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Kyoto University
Tight Graphs and Their Primitive Idempotents*

Arlene A. Pascasio
De La Salle University
Manila, Philippines

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Abstract

In this paper, we prove

Theorem 1. Let $\Gamma$ denote a distance-regular graph with diameter $d \geq 3$. Suppose $E$ and $F$ are primitive idempotents of $\Gamma$, with cosine sequences $\sigma_0, \sigma_1, \ldots, \sigma_d$ and $\rho_0, \rho_1, \ldots, \rho_d$, respectively. Then the following are equivalent.

i) The entry-wise product $E \circ F$ is a scalar multiple of a primitive idempotent of $\Gamma$.

ii) There exists a real number $\epsilon$ such that

$$\sigma_i \rho_i - \sigma_{i-1} \rho_{i-1} = \epsilon (\sigma_{i-1} \rho_i - \sigma_i \rho_{i-1}) \quad (1 \leq i \leq d).$$

Let $\Gamma$ denote a distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \cdots > \theta_d$. In [1], Jurišić, Koolen and Terwilliger proved that the valency $k$ and the intersection numbers $a_1, b_1$ satisfy

$$
\left(\frac{\theta_1}{a_1 + 1} + \frac{k}{a_1 + 1}\right) \left(\frac{\theta_d}{a_1 + 1} + \frac{k}{a_1 + 1}\right) \geq \frac{-a_1 b_1}{(a_1 + 1)^2}.
$$

They called the graph tight whenever $\Gamma$ is not bipartite, and equality holds above. Combining Theorem 1 with some of their results, we obtain

Corollary 2. Let $\Gamma$ denote a nonbipartite distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \cdots > \theta_d$. The following are equivalent.

i) There exist nontrivial primitive idempotents $E, F$ of $\Gamma$ such that (i), (ii) hold in Theorem 1.

ii) $\Gamma$ is tight.

Moreover, if (i), (ii) hold then the eigenvalues of $\Gamma$ associated with $E, F$ are a permutation of $\theta_1, \theta_d$.

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Reference

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