

# Stepwise Synthesis of Partial Specifications preserving Strong $(\Omega_1, \Omega_2)$ -Equivalence

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**abstract :** In this paper we present a *partial* equality called *strong  $(\Omega_1, \Omega_2)$ -equivalence* between two partial specifications  $SP_1$  and  $SP_2$ , denoted by  $SP_1 \sim_{\Omega_1, \Omega_2} SP_2$ , based on an extended bisimulation relation. The parameters  $\Omega_1$  and  $\Omega_2$  are the sets of available actions in  $SP_1$  and  $SP_2$ , respectively. Furthermore we present a synthesis method of the two partial specifications  $SP_1$  and  $SP_2$  into a specification  $SP_{12}$  such that  $SP_{12} \sim_{\Omega_1 \cup \Omega_2, \Omega_1} SP_1$  and  $SP_{12} \sim_{\Omega_1 \cup \Omega_2, \Omega_2} SP_2$ .  $SP_{12}$  is called the *principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification* of  $SP_1$  and  $SP_2$ , and it is denoted by  $SP_{12} \simeq (SP_1 \uparrow_{\Omega_1} \sqcap_{\Omega_2} SP_2)$ . For example, this synthesis method is used for stepwise refinement of partial specifications preserving strong  $(\Omega_1, \Omega_2)$ -equivalence.

## 1 Introduction

A specification of a large system is generally too complex for users (and also designers) to understand, and users are often interested in *some* actions. Therefore it is useful for users to extract partial specifications about their interesting actions from the complete specification by hiding uninteresting actions. On the other hand, it is also needed to stepwise synthesize partial specifications required by many designers.

In this paper, we propose a partial analysis method in models based on labeled transition systems (LTS). A set of interesting actions is called a *filter* ranged over by  $\Omega$ . An action observed through the filter is changed into an internal action  $\tau$ , if the action is not included in the filter.

We explain the partial analysis by using Figure 1. The transition graph  $SP$  shows the specification of a system  $SYS$ . Each node represents a state and each labeled arrow represents a transition by the label which corresponds to an action. In this paper we use the sequential process expressions of a fundamental process algebra CCS[1] for describing specifications as follows:

$$SP \stackrel{\text{def}}{=} a.(b.c.SP + b.\tau.SP) + a.(b.\tau.SP + \tau.c.SP)$$

where the period ‘.’ is a sequential combinator and ‘+’ is a choice combinator.  $\stackrel{\text{def}}{=}$  is used for defining the left side *specification constant* as the right side specification, thus it is a recursive definition.

$SP_1$  is a partial specification of  $SYS$  observed through the filter  $\{a, b\}$ . In this case the two states (3) and (4) in  $SP$  are not distinct in  $SP_1$ , because  $c$  is changed into  $\tau$  through  $\{a, b\}$ . Although it is expected that two sequential transitions by  $\tau$  from (2) to (0) via (5) in  $SP_1$  is reduced to one transition by  $\tau$ , the reduction is not considered in this paper. The reduction of internal actions based on weak equivalence of CCS is the next work, and we first clarify

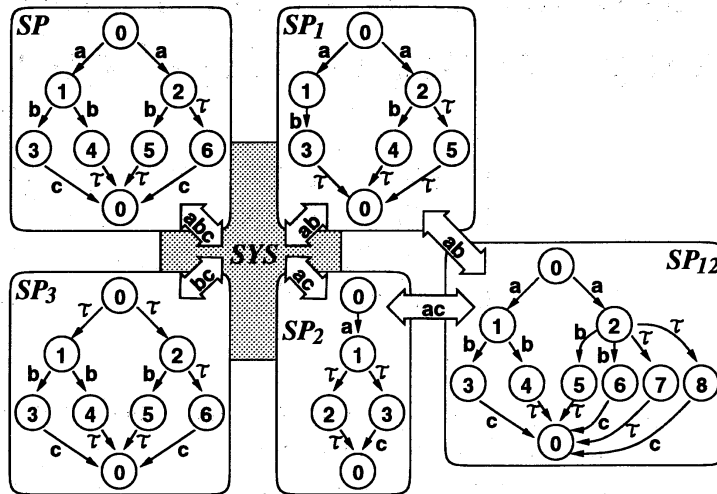


Figure 1: Partial specifications ( $SP_1, SP_2, SP_3$ ) and a synthetic specification  $SP_{12}$

the effect of the filter based on strong equivalence.  $SP_2$  and  $SP_3$  are partial specifications observed through  $\{a, c\}$  and  $\{b, c\}$ , respectively.

We give a *partial* equality called *strong*  $(\Omega_1, \Omega_2)$ -*equivalence* denoted by  $\Omega_1 \sim \Omega_2$ , for relating such partial specifications, where  $\Omega_1$  and  $\Omega_2$  are filters used for getting the left side partial specification and the right side one, respectively. For example,  $SP_1$  and  $SP_2$  are strongly  $(\{a, b\}, \{a, c\})$ -equivalent and it is denoted by

$$SP_1 \{a,b\} \sim \{a,c\} SP_2.$$

Braces and commas of filters are often omitted such that  $\{a, b\}$  is written as  $ab$ .

Next, we discuss how to reconstruct the complete specification by synthesizing partial specifications. For the example of Figure 1, it is expected to produce a synthetic specification  $SP_{12}$  such that

$$SP_{12} \text{ } abc \sim_{ab} SP_1 \text{ and } SP_{12} \text{ } abc \sim_{ac} SP_2. \quad (*_1)$$

In this case, it is a problem that there are many synthetic specifications satisfying  $(*_1)$  and they are always not strongly  $(abc, abc)$ -equivalent. For example, the following specifications satisfy  $(*_1)$ , but  $SP_{12} \text{ } abc \not\sim_{abc} SP'_{12}$ .

$$\begin{aligned} SP_{12} &\stackrel{\text{def}}{=} a.(b.c.SP_{12} + b.\tau.SP_{12}) + a.(b.\tau.SP_{12} + \tau.c.SP_{12} + b.c.SP_{12} + \tau.\tau.SP_{12}) \\ SP'_{12} &\stackrel{\text{def}}{=} a.(b.c.SP'_{12} + b.\tau.SP'_{12}) + a.(b.c.SP'_{12} + \tau.\tau.SP'_{12}) \end{aligned}$$

It is also important to notice that  $SP_{12} \text{ } abc \not\sim_{bc} SP_3$  and  $SP'_{12} \text{ } abc \not\sim_{bc} SP_3$ . This inequality complicates the next synthesis of  $SP_{12}$  and  $SP_3$ , thus stepwise synthesis of specifications is difficult.

