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The Rohlin Property for Actions of $\mathbb{Z}^2$ on UHF Algebras

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1 Introduction

A noncommutative Rohlin type theorem was introduced by A. Connes for the classification of single automorphisms of von Neumann algebras\cite{2, 3}. This was generalized, for example, by A.Ocneanu \cite{19, 20}, for systems of several commuting automorphisms of finite von Neumann algebras, and more generally for actions of discrete amenable groups. On the other hand this was also generalized in the framework of $C^*$-algebras. The Rohlin property for single automorphisms of a certain class of $C^*$-algebras was established \[1, 5, 6, 11, 12, 13, 14\]. In particular a noncommutative Rohlin type theorem for single automorphisms of UHF algebras (and some AF algebras) was shown and the automorphisms with the Rohlin property were classified up to outer conjugacy by A. Kishimoto \cite{13, 14}. Here we present a generalization of the above results for UHF algebras.

2 Rohlin type theorem

Let $N$ be a positive integer. We first define the Rohlin property for actions of $\mathbb{Z}^N$ on unital $C^*$-algebras. This is a simple generalization of that in the case of $N = 1$ \cite{13}. Let $\xi_1, \ldots, \xi_N$ be the canonical basis of $\mathbb{Z}^N$ i.e.

$$\xi_i = (0, \ldots, 0, 1, 0, \ldots, 0),$$

where 1 is in the $i$-th component, and let $I = (1, \ldots, 1)$ throughout this paper. For $m = (m_1, \ldots, m_N)$ and $n = (n_1, \ldots, n_N) \in \mathbb{Z}^N$, $m \leq n$ means $m_i \leq n_i$ for each $i = 1, \ldots, N$.

**Definition 1** Let $N$ be a positive integer. Let $A$ be a unital $C^*$-algebra and let $\alpha$ be an action of $\mathbb{Z}^N$ on $A$ i.e. $\alpha$ is a group homomorphism from $\mathbb{Z}^N$ into the automorphisms $Aut(A)$ of $A$. Then $\alpha$ is said to have the Rohlin property if for any $m \in \mathbb{N}^N$ there exist $m^{(1)}, \ldots, m^{(R)} \in \mathbb{N}^N$ with $m^{(1)}, \ldots, m^{(R)} \geq m$ and which satisfy the following condition: For any $\varepsilon > 0$ and finite subset $F$ of $A$ there exist projections

$$e_g^{(r)} : r = 1, \ldots, R, \ g \in \mathbb{Z}^N/m^{(r)}\mathbb{Z}^N$$
in $A$ satisfying
\[\sum_{r=1}^{R} \sum_{g \in \mathbb{Z}^N/m^{(r)} \mathbb{Z}^N} e_g^{(r)} = 1,\]
\[||[x, e_g^{(r)}]|| < \epsilon,\]
\[||\alpha_g(e_h^{(r)}) - e_{g+h}^{(r)}|| < \epsilon\]
for any $x \in F, r = 1, \ldots, R, g \in \mathbb{Z}^N$ with $0 \leq g \leq m^{(r)} - I$ and $h \in \mathbb{Z}^N/m^{(r)} \mathbb{Z}^N$.

**Remark 2** Let $A$ be a UHF algebra. Then from the standard technique, we can replace the condition (1) by
\[[x, e_g^{(r)}] = 0.\]

An action $\alpha$ of $\mathbb{Z}^N$ on $A$ is determined by an $N$-tuple $(\alpha_{\xi_1}, \ldots, \alpha_{\xi_N})$ of commuting automorphisms of $A$. In terms of $(\alpha_{\xi_1}, \ldots, \alpha_{\xi_N})$ we can restate the Rohlin property for $\alpha$ as follows: For any $n, m \in \mathbb{N}$ with $1 \leq n \leq N$ there exist positive integers $m^{(1)}, \ldots, m^{(R)} \geq m$ which satisfy the following condition: For any $\epsilon > 0$ and finite subset $F$ of $A$ there exist projections
\[e_0^{(r)}, \ldots, e_{m^{(r)}-1}^{(r)} : r = 1, \ldots, R\]
in $A$ satisfying
\[\sum_{r=1}^{R} \sum_{j=0}^{m^{(r)}-1} e_j^{(r)} = 1,\]
\[||[x, e_j^{(r)}]|| < \epsilon\]
for each $r = 1, \ldots, R, j = 0, \ldots, m^{(r)} - 1$ and $x \in F$, and
\[||\alpha_{\xi_n}(e_j^{(r)}) - e_{j+1}^{(r)}|| < \epsilon,\]
\[||\alpha_{\xi_{n'}}(e_j^{(r)}) - e_j^{(r)}|| < \epsilon\]
for each $n' = 1, \ldots, N$ with $n' \neq n, r = 1, \ldots, R$ and $j = 0, \ldots, m^{(r)} - 1$, where $e_{m^{(r)}-1}^{(r)} \equiv e_0^{(r)}$.

