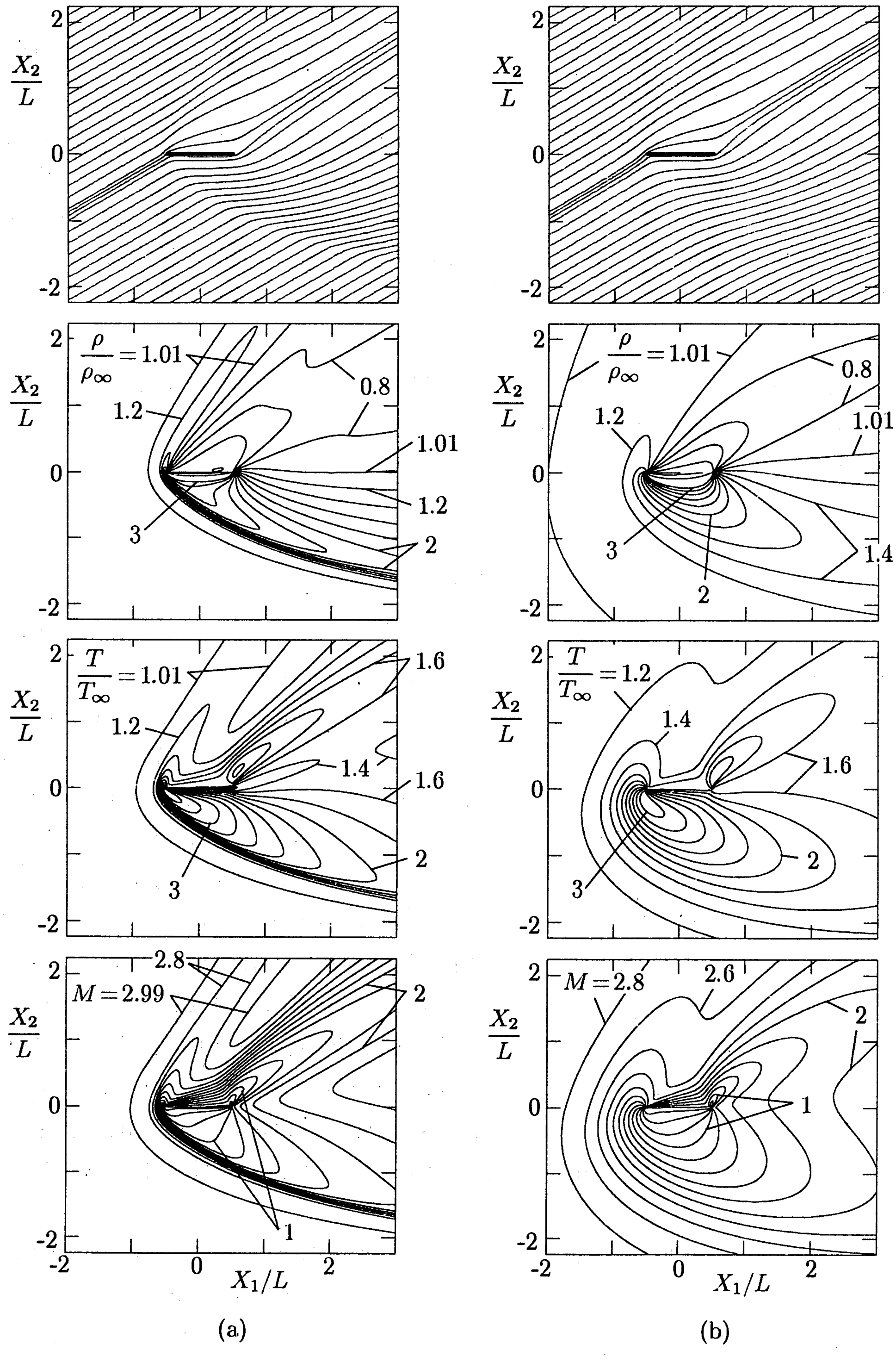

Fig. 1 The streamlines of the flow and the isolines of the density ρ, temperature T, and local Mach number M for $T_w/T_\infty = 1$, $M_\infty = 3$, and $\alpha = 30°$. (a) $Kn = 0.05$, (b) $Kn = 0.5$. (See the main text.) The plate is not shown in the figures for the isolines.