In order to overcome this problem, we attempt to describe a *comprehensive* specification which includes all the synthetic specifications satisfying  $(*_1)$ . Thus a synthetic specification satisfying  $(*_1)$  can be selected from the comprehensive specification. For describing the comprehensive specification, we present a *multi-labeled transition system* abbreviated to a *multi-LTS*. A transition of the multi-LTS has the following form

$$P \xrightarrow{\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle} \langle P_1, P_2, \dots, P_n \rangle$$

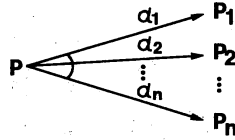


Figure 2: A transition in the multi-labeled transition system (multi-LTS)

and its transition graph is written as shown in Figure 2. It intuitively means that  $P$  can perform an action  $\alpha_1$  then behaves like  $P_1$ ,  $P$  can perform an action  $\alpha_2$  then behaves like  $P_2$ ,  $\dots$ , and/or  $P$  can perform an action  $\alpha_n$  then behaves like  $P_n$ . The ‘and/or’ is important. For example, the following specification  $AB$

$$AB \xrightarrow{\langle a, b \rangle} \langle \mathbf{0}, \mathbf{0} \rangle$$

can perform  $a$  then stops and/or can perform  $b$  then stops.

We present a specification language  $SPEC^V$  based on the multi-LTS.  $SPEC^V$  consists of the sequential combinators of CCS and a new combinator  $\vee$  called an *Andor combinator*. Intuitively  $P \vee Q$  behaves  $P$  or  $Q$  or  $P + Q$ . We often say that  $P \vee Q$  includes  $P$ ,  $Q$ , and  $P + Q$ . For example, the above  $AB$  can be described as follows:

$$AB \stackrel{\text{def}}{=} (a.\mathbf{0}) \vee (b.\mathbf{0})$$

The important difference between  $\vee$  and  $+$  is explained as follows:

- $(a.\mathbf{0}) \vee (b.\mathbf{0})$  can perform  $a$  and/or  $b$ .
- $(a.\mathbf{0}) + (b.\mathbf{0})$  can perform  $a$  and  $b$ .

If a specification contains no Andor combinators, then it is called a *ground specification*. Thus ground specification can be described in CCS. For example,  $(a.\mathbf{0}) + (b.\mathbf{0})$  is a ground specification. A specification of a practical system is always a ground specification. Specifications containing Andor combinators are used for describing medium specifications during design process.

Strong  $(\Omega_1, \Omega_2)$ -equivalence previously introduced for ground specifications is extended to  $SPEC^V$ . Intuitively, two specifications  $P$  and  $Q$  are strongly  $(\Omega_1, \Omega_2)$ -equivalent, denoted by  $P \Omega_1 \sim_{\Omega_2} Q$ , if  $P_0 \Omega_1 \sim_{\Omega_2} Q_0$  for *some* ground specifications  $P_0$  and  $Q_0$  included in  $P$  and  $Q$ , respectively. For example,

$$(a.\mathbf{0}) \vee (b.\mathbf{0}) \stackrel{abc \sim_{abc}}{\sim} (a.\mathbf{0}) \vee (c.\mathbf{0}),$$

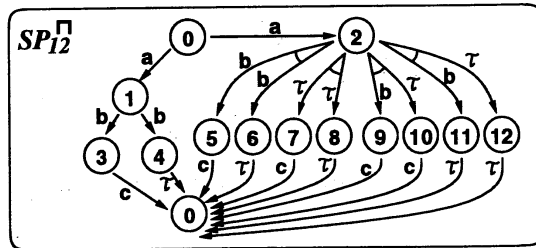
because both of them include a ground specification  $(a.\mathbf{0})$ .

Another equality called *strong  $(\Omega_1, \Omega_2)$ -full-equivalence* is also given. Intuitively, two specifications  $P$  and  $Q$  are strongly  $(\Omega_1, \Omega_2)$ -full-equivalent, denoted by  $P \Omega_1 \simeq_{\Omega_2} Q$ , if  $P_0 \Omega_1 \sim_{\Omega_2} Q_0$  for *any* ground specifications  $P_0$  and  $Q_0$  included in  $P$  and  $Q$ , respectively. For example,

$$\begin{aligned} (a.\mathbf{0}) \vee (b.\mathbf{0}) &\stackrel{abc \simeq_{abc}}{\simeq} (b.\mathbf{0}) \vee (a.\mathbf{0}), \\ (a.\mathbf{0}) \vee (b.\mathbf{0}) &\stackrel{abc \not\simeq_{abc}}{\not\simeq} (a.\mathbf{0}) \vee (c.\mathbf{0}), \\ (a.\mathbf{0}) \vee (b.\mathbf{0}) &\stackrel{abc \not\simeq_{abc}}{\not\simeq} ((a.\mathbf{0}) \vee (b.\mathbf{0})) + (a.\mathbf{0}). \end{aligned}$$

The reason of the last inequality is that  $(a.\mathbf{0}) \vee (b.\mathbf{0})$  includes a ground specification  $(b.\mathbf{0})$  but  $((a.\mathbf{0}) \vee (b.\mathbf{0})) + (a.\mathbf{0})$  does not include  $(b.\mathbf{0})$ .

Then we present a relation called the *principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification*  $P$  of two specifications  $Q_1$  and  $Q_2$ , denoted by  $P \simeq (Q_1 \Omega_1 \sqcap_{\Omega_2} Q_2)$ . For the example of Figure 1,

Figure 3: The transition graph of  $SP_{12}^{\square}$ 

the following specification  $SP_{12}^{\square}$  is the principal strong  $(ab, ac)$ -synthetic specification of  $SP_1$  and  $SP_2$ ,

$$\begin{aligned} SP_{12}^{\square} \stackrel{\text{def}}{=} & a.(b.c.SP_{12} + b.\tau.SP_{12}) \\ & + a.((b.c.SP_{12} \vee b.\tau.SP_{12}) + (\tau.c.SP_{12} \vee \tau.\tau.SP_{12})) \\ & + (b.c.SP_{12} \vee \tau.c.SP_{12}) + (b.\tau.SP_{12} \vee \tau.\tau.SP_{12}), \end{aligned}$$

thus  $SP_{12}^{\square} \simeq (SP_1 \text{ ab} \square_{ac} SP_2)$ . Figure 3 shows the transition graph of  $SP_{12}^{\square}$ . In Section 4 it is shown how to check whether a specification is the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification of two specifications, and how to produce the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification from two specifications. The principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification is *uniquely* decided.

The main property of the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification is shown in Proposition 4.1. For the example of Figure 1, the following relation holds by Proposition 4.1. For any ground specification  $SP_0$ ,

$$SP_0 \text{ abc} \sim_{abc} SP_{12}^{\square} \quad \text{iff} \quad SP_0 \text{ abc} \sim_{ab} SP_1 \quad \text{and} \quad SP_0 \text{ abc} \sim_{ac} SP_2.$$

Thus, all the ground specifications included in  $SP_{12}^{\square}$  satisfy both  $SP_1$  and  $SP_2$ , and they are all the ground specifications satisfying both  $SP_1$  and  $SP_2$ . Furthermore  $SP_3$  can be synthesized to  $SP_{12}^{\square}$  by  $SP_{123}^{\square} \simeq (SP_{12}^{\square} \text{ abc} \square_{bc} SP_3)$ . Then the following relation holds by Theorem 4.4, because  $SP_{12}^{\square} \text{ abc} \sim_{bc} SP_3$ . For any ground specification  $SP_0$ ,

$$SP_0 \text{ abc} \sim_{abc} SP_{123}^{\square} \quad \text{iff} \quad SP_0 \text{ abc} \sim_{ab} SP_1, \quad SP_0 \text{ abc} \sim_{ac} SP_2, \quad \text{and} \quad SP_0 \text{ abc} \sim_{bc} SP_3.$$

Thus stepwise refinement of specifications is possible.