In [13] A. Kishimoto introduced a notion of uniform outerness for automorphisms of $C^*$-algebras and he showed, if the algebras are UHF, this notion is equivalent to the usual outerness for automorphisms of the von Neumann algebras obtained through GNS representations associated with traces.
**Definition 3** Let $A$ be a unital $C^*$-algebra and let $\alpha$ be an automorphism of $A$. Then $\alpha$ is said to be uniformly outer if for any $a \in A$, nonzero projection $p \in A$ and $\varepsilon > 0$ there exist projections $p_1, \ldots, p_n$ in $A$ such that

$$p = \sum_{i=1}^{n} p_i,$$

$$\|p_i a \alpha(p_i)\| < \varepsilon$$

for $i = 1, \ldots, n$.

**Theorem 4** [13] Let $A$ be a UHF algebra and let $\alpha$ be an automorphism of $A$. Then the following conditions are equivalent:

1. $\alpha$ is uniformly outer.
2. The weak extension of $\alpha$ to an automorphism of $\pi_\tau(A)''$ is outer, where $\tau$ denotes a unique tracial state on $A$ and $\pi_\tau$ is the GNS representation of $A$ associated with $\tau$.

We recall a Rohlin type theorem for automorphisms of UHF algebras.

**Theorem 5** [14] Let $\alpha$ be an automorphism of a UHF algebra $A$. Then the following conditions are equivalent:

1. $\alpha$ has the Rohlin property.
2. $\alpha^m$ is uniformly outer for any $m \in \mathbb{Z} \setminus \{0\}$.

We show the two-dimensional version of the above theorem, namely

**Theorem 6** Let $\alpha$ be an action of $\mathbb{Z}^2$ on a UHF algebra $A$. Then the following conditions are equivalent:

1. $\alpha$ has the Rohlin property.
2. $\alpha_g$ is uniformly outer for any $g \in \mathbb{Z}^2 \setminus \{0\}$.

Once we establish this theorem, we have immediately

**Corollary 7** Let $\alpha$ be an action of $\mathbb{Z}^2$ on a UHF algebra $A$. Then the following conditions are equivalent:

1. $\alpha$ has the Rohlin property as an action of $\mathbb{Z}^2$ on $A$.
2. $\alpha_g$ has the Rohlin property as an automorphism of $A$ for any $g \in \mathbb{Z}^2 \setminus \{0\}$. 
3 Conjugacy

We introduce three types of conjugacy for actions of $\mathbb{Z}^N$ on $C^*$-algebras and discuss their relation.

Definition 8 Let $\alpha$ be an action of $\mathbb{Z}^N$ on a unital $C^*$-algebra $A$ and let $u$ be a mapping from $\mathbb{Z}^N$ into the unitaries $U(A)$ of $A$. Then $u$ is said to be a 1-cocycle for $\alpha$ if
\[
 u_{g+h} = u_g \alpha_g(u_h)
\]
for any $g, h \in \mathbb{Z}^N$.

If $\alpha$ and $\beta$ are actions of $\mathbb{Z}^N$ on a unital $C^*$-algebra $A$, then for each $\epsilon > 0$ and $\gamma \in \text{Aut}(A)$ we write $\alpha \gamma^\epsilon \beta$ if
\[
 ||\alpha_{\xi_i} - \gamma \circ \beta_{\xi_i} \circ \gamma^{-1}|| \leq \epsilon
\]
for $i = 1, \ldots, N$ and write $\alpha \cong \beta$ (or $\alpha \cong \beta$ simply) instead of $\alpha \gamma^0 \beta$.