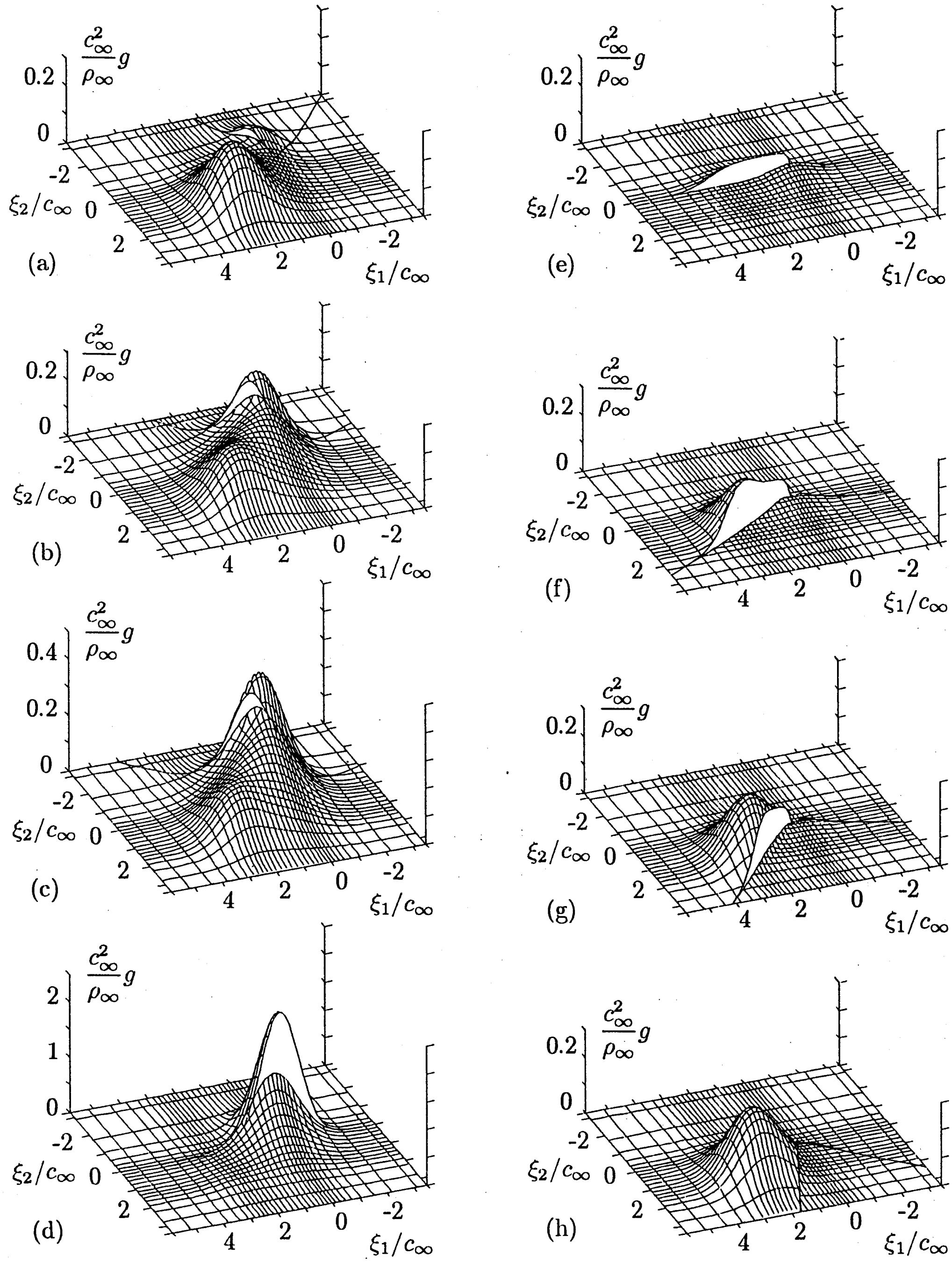

Fig. 2 The marginal velocity distribution function g at eight points along the line $X_1/L = -0.248$ for $T_w/T_\infty = 1$, $M_\infty = 3$, $\alpha = 30°$, and $Kn = 0.5$. (a) $X_2/L = -0.697$, (b) $X_2/L = -0.3$, (c) $X_2/L = -0.204$, (d) $X_2/L = 0_-$, (e) $X_2/L = 0_+$, (f) $X_2/L = 0.108$, (g) $X_2/L = 0.204$, (h) $X_2/L = 0.507$. Here, $c_\infty = (2RT_\infty)^{1/2}$. The lines $\xi_1 = \text{const}$ and $\xi_2 = \text{const}$ (corresponding to every three computational lattice lines) and the discontinuity lines are drawn on the surface, and the discontinuity is expressed as vertical cliffs.

$X_2 > 0$ and $\xi_2 < 0$ for $X_2 < 0$)] is appreciable in Figs. 2(b), 2(c), 2(f), and 2(g). The decay and the shift of the location of the discontinuity with the distance from the plate are clearly demonstrated by these figures.