The outline of this paper is as follows: In Section 2, we define the syntax and the semantics of  $SPEC^{\vee}$ . In Section 3, strong  $(\Omega_1, \Omega_2)$ -equivalence is defined. In Section 4, principal strong  $(\Omega_1, \Omega_2)$ -synthetic specifications are defined. In Section 5, we discuss partial analysis and synthesis methods already proposed. In Section 6, we conclude this paper.

## 2 Definition of specifications

We use expressions like CCS for describing behavior of specifications. They are sequential processes of CCS with Andor combinators introduced in Section 1. We call the specification language  $SPEC^{\vee}$ . In this section, we define the syntax and the semantics of  $SPEC^{\vee}$ .

We assume that an infinite set of *names*  $\mathcal{N}$  ranged over by  $a, b, \dots$ , is given. The set of actions  $Act$  ranged over by  $\alpha, \beta, \dots$ , is defined as  $Act = \mathcal{N} \cup \{\tau\}$ , where  $\tau$  is a special action called an *internal action* not included in  $\mathcal{N}$ . We also assume that a set of *specification constants* (also called *Constants*)  $\mathcal{K}$  ranged over by  $A, B, \dots$ , is given. Then, the syntax of  $SPEC^{\vee}$  is defined.

**Definition 2.1** *The set of specifications  $\mathcal{P}$  is the smallest set including the following expressions:*

$$\begin{aligned} A &: \text{Constant } (A \in \mathcal{K}) \\ \mathbf{0} &: \text{Inaction} \\ \alpha.P &: \text{Prefix } (\alpha \in \text{Act}) \\ P + Q &: \text{Choice} \\ P \vee Q &: \text{Andor} \end{aligned}$$

where  $P$  and  $Q$  are already in  $\mathcal{P}$ . ■

A Constant is a specification whose meaning is given by a defining equation. In fact, we assume that for every Constant  $A \in \mathcal{K}$ , there is a defining equation of the form  $A \stackrel{\text{def}}{=} P$ , where  $P \in \mathcal{P}$ . To avoid too many parentheses, combinators have binding power in the following order: Prefix  $>$  Choice  $>$  Andor.

We also define *ground specifications*  $\mathcal{P}_0$  with the following syntax:

$$P_0 ::= A_0 \mid \mathbf{0} \mid \alpha.P_0 \mid P_0 + P_0$$

where  $A_0 \in \mathcal{K}_0 \subseteq \mathcal{K}$  and  $\alpha \in \text{Act}$ . The set of ground specifications is denoted by  $\mathcal{P}_0$ . We assume that for every Constant  $A_0 \in \mathcal{K}_0$ , there is a defining equation of the form  $A_0 \stackrel{\text{def}}{=} P_0$ , where  $P_0 \in \mathcal{P}_0$ .

We use the following short notations:

$$\begin{aligned} \Sigma\{P_i : i \in I\} &\equiv \begin{cases} \mathbf{0} & (I = \emptyset) \\ P_1 + P_2 + \dots + P_n & (I = \{1, 2, \dots, n\}) \end{cases} \\ \vee\{P_i : i \in I\} &\equiv \begin{cases} \mathbf{0} & (I = \emptyset) \\ P_1 \vee P_2 \vee \dots \vee P_n & (I = \{1, 2, \dots, n\}) \end{cases} \end{aligned}$$

Next, in order to define the multi-labeled transition system introduced in Section 1, an operator  $\langle \rangle$  for any set  $S$  is defined as

$$\langle S \rangle = \{\langle e_1, e_2, \dots, e_n \rangle : e_i \in S, n \geq 1\}.$$

$\langle S \rangle$  is called the *multi-set* of  $S$ . Three functions over a multi-set  $\langle S \rangle$  are defined.

**Definition 2.2** *Let  $s, s' \in \langle S \rangle$ ,  $e_i, e'_i \in S$ , and  $n, i \in \{1, 2, 3, \dots\}$ .*

- $(\#s)$  is the length of  $s$ . Thus  $\#\langle e_1, e_2, \dots, e_n \rangle = n$ .
- $(s \triangleleft i)$  is the  $i$ th element of  $s$  ( $i \in [1, \#s]$ ). Thus  $\langle e_1, e_2, \dots, e_n \rangle \triangleleft i = e_i$ .
- $(s; s')$  is the concatenation of  $s$  and  $s'$ .

Thus  $\langle e_1, e_2, \dots, e_n \rangle; \langle e'_1, e'_2, \dots, e'_m \rangle = \langle e_1, e_2, \dots, e_n, e'_1, e'_2, \dots, e'_m \rangle$ .

where  $[1, n]$  is an abbreviation of an integer set  $\{1, 2, \dots, n\}$ . ■

Then we define multi-labeled transition systems (multi-LTS).

**Definition 2.3** *A multi-LTS is a triple  $(S, L, \rightarrow)$  where*

1.  $S$  is a set of states,
2.  $L$  is a set of labels,
3.  $\rightarrow$  is a set of transition relations such that

$$\rightarrow \subseteq \{(e, u, s) : e \in S, u \in \langle L \rangle, s \in \langle S \rangle, \#u = \#s\}$$

We write  $e \xrightarrow{u} s$  for  $(e, u, s) \in \rightarrow$ . ■

The semantics of  $SPEC^V$  is given by the multi-LTS  $(\mathcal{P}, Act, \rightarrow)$ , where  $\rightarrow$  is defined in Definition 2.4. The set of *multi-specifications*  $\langle \mathcal{P} \rangle$  is ranged over by  $M, N, \dots$  and the set of *multi-actions*  $\langle Act \rangle$  is ranged over by  $\mu, \nu, \dots$ .

**Definition 2.4** *The transition relation  $\rightarrow$  between a specification  $P$ , a multi-action  $\mu$ , and a multi-specification  $M$  is the smallest relation satisfying the following inference rules. Each rule is to be read as follows: if the transition relation(s) above the line are inferred and the side condition(s) are satisfied, then the transition relation below the line can be also inferred.*

$$\begin{array}{l}
\text{Act} \frac{}{\alpha.P \xrightarrow{(\alpha)} \langle P \rangle} \qquad \text{Choice}_1 \frac{P \xrightarrow{\mu} M}{P + Q \xrightarrow{\mu} M} \\
\text{Con} \frac{P \xrightarrow{\mu} M}{A \xrightarrow{\mu} M} (A \stackrel{\text{def}}{=} P) \qquad \text{Choice}_2 \frac{Q \xrightarrow{\mu} N}{P + Q \xrightarrow{\mu} N} \\
\text{Andor} \frac{P \xrightarrow{\mu} M \quad Q \xrightarrow{\nu} N}{P \vee Q \xrightarrow{\mu; \nu} M; N}
\end{array}$$

where ‘;’ used in **Andor** is the concatenation function given above. ■

### 3 Partial equalities

In this section two kinds of partial equalities in  $SPEC^V$  are defined based on extended bisimulation relations.