Definition 9 Let $\alpha$ and $\beta$ be as above. Then
(1) $\alpha$ and $\beta$ are said to be approximately conjugate if for any $\epsilon > 0$ there exists an automorphism $\gamma$ of $A$ such that $\alpha \gamma^\epsilon \beta$.
(2) $\alpha$ and $\beta$ are said to be cocycle conjugate if there exist an automorphism $\gamma$ of $A$ and a 1-cocycle $u$ for $\alpha$ such that
\[
 \text{Ad} u_g \circ \alpha_g = \gamma \circ \beta_g \circ \gamma^{-1}
\]
for any $g \in \mathbb{Z}^N$.
(3) $\alpha$ and $\beta$ are said to be outer conjugate if there exist an automorphism $\gamma$ of $A$ and unitaries $u_1, \ldots, u_N$ in $A$ such that
\[
 \text{Ad} u_i \circ \alpha_{\xi_i} = \gamma \circ \beta_{\xi_i} \circ \gamma^{-1}
\]
for $i = 1, \ldots, N$.

Of course cocycle conjugacy implies outer conjugacy. Moreover one has

Proposition 10 Let $A$ be a simple, separable unital $C^*$-algebra with a unique tracial state $\tau$ and let $\alpha, \beta$ be actions of $\mathbb{Z}^N$ on $A$. If $\alpha$ and $\beta$ are approximately conjugate then they are cocycle conjugate.

Remark 11 If $\alpha$ and $\beta$ are automorphisms of a UHF algebra with the Rohlin property and they are outer conjugate, then they are approximately conjugate by the stability property. Hence the three notions of conjugacy defined above are equivalent for those automorphisms. But outer conjugacy does not always imply approximate conjugacy for $\mathbb{Z}^N$ actions. See Remark 18 for a counter example.
4 Product type actions

We discuss product type actions of $\mathbb{Z}^2$ on UHF algebras. As in the case of single automorphisms, the Rohlin property for these actions is deeply related to a notion of uniform distribution of points in $\mathbb{T}^2$. We first state this notion as a proposition whose proof is found in [1].

**Proposition 12** Let $(S_k | k \in \mathbb{N})$ be a sequence of finite sequences in $\mathbb{T}^N$ i.e.

$$S_k = (s_{k,p} | p = 1, \ldots, n_k),$$

$s_{k,p} \in \mathbb{T}^N$

for each $k \in \mathbb{N}$ and $p = 1, \ldots n_k$. Then the following conditions on $(S_k | k \in \mathbb{N})$ are equivalent.

(1)

$$\lim_{k \to \infty} \frac{1}{n_k} \sum_{p=1}^{n_k} f(s_{k,p}) = \int_{\mathbb{T}^N} f(s) ds$$

for any $f \in C(\mathbb{T}^N)$, where $ds$ denotes the normalized Haar measure on $\mathbb{T}^N$.

(2)

$$\lim_{k \to \infty} \frac{1}{n_k} \sum_{p=1}^{n_k} s_{k,p}^l = 0$$

for any $l = (l_1, \ldots, l_N) \in \mathbb{Z}^N \setminus \{0\}$, where $s^l$ denotes $s_{1}^{l_1} \cdots s_{N}^{l_N}$ for each $s = (s_1, \ldots, s_N) \in \mathbb{T}^N$.

(3)

$$\lim_{k \to \infty} \frac{1}{n_k} \nu_k \left( \prod_{i=1}^{N} [\theta_{1}^{(i)}, \theta_{2}^{(i)}] \right) = (2\pi)^{-N} \prod_{i=1}^{N} (\theta_{2}^{(i)} - \theta_{1}^{(i)})$$

for any $0 \leq \theta_{1}^{(i)} \leq \theta_{2}^{(i)} < 2\pi$, where $\nu_k$ is defined by

$$\nu_k(S) \equiv \# \{ p \mid 1 \leq p \leq n_k \text{ and } \arg(s_{k,p}) \in S \}$$

for each subset $S$ of $\prod_{i=1}^{N} [0, 2\pi)$ and $\# F$ denotes the cardinality of the set $F$.

