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Kyoto University
Upper and Lower Bounds Computations of Drag Coefficients

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1 Introduction

In the recent paper we have derived an error estimation for drag coefficients and presented a numerical method by an extrapolation for a precise computation of drag coefficients [1]. In this paper we show another numerical method having upper and lower bounds. Our idea is to control a parameter appearing in the stabilized finite element method. Here we consider only two-dimensional problems for the simplicity of the notation. For the details of the method we refer to the paper [2].

2 A numerical method for drag coefficients

Let $G$ be a two-dimensional body in a velocity field. Let $U$ be the representative velocity and $\rho$ be the density of the fluid. The drag coefficient of $G$ is defined by

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \ell},$$

where $D$ is the component parallel to $U$ of the total force exerted on $G$ by the fluid and $\ell$ is the length of $G$ orthogonal to $U$.

We suppose the velocity field is governed by the stationary Navier-Stokes equation,

$$(u \cdot \text{grad})u + \frac{1}{Re} \Delta u + \text{grad} p = 0,$$
$$\text{div} u = 0,$$

where $u = (u_1, u_2)^T$ is the velocity, $p$ is the pressure, and $Re$ is the Reynolds number.

We consider the stabilized finite element method [3]. Let $V_h(g)$ be an affine finite element space satisfying the velocity boundary condition $u = g$, and $Q_h$ be the finite element space for the pressure. We set $V_h = V_h(0)$. We seek the finite element solution $(u_h, p_h) \in V_h(g) \times Q_h$ such that

$$A_h(u_h)((u_h, p_h), (v_h, q_h)) = 0 \quad (\forall (v_h, q_h) \in V_h),$$
where $\mathcal{V}_h$ is the product space
$$\mathcal{V}_h = V_h \times Q_h,$$
and $\mathcal{A}_h(u_h)$ is a bilinear form in $\mathcal{V}_h$ defined by
$$\mathcal{A}_h(u_h)((u_h, p_h), (v_h, q_h)) = a_1(u_h, u_h, v_h) + a(u_h, v_h) + b(u_h, q_h) + C_h(u_h)((u_h, p_h); (v_h, q_h)),$$
$$C_h(u)((u_h, p_h), (v_h, q_h)) = \sum_K \tau_K \int_K \{(w \cdot \text{grad})u_h + \frac{1}{Re} Lu_h + \text{grad} p_h\}\{(w \cdot \text{grad})v_h + \frac{1}{Re} Lv_h - \text{grad} q_h\}dx.$$
Here $a_1, a, b$ are the trilinear form, the bilinear forms derived from the nonlinear convection term, the diffusion term and the divergence term, respectively. The summation is taken for all elements $K$ and $\tau_K$ is the stabilization parameter defined by
$$\tau_K = \begin{cases} h_K^2 Re/(4c_0^2) & \text{when } Re_K < 1, \\ h_K/(2|w_K|) & \text{when } Re_K \geq 1, \end{cases}$$
where $Re_K$ is an element Reynolds number
$$Re_K = \frac{h_K|w_K|Re}{2c_0^2}, \quad (1)$$
h$_K$ is the diameter of element $K$, $w_K$ is a representative velocity of $w$ in $K$, e.g., the value of $w$ at the centroid of $K$, and $c_0$ is a positive constant independent of $h$.

We define an approximate drag coefficient $C_{D}^{hs}[2]$ by
$$C_{D}^{hs} = -\frac{2}{\rho U^2} \{ a_1(u_h, u_h, \phi_h) + a(u_h, \phi_h) + b(\phi_h, p_h) + C_h(u_h)((u_h, p_h), (\phi_h, 0)) \},$$
where $\phi_h = (0, \phi_{2h})$ be a function in the velocity finite element space defined by
$$\phi_{2h} = \begin{cases} 1 & \text{at all nodal points on the boundary of } G, \\ 0 & \text{at the other nodal points}. \end{cases}$$

### Table 1: Drag coefficients of the circle

<table>
<thead>
<tr>
<th>$Re$</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
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<tbody>
<tr>
<td>$C_D$</td>
<td>3.074</td>
<td>2.190</td>
<td>1.832</td>
<td>1.626</td>
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3 Upper and lower bounds computations

In (1) we have a parameter $c_0$. We can control it to obtain upper and lower bounds of drag coefficients. Table 1 shows drag coefficients of the unit circle obtained by this method [2].
References