First, we define a function for filtering actions.

**Definition 3.1** *Filter function  $(/ : Act \times 2^{\mathcal{N}} \rightarrow Act)$  is defined as follows: For any  $\alpha \in Act$  and  $\Omega \subseteq \mathcal{N}$ ,*

$$\alpha/\Omega = \begin{cases} \alpha & (\alpha \in \Omega) \\ \tau & (\text{otherwise}) \end{cases}$$
■

If  $\alpha \in \Omega$ , then  $\alpha$  is called a *valid action* for  $\Omega$ , otherwise  $\alpha$  is called an *invalid action* for  $\Omega$ .

Next we define a function called *Selective function* to select the valid action from two actions observed through two filters. If two actions are inconsistent with each other, then Selective function returns the symbol  $\perp$  ( $\notin Act$ ).

**Definition 3.2** *Selective function  $\bullet : Act \times 2^{\mathcal{N}} \times Act \times 2^{\mathcal{N}} \rightarrow Act \cup \{\perp\}$  is defined as follows: For any  $\alpha_i \in Act$  and  $\Omega_i \subseteq \mathcal{N}$ ,*

$$(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \begin{cases} \alpha_1/\Omega_1 & (\alpha_1/\Omega_1 = \alpha_2/\Omega_2) & (C1) \\ \alpha_1 & (\alpha_1 \in \Omega_1 - \Omega_2, \alpha_2 \notin \Omega_2) & (C2) \\ \alpha_2 & (\alpha_2 \in \Omega_2 - \Omega_1, \alpha_1 \notin \Omega_1) & (C3) \\ \perp & (\text{otherwise}) & (C4) \end{cases}$$
■

(C2) represents a selection of the valid action  $\alpha_1$ . The condition  $(\alpha_1 \in \Omega_1 - \Omega_2)$  of (C2) represents that  $\alpha_1$  must be invalid for  $\Omega_2$ , in spite of  $\alpha_2 \notin \Omega_2$ , because  $\alpha_2$  should be equal

to  $\alpha_1$  if  $\alpha_1 \in \Omega_2$ . (C4) represents the three cases that (1)  $\alpha_1 \in \Omega_1 \cap \Omega_2$  and  $\alpha_1 \neq \alpha_2$  or (2)  $\alpha_2 \in \Omega_1 \cap \Omega_2$  and  $\alpha_1 \neq \alpha_2$  or (3)  $\alpha_1 \in \Omega_1$ ,  $\alpha_2 \in \Omega_2$ , and  $\alpha_1 \neq \alpha_2$ .

For example, the following applications show properties of Selective function.

$$\begin{array}{l|l} (a, ab) \bullet (a, ac) = a & \text{by (C1)} & (b, ab) \bullet (e, ac) = b & \text{by (C2)} \\ (d, ab) \bullet (e, ac) = \tau & \text{by (C1)} & (a, ab) \bullet (\tau, ac) = \perp & \text{by (C4)} \\ (\tau, ab) \bullet (e, ac) = \tau & \text{by (C1)} & (b, ab) \bullet (c, ac) = \perp & \text{by (C4)} \end{array}$$

Then,  $(\Omega_1, \Omega_2)$ -consistency about actions is defined as follows. If two actions  $\alpha_1$  and  $\alpha_2$  observed through two filters  $\Omega_1$  and  $\Omega_2$ , respectively, are consistent with each other by means of Selective function, then they are  $(\Omega_1, \Omega_2)$ -consistent and it is denoted by  $(\alpha_1 \Omega_1 \dot{\equiv} \Omega_2 \alpha_2)$ , thus

$$\alpha_1 \dot{\equiv} \Omega_2 \alpha_2 = \{(\alpha_1, \alpha_2) : (\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) \neq \perp\}$$

We show several properties of Synthetic function.

**Proposition 3.1** For any  $\alpha_i \in \text{Act}$ ,  $\Omega_i \subseteq \mathcal{N}$ ,

- (1) If  $(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha$ , then  $(\alpha_1, \Omega_1) \bullet (\alpha, \Omega_{12}) = \alpha$ .
- (2) If  $(\alpha_1 \Omega_1 \dot{\equiv} \Omega_2 \alpha)$ ,  $(\alpha \Omega \dot{\equiv} \Omega_2 \alpha_2)$ , and  $\Omega_1 \subseteq \Omega$ , then  $(\alpha_1 \Omega_1 \dot{\equiv} \Omega_2 \alpha_2)$ .
- (3) If  $(\alpha_1 \Omega_1 \dot{\equiv} \Omega_2 \alpha)$ ,  $(\alpha \Omega \dot{\equiv} \Omega_2 \alpha_2)$ , and  $\Omega_{12} \subseteq \Omega$ , then  $(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha / \Omega_{12}$ .
- (4)  $((\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2), \Omega_{12}) \bullet (\alpha_3, \Omega_3) = (\alpha_1, \Omega_1) \bullet ((\alpha_2, \Omega_2) \bullet (\alpha_3, \Omega_3), \Omega_{23})$

where  $\Omega_{12} = \Omega_1 \cup \Omega_2$  and  $\Omega_{23} = \Omega_2 \cup \Omega_3$ . ■

(1) means that the selected action  $\alpha$  is consistent with  $\alpha_1$ . (2) is conditional transitivity. (3) is useful for synthesis of two partial specifications. (4) means that the selected action does not depend on the order of the selections.

Now we define two kinds of partial equalities by using  $(\Omega_1, \Omega_2)$ -consistency.

**Definition 3.3** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ . A binary relation  $\mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$  over specifications is a strong  $(\Omega_1, \Omega_2)$ -bisimulation, if  $(P, Q) \in \mathcal{S}$  implies that

- (i) whenever  $P \xrightarrow{\mu} M$  then, for some  $N$  and  $\nu$ ,  $Q \xrightarrow{\nu} N$  and for some  $i$  and  $j$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ ,
- (ii) whenever  $Q \xrightarrow{\nu} N$  then, for some  $M$  and  $\mu$ ,  $P \xrightarrow{\mu} M$  and for some  $i$  and  $j$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ . ■

**Definition 3.4** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ .  $P$  and  $Q$  are strongly  $(\Omega_1, \Omega_2)$ -equivalent, written  $P \Omega_1 \sim \Omega_2 Q$ , if  $(P, Q) \in \mathcal{S}$  for some strong  $(\Omega_1, \Omega_2)$ -bisimulation  $\mathcal{S}$ . ■

**Definition 3.5** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ . A binary relation  $\mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$  over specifications is a strong  $(\Omega_1, \Omega_2)$ -full-bisimulation, if  $(P, Q) \in \mathcal{S}$  implies that