Moreover suppose that $(n_k | k \in \mathbb{N})$ have the following property: For any $n \in \mathbb{N}$ there exists positive integer $k_0$ such that for any positive integer $k \geq k_0$ there exist positive integers $n_k^{(1)}, \ldots, n_k^{(N)} \geq n$ which satisfy $n_k = n_k^{(1)} \cdots n_k^{(N)}$. Then the above conditions are also equivalent to

(4) For any $\epsilon > 0$ there exist positive integers $k_0$ and $n_0$ such that for any $k, m, n_k^{(1)}, \ldots, n_k^{(N)} \in \mathbb{N}$ satisfying $k \geq k_0$, $n_k^{(1)}, \ldots, n_k^{(N)} \geq n_0$ and $n_k =$
$m \cdot n_{k}^{(1)} \cdots n_{k}^{(N)}$ there exists an $m$ to 1 surjection $\varphi$ from \(\{1, \ldots, n_{k}\}\) onto \(\{1, \ldots, n_{k}^{(1)}\} \times \cdots \times \{1, \ldots, n_{k}^{(N)}\}\) satisfying

\[
|s_{k,p} - (\exp(2\pi i \cdot (\varphi(p)_{1}/n_{k}^{(1)}), \ldots, \exp(2\pi i \cdot (\varphi(p)_{N}/n_{k}^{(N)})))| < \epsilon
\]

(2)

for any $k$ and $p$, where $|s| \equiv \max\{|s_{p}| : 1 \leq p \leq N\}$ for each $s \in T^{N}$ and $(\varphi(p))_{i}$ denotes the $i$-th component of $\varphi(p)$.

If $S_{k}$ satisfies the estimate (2) for some $\varphi$ as above, then $S_{k}$ is said to be \((n_{k}^{(1)}, \ldots, n_{k}^{(N)}; \epsilon)\)-distributed. If one of the conditions of the above proposition holds then $(S_{k} | k \in \mathbb{N})$ is said to be uniformly distributed. Now we state

**Definition 13** Let $A$ be a UHF algebra and let $\alpha$ be an action of $\mathbb{Z}^{N}$ on $A$. Then $\alpha$ is said to be a product type action if there exist a sequence \((m_{k} | k \in \mathbb{N})\) of positive integers such that $A \cong \otimes_{k=1}^{\infty}M_{m_{k}}(\mathbb{C})$ and

\[
\alpha_{g}(A_{k}) = A_{k}
\]

for any $g \in \mathbb{Z}^{N}$ and $k \in \mathbb{N}$, where $A_{k}$ denotes the $C^{*}$-subalgebra of $A$ corresponding to $M_{m_{k}}(\mathbb{C}) \otimes (\otimes_{l \neq k} \mathbb{C}1_{m_{k}})$.

**Remark 14** In the situation above, if $N = 2$ then one finds unitaries $u_{k}^{(1)}, u_{k}^{(2)}$ in $A_{k}$ and $\lambda_{k} \in T$ such that

\[
\alpha_{(p,q)}[A_{k}] = Ad u_{k}^{(1)p} u_{k}^{(2)q},
\]

\[
u_{k}^{(1)} u_{k}^{(2)} = \lambda_{k} u_{k}^{(2)} u_{k}^{(1)}
\]

for any $p, q$. Since $u_{k}^{(1)}, u_{k}^{(2)}$ are unique up to a constant multiple, $\lambda_{k}$ is unique. In addition $\lambda_{k}^{m_{k}} = 1$.

Let $n \in \mathbb{N}$ and let $U, V \in U(M_{n}(\mathbb{C}))$ satisfying $UV = VU$. Then we set Sp($U$) to be a sequence consisting of the eigenvalues of $U$ repeated as often as multiplicity indicates and Sp($U, V$) is a sequence consisting of the pairs of eigenvalues of $U$ and $V$ with a common eigenvector, repeated as often as multiplicity indicates. Then the Rohlin property for the product type actions of $\mathbb{Z}^{2}$ on $A$ with $\lambda_{k} = 1$ is characterized as follows.

**Proposition 15** Let $A$ be a UHF algebra and let $\alpha$ be a product type action of $\mathbb{Z}^{2}$ on $A$ with \((m_{k} | k \in \mathbb{N}), (u_{k}^{(1)} | k \in \mathbb{N}), (u_{k}^{(2)} | k \in \mathbb{N}), (\lambda_{k} | k \in \mathbb{N})\) as above. If $\lambda_{k} = 1$ for each $k \in \mathbb{N}$ then the following conditions are equivalent:

1. $\alpha$ has the Rohlin property.
(2) \( \left( \text{Sp}(\otimes_{k=m}^{n} u_{k}^{(1)}, \otimes_{k=m}^{n} u_{k}^{(2)}) \mid n = m, m+1, \ldots \right) \) is uniformly distributed for any \( m \in \mathbb{N} \).