Let $(\hat{F}_1, \hat{F}_2, 0)$ be the force on the surface of the plate per unit area and $\hat{E}$ be the energy transferred to it per unit time and unit area. The distributions of $\hat{F}_1$, $\hat{F}_2$, and $\hat{E}$ on the upper and lower surfaces for $\alpha = 30°$ and $Kn = 0.05$ and 0.5 are shown in Fig. 3, where $\hat{F}_1^+$, $\hat{F}_2^+$, and $\hat{E}^+$ stand for $\hat{F}_1$, $\hat{F}_2$, and $\hat{E}$ on the upper surface, and $\hat{F}_1^-$, $\hat{F}_2^-$, and $\hat{E}^-$ those on the lower surface. Let us denote by $(F_1, F_2, 0)$ the total force acting on the plate and by E the total energy transferred to it per unit time, per unit spanwise length. The drag $F_D = F_1 \cos\alpha + F_2 \sin\alpha$ and the lift $F_L = -F_1 \sin\alpha + F_2 \cos\alpha$ on the plate (per unit spanwise length) and E for $Kn = 0.05$ and 0.5, and $\alpha = 0°$, $10°$, and $30°$ are given in Table 1. These are mainly determined by the distributions of $\hat{F}_1$, $\hat{F}_2$, and $\hat{E}$ on the lower surface because those in the upper surface are much smaller in magnitude (cf. Fig. 3).

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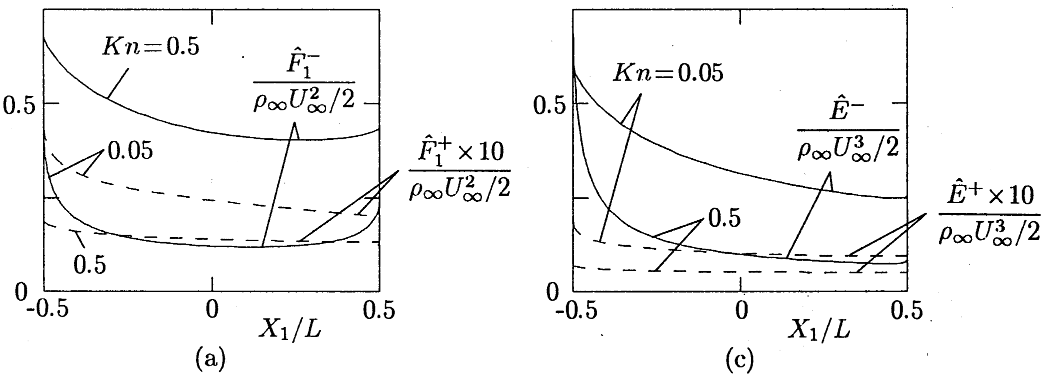

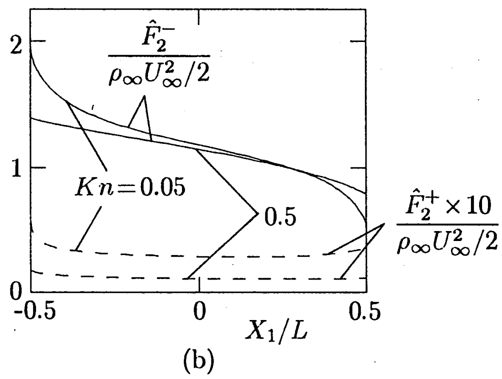

Fig. 3 The distributions of the X_1- and X_2-components of the force, $\hat{F}_1$ and $\hat{F}_2$, on the surface of the plate per unit area and of the energy $\hat{E}$ transferred to it per unit time and unit area for $T_w/T_\infty = 1$, $M_\infty = 3$, $\alpha = 30°$, and $Kn = 0.05$ and $Kn = 0.5$. (a) $\hat{F}_1$, (b) $\hat{F}_2$, (c) $\hat{E}$. Here, the superscript + (−) indicates the value on the upper (lower) surface.

Table 1 The drag F_D and lift F_L on the plate (per unit spanwise length) and the energy E transferred to it (per unit time and unit spanwise length) for $T_w/T_\infty = 1$ and $M_\infty = 3$.

	$(\frac{1}{2}\rho_\infty U_\infty^2 L)^{-1}F_D$		$(\frac{1}{2}\rho_\infty U_\infty^2 L)^{-1}F_L$		$(\frac{1}{2}\rho_\infty U_\infty^3 L)^{-1}E$	
α(deg)	$Kn = 0.05$	$Kn = 0.5$	$Kn = 0.05$	$Kn = 0.5$	$Kn = 0.05$	$Kn = 0.5$
0 [7]	0.265	0.438	0	0	0.133	0.222
10	0.310	0.510	0.310	0.315	0.132	0.241
30	0.730	0.967	0.912	0.732	0.146	0.347