- (i) whenever  $P \xrightarrow{\mu} M$  then, for some  $N$  and  $\nu$ ,  $Q \xrightarrow{\nu} N$  and
  - (a) for all  $i \in [1, \#\mu]$ , for some  $j$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ ,
  - (b) for all  $j \in [1, \#\nu]$ , for some  $i$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ ,
- (ii) whenever  $Q \xrightarrow{\nu} N$  then, for some  $M$  and  $\mu$ ,  $P \xrightarrow{\mu} M$  and
  - (a) for all  $i \in [1, \#\mu]$ , for some  $j$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ ,
  - (b) for all  $j \in [1, \#\nu]$ , for some  $i$ ,  $(\mu \triangleleft i \Omega_1 \dot{\equiv} \Omega_2 \nu \triangleleft j)$  and  $(M \triangleleft i, N \triangleleft j) \in \mathcal{S}$ . ■

**Definition 3.6** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ .  $P$  and  $Q$  are strongly  $(\Omega_1, \Omega_2)$ -full-equivalent, written  $P \Omega_1 \simeq_{\Omega_2} Q$ , if  $(P, Q) \in \mathcal{S}$  for some strong  $(\Omega_1, \Omega_2)$ -full-bisimulation  $\mathcal{S}$ . ■

The following relations clearly hold from the definitions.

- For any  $P, Q \in \mathcal{P}$ ,  $P \Omega_1 \simeq_{\Omega_2} Q \Rightarrow P \Omega_1 \sim_{\Omega_2} Q$
- For any  $P_0, Q_0 \in \mathcal{P}_0$ ,  $P_0 \Omega_1 \simeq_{\Omega_2} Q_0 \Leftrightarrow P_0 \Omega_1 \sim_{\Omega_2} Q_0$
- For any  $P_0, Q_0 \in \mathcal{P}_0$ ,  $P_0 \mathcal{N} \sim_{\mathcal{N}} Q_0 \Leftrightarrow P_0 \sim Q_0$

where  $\sim$  is strong equivalence defined in [1]. Although neither  $\Omega_1 \sim_{\Omega_2}$  nor  $\Omega_1 \simeq_{\Omega_2}$  is an equivalence relation, the following conditional reflexive, symmetric, and transitive laws hold for  $\Omega_1 \sim_{\Omega_2}$ .

**Proposition 3.2** For any  $P, Q \in \mathcal{P}$ ,  $\Omega_i \subseteq \mathcal{N}$ , and  $R_0 \in \mathcal{P}_0$ ,

- (1)  $P \Omega_1 \sim_{\Omega_2} P$
- (2) If  $P \Omega_1 \sim_{\Omega_2} Q$ , then  $Q \Omega_2 \sim_{\Omega_1} P$
- (3) If  $P \Omega_1 \sim_{\Omega} R_0$ ,  $R_0 \Omega \sim_{\Omega_2} Q$ , and  $\Omega_1 \subseteq \Omega$ , then  $P \Omega_1 \sim_{\Omega_2} Q$ . ■

Proposition 3.2 also holds if  $\Omega_1 \sim_{\Omega_2}$  is replaced to  $\Omega_1 \simeq_{\Omega_2}$ . Furthermore a medium  $R_0$  of (3) does not have to be a *ground* specification for  $\Omega_1 \simeq_{\Omega_2}$ , thus for any  $R \in \mathcal{P}$ , if  $P \Omega_1 \simeq_{\Omega} R$ ,  $R \Omega \simeq_{\Omega_2} Q$ , and  $\Omega_1 \subseteq \Omega$ , then  $P \Omega_1 \simeq_{\Omega_2} Q$ .

## 4 Synthetic specification

In this section, we give a synthesis method of specifications preserving  $\Omega_1 \sim_{\Omega_2}$ . We explain how to synthesize specifications by using the following example.

$$\begin{aligned} A_c &\equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} & B_2 &\equiv a.\tau.\mathbf{0} + \tau.\tau.\mathbf{0} + \tau.c.\mathbf{0} \\ B_1 &\equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.\tau.\mathbf{0} & B_3 &\equiv \tau.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} \end{aligned} \quad (*_2)$$

where  $\equiv$  represents syntactic identity.  $B_1, B_2$ , and  $B_3$  are partial specifications of the complete specification  $A_c$ , thus for any  $i, j \in \{1, 2, 3\}$ ,

$$A_c \Omega \sim_{\Omega_i} B_i \text{ and } B_i \Omega_i \sim_{\Omega_j} B_j$$

where  $\Omega = \{a, b, c\}$ ,  $\Omega_1 = \{a, b\}$ ,  $\Omega_2 = \{a, c\}$ , and  $\Omega_3 = \{b, c\}$ .

At first, we consider *only* ground specifications for simplicity. A simple synthesis method is to use the following combinator.

$$\text{SYN} \frac{Q_1 \xrightarrow{\langle \alpha_1 \rangle} \langle Q'_1 \rangle \quad Q_2 \xrightarrow{\langle \alpha_2 \rangle} \langle Q'_2 \rangle}{Q_1 \Omega_1 \parallel_{\Omega_2} Q_2 \xrightarrow{\langle \alpha \rangle} \langle Q'_1 \Omega_1 \parallel_{\Omega_2} Q'_2 \rangle} ((\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha)$$

This synthetic specification  $Q_1 \Omega_1 \parallel_{\Omega_2} Q_2$  performs valid actions of  $Q_1$  and  $Q_2$ , but the following expected equation does not always hold.

$$Q_1 \Omega_1 \parallel_{\Omega_2} Q_2 \Omega_{12} \sim_{\Omega_i} Q_i \quad (i \in \{1, 2\}) \quad (*_3)$$

where  $\Omega_{12} = \Omega_1 \cup \Omega_2$ . For the example  $(*_2)$ , the result of synthesis of  $B_1$  and  $B_2$  by  $\Omega_1 \parallel_{\Omega_2}$  is shown as follows:



$$B_1 ab \parallel_{ac} B_2 \sim a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + b.c.\mathbf{0} + \tau.\tau.\mathbf{0} + b.\mathbf{0} + \tau.\mathbf{0}$$

The last part ( $b.\mathbf{0} + \tau.\mathbf{0}$ ) is important and breaks the expected equation ( $*_3$ ).

An easy method to guarantee ( $*_3$ ) may be to modify the combinator  $\Omega_1 \parallel \Omega_2$  as follows:

$$\mathbf{GLB} \frac{Q_1 \xrightarrow{\langle \alpha_1 \rangle} \langle Q'_1 \rangle \quad Q_2 \xrightarrow{\langle \alpha_2 \rangle} \langle Q'_2 \rangle}{Q_1 \Omega_1 \parallel \Omega_2 Q_2 \xrightarrow{\langle \alpha \rangle} \langle Q'_1 \Omega_1 \parallel \Omega_2 Q'_2 \rangle} \left( \begin{array}{l} (\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha, \\ Q'_1 \Omega_1 \sim \Omega_2 Q'_2 \end{array} \right),$$

but this definition circulates, because  $\Omega_1 \sim \Omega_2$  should be defined in terms of our transition relations. Therefore, a relation is defined instead of the combinator.