In \cite{14} A.Kishimoto showed for each UHF algebra \( A \)

(1) For any product type actions \( \alpha \) and \( \beta \) of \( \mathbb{Z} \) on \( A \) with the Rohlin property, \( \alpha \) and \( \beta \) are approximately conjugate.

(2) For any action \( \alpha \) of \( \mathbb{Z} \) on \( A \) with the Rohlin property and \( \varepsilon > 0 \) there exist a product type action \( \beta \) of \( \mathbb{Z} \) on \( A \) with the Rohlin property and an automorphism \( \gamma \) of \( A \) such that \( \alpha \overset{\gamma \varepsilon}{\approx} \beta \).

In particular there is one and only one approximate conjugacy class of actions of \( \mathbb{Z} \) on \( A \) with the Rohlin property. In the case of \( N = 2 \) we do not know whether (2) is valid or not. In the rest of this paper we state several results for (1).

**Theorem 16** Let \( A \) be a UHF algebra and let \( \alpha \) and \( \beta \) be product type actions of \( \mathbb{Z}^{2} \) on \( A \) with the Rohlin property. Take \((m_{k} \mid k \in \mathbb{N})\), \((\lambda_{k} \mid k \in \mathbb{N})\) for \( \alpha \) as in Definition 13 and Remark 14. Also take \((n_{l} \mid l \in \mathbb{N})\), \((\mu_{l} \mid l \in \mathbb{N})\) for \( \beta \) similarly. If \( \lambda_{k} = \mu_{l} = 1 \) for each \( k, l \in \mathbb{N} \) then \( \alpha \) and \( \beta \) are approximately conjugate.

We discuss product type actions for two classes of UHF algebras. Let \((p_{k} \mid k \in \mathbb{N})\) be the increasing sequence of all the prime numbers. For a sequence \((i_{k} \mid k \in \mathbb{N})\) of nonnegative integers such that \( \sum_{k=1}^{\infty} i_{k} = \infty \), let \( q_{k} \equiv p_{k}^{i_{k}} \) and let \( A \equiv \otimes_{k=1}^{\infty} M_{q_{k}}(\mathbb{C}) \). We regard \( M_{q_{k}}(\mathbb{C}) \) as a \( \mathbb{C}^{*} \)-subalgebra of \( A \). We consider the class of product type actions \( \alpha \) of \( \mathbb{Z}^{2} \) on \( A \) factorized as \( \otimes_{k=1}^{\infty} M_{q_{k}}(\mathbb{C}) \).

Thus \( \alpha \) is defined in terms of unitaries \( u_{k}^{(1)}, u_{k}^{(2)} \) in \( M_{q_{k}}(\mathbb{C}) \) and \( \lambda_{k} \in \mathbb{T} \) with \( u_{k}^{(1)}u_{k}^{(2)} = \lambda_{k}u_{k}^{(2)}u_{k}^{(1)} \) by

\[
\alpha_{(p,q)}(M_{q_{k}}(\mathbb{C})) = \text{Ad} u_{k}^{(1)}u_{k}^{(2)}q_{k}.\]

Since \( \lambda_{q_{k}} = 1 \), we may regard \( \lambda_{k} \) as an element of \( G_{k} \equiv \mathbb{Z}/q_{k}\mathbb{Z} \). We let \([\alpha] \) be the sequence \((\lambda_{k} \mid k \in \mathbb{N})\) which is also considered as an element of \( \prod_{k=1}^{\infty} G_{k} \).

We define an equivalence relation in \( \prod_{k=1}^{\infty} G_{k} \) by: \( g \sim h \) if there is an \( n \) such that \( g_{k} = h_{k} \) for \( k \geq n \). Let \( 0 \) be the trivial sequence \((0,0,\ldots)\). We note that for every \( g \in \prod_{k=1}^{\infty} G_{k} \) there is an action \( \alpha \) of \( \mathbb{Z}^{2} \) in the above class such that \([\alpha] = g \).

**Theorem 17** (1) If \( \alpha \) is an action of \( \mathbb{Z}^{2} \) in the above class and \([\alpha] \neq 0 \), then \( \alpha \) has the Rohlin property.

(2) If \( \alpha \) and \( \beta \) are actions of \( \mathbb{Z}^{2} \) in the above class and satisfy the Rohlin property, then the following are equivalent:
\( (2.1) \ [\alpha] \sim [\beta] \).

\( (2.2) \) \( \alpha \) and \( \beta \) are outer conjugate with each other.

**Remark 18** Let \( A \equiv M_3(\mathbb{C}) \otimes M_{2\infty}(\mathbb{C}) \) and define \( n \times n \) unitary matrices \( u_n, v_n \) by

\[
\begin{bmatrix}
1 & \omega_n & \cdots \\
\omega_n & \ddots & \ddots \\
\ldots & \ddots & \ddots & 1 \\
\omega_n & \ldots & 1 & 0
\end{bmatrix},
\begin{bmatrix}
0 & \cdots & 0 & 1 \\
1 & \ddots & \ddots & 0 \\
\ldots & \ddots & \ddots & 0 \\
1 & 0 & \ldots & \ddots
\end{bmatrix}
\]

where \( \omega_n \equiv \exp(2\pi i \cdot \frac{1}{n}) \) for each \( n \in \mathbb{N} \). Using these matrices we define actions \( \alpha, \beta \) of \( \mathbb{Z}^2 \) on \( A \) by

\[
\alpha_{\xi_1} \equiv Ad u_3 \otimes \left( \bigotimes_{k=1}^{\infty} Ad u_{2^k} \right), \quad \alpha_{\xi_2} \equiv Ad v_3 \otimes \left( \bigotimes_{k=1}^{\infty} Ad v_{2^k} \right),
\]

\[
\beta_{\xi_1} \equiv id_{M_3(\mathbb{C})} \otimes \left( \bigotimes_{k=1}^{\infty} Ad u_{2^k} \right), \quad \beta_{\xi_1} \equiv id_{M_3(\circ)} \otimes \left( \bigotimes_{k=1}^{\infty} Ad v_{2^k} \right).
\]

Then in the same way as the proof of Theorem 17, we can show that \( \alpha, \beta \) have the Rohlin property and they are not approximately conjugate. But they are clearly outer conjugate.

Let \( \{ q_k \mid k \in K \} \) be a finite or infinite set of prime numbers. We next consider product type actions of \( \mathbb{Z}^2 \) on the UHF algebra

\[
\bigotimes_{k \in K} M_{q_k}^\infty(\mathbb{C}),
\]

where \( M_{q_k}^\infty(\mathbb{C}) \) is understood as \( \otimes_{n=1}^{\infty} M_{q_k}(\mathbb{C}) \).

**Theorem 19** Let \( A \) be a UHF algebra as above and let \( \alpha, \beta \) be product type actions of \( \mathbb{Z}^2 \) on \( A \). If \( \alpha \) and \( \beta \) have the Rohlin property then \( \alpha \) and \( \beta \) are approximately conjugate with each other.

Now we sum up the results we have stated so far.

**Theorem 20** Let \( \{ p_k \mid k \in \mathbb{N} \} \) be the increasing sequence of all the prime numbers. For any UHF algebra \( A \) there exist one and only one sequence \( \{ i_k \mid k \in \mathbb{N} \} \) of nonnegative integers and \( \infty \) such that \( A \cong \bigotimes_{k=1}^{\infty} M_{p_k}^{i_k}(\mathbb{C}) \), where \( M_{p_k}^{\infty}(\mathbb{C}) \) is understood as \( \otimes_{n=1}^{\infty} M_{p_k}(\mathbb{C}) \) ([10]). Then it follows:
(1) If $\#\{k \in \mathbb{N} \mid 1 \leq i_k < \infty \} = \infty$ then there are infinitely many outer conjugacy classes of product type actions of $\mathbb{Z}^2$ on $A$ with the Rohlin property.

(2) If $\#\{k \in \mathbb{N} \mid 1 \leq i_k < \infty \} < \infty$ and $A$ is infinite-dimensional then there is one and only one outer conjugacy class of product type actions of $\mathbb{Z}^2$ on $A$ with the Rohlin property.

(3) If $A$ is finite-dimensional then there is no action of $\mathbb{Z}^2$ on $A$ with the Rohlin property.

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