**Definition 4.1** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ . A triadic relation  $\mathcal{T}$  over ground specifications is a strong  $(\Omega_1, \Omega_2)$ -GLB relation, if  $(P, Q_1, Q_2) \in \mathcal{T}$  implies that

- (i) whenever  $P \xrightarrow{\langle \alpha \rangle} \langle P' \rangle$  then, for some  $Q'_1, Q'_2, \alpha_1$ , and  $\alpha_2$ ,  
 $Q_1 \xrightarrow{\langle \alpha_1 \rangle} \langle Q'_1 \rangle$ ,  $Q_2 \xrightarrow{\langle \alpha_2 \rangle} \langle Q'_2 \rangle$ ,  $(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha$ ,  $Q'_1 \Omega_1 \sim \Omega_2 Q'_2$ , and  $(P', Q'_1, Q'_2) \in \mathcal{T}$ ,
- (ii) whenever  $Q_1 \xrightarrow{\langle \alpha_1 \rangle} \langle Q'_1 \rangle$ ,  $Q_2 \xrightarrow{\langle \alpha_2 \rangle} \langle Q'_2 \rangle$ ,  $(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha \neq \perp$ , and  $Q'_1 \Omega_1 \sim \Omega_2 Q'_2$ ,  
then, for some  $P'$ ,  $P \xrightarrow{\langle \alpha \rangle} \langle P' \rangle$  and  $(P', Q'_1, Q'_2) \in \mathcal{T}$ .

■

**Definition 4.2** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ .  $P$  is the strong  $(\Omega_1, \Omega_2)$ -GLB specification of  $Q_1$  and  $Q_2$ , written  $P \sim (Q_1 \Omega_1 \parallel \Omega_2 Q_2)$ , if  $(P, Q_1, Q_2) \in \mathcal{T}$  for some strong  $(\Omega_1, \Omega_2)$ -GLB relation  $\mathcal{T}$ .

■

The strong  $(\Omega_1, \Omega_2)$ -GLB specification  $P$  of  $Q_1$  and  $Q_2$  is uniquely decided from  $Q_1$  and  $Q_2$  up to  $\mathcal{N} \sim \mathcal{N}$ , and the following relation holds.

$$\text{If } P \sim (Q_1 \Omega_1 \parallel \Omega_2 Q_2) \text{ and } Q_1 \Omega_1 \sim \Omega_2 Q_2, \text{ then } P \Omega_1 \cup \Omega_2 \sim \Omega_i Q_i \text{ (} i \in \{1, 2\} \text{)} \quad (*_4)$$

For the example ( $*_2$ ), we can obtain the strong  $(ab, ac)$ -GLB specification  $A_{12}$  such that  $A_{12} \sim (B_1 ab \parallel_{ac} B_2)$ , considering Definition 4.2.

$$A_{12} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + b.c.\mathbf{0} + \tau.\tau.\mathbf{0}$$

Then  $A_{12} abc \sim_{ab} B_1$  and  $A_{12} abc \sim_{ac} B_2$  by ( $*_4$ ).

Unfortunately strong  $(\Omega_1, \Omega_2)$ -GLB specifications are not useful for synthesis of three or more specifications, namely stepwise refinement. For example, ( $*_4$ ) can not be used for the synthesis of  $A_{12}$  and  $B_3$ , because  $A_{12} abc \not\sim_{bc} B_3$ .

The problem is that there are many synthetic specifications like  $A'$  satisfying  $A' abc \sim_{ab} B_1$  and  $A' abc \sim_{ac} B_2$ . The complete specification  $A_c$  is one of them. Therefore it is expected to produce a comprehensive specification which includes all the synthetic specifications like  $A'$ . For the purpose a multi-LTS is needed and  $SPEC^\vee$  is used.

Now we define principal strong  $(\Omega_1, \Omega_2)$ -synthetic specifications in  $SPEC^\vee$ .

**Definition 4.3** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ . A triple relation  $\mathcal{T} \subseteq \mathcal{P}^3$  over specifications is a principal strong  $(\Omega_1, \Omega_2)$ -synthetic relation, if  $(P, Q_1, Q_2) \in \mathcal{T}$  implies that

(i) for all  $M$  and  $\mu$ , whenever  $P \xrightarrow{\mu} M$  then,

(1) for some  $(N_1, \nu_1)$ ,  $Q_1 \xrightarrow{\nu_1} N_1$  and

(a) for all  $(N'_2, \nu'_2, j', k')$ ,

whenever  $Q_2 \xrightarrow{\nu'_2} N'_2$ ,  $\nu_1 \triangleleft j' \Omega_1 \doteq \Omega_2 \nu'_2 \triangleleft k'$ , and  $N_1 \triangleleft j' \Omega_1 \sim \Omega_2 N'_2 \triangleleft k'$ , then for some  $i$ ,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \mu \triangleleft i$  and  $(M \triangleleft i, N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ ,

and

(b) for all  $i' \in [1, \#\mu]$ , for some  $(N'_2, \nu'_2, j', k')$ ,  $N_1 \triangleleft j' \Omega_1 \sim \Omega_2 N'_2 \triangleleft k'$ ,

$Q_2 \xrightarrow{\nu'_2} N'_2$ ,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \mu \triangleleft i'$ , and  $(M \triangleleft i', N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ ,

or

(2) for some  $(N_2, \nu_2)$ ,  $Q_2 \xrightarrow{\nu_2} N_2$  and

(a) for all  $(N'_1, \nu'_1, j', k')$ ,

whenever  $Q_1 \xrightarrow{\nu'_1} N'_1$ ,  $\nu'_1 \triangleleft j' \Omega_1 \doteq \Omega_2 \nu_2 \triangleleft k'$ , and  $N'_1 \triangleleft j' \Omega_1 \sim \Omega_2 N_2 \triangleleft k'$ , then for some  $i$ ,  $(\nu'_1 \triangleleft j', \Omega_1) \bullet (\nu_2 \triangleleft k', \Omega_2) = \mu \triangleleft i$  and  $(M \triangleleft i, N'_1 \triangleleft j', N_2 \triangleleft k') \in \mathcal{T}$ ,

and

(b) for all  $i' \in [1, \#\mu]$ , for some  $(N'_1, \nu'_1, j', k')$ ,  $N'_1 \triangleleft j' \Omega_1 \sim \Omega_2 N_2 \triangleleft k'$ ,

$Q_1 \xrightarrow{\nu'_1} N'_1$ ,  $(\nu'_1 \triangleleft j', \Omega_1) \bullet (\nu_2 \triangleleft k', \Omega_2) = \mu \triangleleft i'$ , and  $(M \triangleleft i', N'_1 \triangleleft j', N_2 \triangleleft k') \in \mathcal{T}$ ,

and

(ii) for all  $(N_1, N_2, \nu_1, \nu_2, j, k)$ ,

whenever  $Q_1 \xrightarrow{\nu_1} N_1$ ,  $Q_2 \xrightarrow{\nu_2} N_2$ ,  $\nu_1 \triangleleft j \Omega_1 \doteq \Omega_2 \nu_2 \triangleleft k$ ,  $N_1 \triangleleft j \Omega_1 \sim \Omega_2 N_2 \triangleleft k$ , then

(1) for some  $(M, \mu)$ ,  $P \xrightarrow{\mu} M$  and

(a) for all  $(N'_2, \nu'_2, j', k')$ ,

whenever  $Q_2 \xrightarrow{\nu'_2} N'_2$ ,  $\nu_1 \triangleleft j' \Omega_1 \doteq \Omega_2 \nu'_2 \triangleleft k'$ , and  $N_1 \triangleleft j' \Omega_1 \sim \Omega_2 N'_2 \triangleleft k'$ , then for some  $i$ ,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \mu \triangleleft i$  and  $(M \triangleleft i, N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ ,

and

(b) for all  $i' \in [1, \#\mu]$ , for some  $(N'_2, \nu'_2, j', k')$ ,  $N_1 \triangleleft j' \Omega_1 \sim \Omega_2 N'_2 \triangleleft k'$ ,

$Q_2 \xrightarrow{\nu'_2} N'_2$ ,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \mu \triangleleft i'$ , and  $(M \triangleleft i', N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ ,

and

(2) for some  $(M, \mu)$ ,  $P \xrightarrow{\mu} M$  and

(a) for all  $(N'_1, \nu'_1, j', k')$ ,

whenever  $Q_1 \xrightarrow{\nu'_1} N'_1$ ,  $\nu'_1 \triangleleft j' \Omega_1 \doteq \Omega_2 \nu_2 \triangleleft k'$ , and  $N'_1 \triangleleft j' \Omega_1 \sim \Omega_2 N_2 \triangleleft k'$ , then for some  $i$ ,  $(\nu'_1 \triangleleft j', \Omega_1) \bullet (\nu_2 \triangleleft k', \Omega_2) = \mu \triangleleft i$  and  $(M \triangleleft i, N'_1 \triangleleft j', N_2 \triangleleft k') \in \mathcal{T}$ ,

and

(b) for all  $i' \in [1, \#\mu]$ , for some  $(N'_1, \nu'_1, j', k')$ ,  $N'_1 \triangleleft j' \Omega_1 \sim \Omega_2 N_2 \triangleleft k'$ ,

$Q_1 \xrightarrow{\nu'_1} N'_1$ ,  $(\nu'_1 \triangleleft j', \Omega_1) \bullet (\nu_2 \triangleleft k', \Omega_2) = \mu \triangleleft i'$ , and  $(M \triangleleft i', N'_1 \triangleleft j', N_2 \triangleleft k') \in \mathcal{T}$ . ■

**Definition 4.4** Let  $\Omega_1, \Omega_2 \subseteq \mathcal{N}$ .  $P$  is the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification of  $Q_1$  and  $Q_2$ , written  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ , if  $(P, Q_1, Q_2) \in \mathcal{T}$  for some principal strong  $(\Omega_1, \Omega_2)$ -synthetic relation  $\mathcal{T}$ . ■

Definition 4.3 bases on the idea of Definition 4.1, thus selection of valid actions and preservation of  $\Omega_1 \sim \Omega_2$ .

Then the expected relation is obtained.

**Proposition 4.1** Let  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ ,  $Q_1 \Omega_1 \sim_{\Omega_2} Q_2$ , and  $\Omega_{12} = \Omega_1 \cup \Omega_2$ . For any ground specifications  $P_0 \in \mathcal{P}_0$ ,

$$P_0 \Omega_{12} \sim_{\Omega_{12}} P \quad \text{iff} \quad P_0 \Omega_{12} \sim_{\Omega_1} Q_1 \quad \text{and} \quad P_0 \Omega_{12} \sim_{\Omega_2} Q_2.$$

**Proof**

( $\Rightarrow$ ) We show that the following  $\mathcal{S}$  is a strong  $(\Omega_{12}, \Omega_1)$ -bisimulation.

$$\mathcal{S} = \{(P_0, Q_1) : P_0 \in \mathcal{P}_0, \exists P, Q_2 \in \mathcal{P}, P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2), Q_1 \Omega_1 \sim_{\Omega_2} Q_2, P_0 \Omega_{12} \sim_{\Omega_{12}} P\}$$

Let  $(P_0, Q_1) \in \mathcal{S}$ . Thus,  $P_0$  is a ground specification, and for some  $P$  and  $Q_2$ ,  $Q_1 \Omega_1 \sim_{\Omega_2} Q_2$ ,  $P_0 \Omega_{12} \sim_{\Omega_{12}} P$ , and  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ .

- (i) Let  $P_0 \xrightarrow{(\alpha)} \langle P'_0 \rangle$ . Since  $P_0 \Omega_{12} \sim_{\Omega_{12}} P$ , for some  $M$  and  $\mu$ , we have that  $P \xrightarrow{\mu} M$ , for some  $i$ ,  $(\alpha \Omega_{12} \dot{=} \mu \triangleleft i)$  and  $P'_0 \Omega_{12} \sim_{\Omega_{12}} M \triangleleft i$ . This implies the following either case.
  - By Definition 4.3(i)(1)(b), since  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ , for some  $N_1$  and  $\nu_1$ , we have  $Q_1 \xrightarrow{\nu_1} N_1$  and for some  $N_2$ ,  $\nu_2$ ,  $j$ , and  $k$ ,  $Q_2 \xrightarrow{\nu_2} N_2$ ,  $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \mu \triangleleft i$ ,  $N_1 \triangleleft j \Omega_1 \sim_{\Omega_2} N_2 \triangleleft k$ , and  $M \triangleleft i \simeq (N_1 \triangleleft j \Omega_1 \sqcap \Omega_2 N_2 \triangleleft k)$ . Thus  $(P'_0, N_1 \triangleleft j) \in \mathcal{S}$ . Here, by Proposition 3.1(1),  $(\nu_1 \triangleleft j \Omega_1 \dot{=} \mu \triangleleft i)$ . Hence, by Proposition 3.1(2),  $(\alpha \Omega_{12} \dot{=} \nu_1 \triangleleft j)$ .
  - The case by Definition 4.3(i)(2)(b) can be shown by the same argument as the above case.
- (ii) Let  $Q_1 \xrightarrow{\nu_1} N_1$ . Since  $Q_1 \Omega_1 \sim_{\Omega_2} Q_2$ , for some  $N_2$  and  $\nu_2$ , we have that  $Q_2 \xrightarrow{\nu_2} N_2$ , for some  $j$  and  $k$ ,  $(\nu_1 \triangleleft j \Omega_1 \dot{=} \nu_2 \triangleleft k)$  and  $N_1 \triangleleft j \Omega_1 \sim_{\Omega_2} N_2 \triangleleft k$ . By Definition 4.3(ii)(1), since  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ , for some  $M$  and  $\mu$ , we have  $P \xrightarrow{\mu} M$ . Since  $P_0 \Omega_{12} \sim_{\Omega_{12}} P$ , for some  $P'_0$  and  $\alpha$ , we have that  $P_0 \xrightarrow{(\alpha)} \langle P'_0 \rangle$ , for some  $i \in [1, \#\mu]$ ,  $(\alpha \Omega_{12} \dot{=} \mu \triangleleft i)$  and  $P'_0 \Omega_{12} \sim_{\Omega_{12}} M \triangleleft i$ . By Definition 4.3(ii)(1)(b), for some  $N'_2$  and  $\nu'_2$ ,  $j'$ , and  $k'$ ,  $Q_2 \xrightarrow{\nu'_2} N'_2$ ,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \mu \triangleleft i$ ,  $N_1 \triangleleft j' \Omega_1 \sim_{\Omega_2} N'_2 \triangleleft k'$ , and  $M \triangleleft i \simeq (N_1 \triangleleft j' \Omega_1 \sqcap \Omega_2 N'_2 \triangleleft k')$ . Thus,  $(P'_0, N_1 \triangleleft j') \in \mathcal{S}$ . Here, by Proposition 3.1(1),  $(\nu_1 \triangleleft j' \Omega_1 \dot{=} \mu \triangleleft i)$ . Hence, by Proposition 3.1(2),  $(\alpha \Omega_{12} \dot{=} \nu_1 \triangleleft j')$ .

( $\Leftarrow$ ) We show that the following  $\mathcal{S}$  is a strong  $(\Omega_{12}, \Omega_{12})$ -bisimulation.

$$\mathcal{S} = \{(P_0, P) : P_0 \in \mathcal{P}_0, \exists Q_1, Q_2 \in \mathcal{P}, P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2), P_0 \Omega_{12} \sim_{\Omega_1} Q_1, P_0 \Omega_{12} \sim_{\Omega_2} Q_2\}$$

Let  $(P_0, P) \in \mathcal{S}$ . Thus,  $P_0$  is a ground specification, and for some  $Q_1$  and  $Q_2$ ,  $P_0 \Omega_{12} \sim_{\Omega_1} Q_1$ ,  $P_0 \Omega_{12} \sim_{\Omega_2} Q_2$ , and  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ .

- (i) Let  $P_0 \xrightarrow{(\alpha)} \langle P'_0 \rangle$ . For each  $n \in \{1, 2\}$ , since  $P_0 \Omega_{12} \sim_{\Omega_n} Q_n$ , for some  $N_n$  and  $\nu_n$ , we have that  $Q_n \xrightarrow{\nu_n} N_n$ , for some  $j$  and  $k$ ,  $(\alpha \Omega_{12} \dot{=} \nu_1 \triangleleft j)$ ,  $(\alpha \Omega_{12} \dot{=} \nu_2 \triangleleft k)$ ,  $P'_0 \Omega_{12} \sim_{\Omega_1} N_1 \triangleleft j$ , and  $P'_0 \Omega_{12} \sim_{\Omega_2} N_2 \triangleleft k$ . By Proposition 3.1(2),  $(\nu_1 \triangleleft j \Omega_1 \dot{=} \nu_2 \triangleleft k)$ . By Proposition 3.2(3),  $N_1 \triangleleft j \Omega_1 \sim_{\Omega_2} N_2 \triangleleft k$ . Thus, by Definition 4.3(ii)(1)(a), since  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ , for some  $M$  and  $\mu$ , we have  $P \xrightarrow{\mu} M$  and for some  $i$ ,  $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \mu \triangleleft i$  and  $M \triangleleft i \simeq (N_1 \triangleleft j \Omega_1 \sqcap \Omega_2 N_2 \triangleleft k)$ . Thus  $(P'_0, M \triangleleft i) \in \mathcal{S}$ . Here, by Proposition 3.1(3),  $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \alpha / \Omega_{12}$ . Hence,  $(\alpha \Omega_{12} \dot{=} \mu \triangleleft i)$ , because  $\alpha / \Omega_{12} = \mu \triangleleft i$ .
- (ii) Let  $P \xrightarrow{\mu} M$ . This implies the following either case.

- By Definition 4.3(i)(1), since  $P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ , for some  $N_1$  and  $\nu_1$ , we have  $Q_1 \xrightarrow{\nu_1} N_1$ . Since  $P_0 \Omega_{12} \sim_{\Omega_1} Q_1$ , for some  $\alpha$  and  $P'_0$ , we have that  $P_0 \xrightarrow{\langle \alpha \rangle} \langle P'_0 \rangle$ , for some  $j$ ,  $(\alpha \Omega_{12} \doteq_{\Omega_1} \nu_1 \triangleleft j)$  and  $P'_0 \Omega_{12} \sim_{\Omega_1} N_1 \triangleleft j$ . Since  $P_0 \Omega_{12} \sim_{\Omega_2} Q_2$ , for some  $N_2$  and  $\nu_2$ , we have that  $Q_2 \xrightarrow{\nu_2} N_2$ , for some  $k$ ,  $(\alpha \Omega_{12} \doteq_{\Omega_2} \nu_2 \triangleleft k)$  and  $P'_0 \Omega_{12} \sim_{\Omega_2} N_2 \triangleleft k$ . By Proposition 3.1(3),  $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \alpha / \Omega_{12}$ . By Proposition 3.2(3),  $N_1 \triangleleft j \Omega_1 \sim_{\Omega_2} N_2 \triangleleft k$ . By Definition 4.3(i)(1)(a), for some  $i$ ,  $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \mu \triangleleft i$  and  $M \triangleleft i \simeq (N_1 \triangleleft j \Omega_1 \sqcap \Omega_2 N_2 \triangleleft k)$ . Thus,  $(P'_0, M \triangleleft i) \in \mathcal{S}$  and  $(\alpha \Omega_{12} \doteq_{\Omega_{12}} \mu \triangleleft i)$ .
- The case by Definition 4.3(i)(2) can be shown by a symmetric argument with the above case. ■

Next, we give two important propositions. Proposition 4.2 shows that the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification  $P$  of two specifications  $Q_1$  and  $Q_2$  is uniquely decided from  $Q_1$  and  $Q_2$  up to  $\mathcal{N} \simeq_{\mathcal{N}}$ .

**Proposition 4.2** *Let  $P_{12} \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ . For any specification  $P \in \mathcal{P}$ ,*

$$P_{12} \mathcal{N} \simeq_{\mathcal{N}} P \quad \text{iff} \quad P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2).$$

**Proof** We show that the following  $\mathcal{T}$  is a principal strong  $(\Omega_1, \Omega_2)$ -synthetic relation and  $\mathcal{S}$  is a strong  $(\mathcal{N}, \mathcal{N})$ -full-bisimulation.

$$\begin{aligned} \mathcal{T} &= \{(P, Q_1, Q_2) : \exists P_{12} \in \mathcal{P}, P_{12} \mathcal{N} \simeq_{\mathcal{N}} P, P_{12} \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)\} \\ \mathcal{S} &= \{(P_{12}, P) : \exists Q_1, Q_2 \in \mathcal{P}, P \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2), P_{12} \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)\} \end{aligned}$$

The proof is omitted because of lack of space. ■

Proposition 4.3 shows how to produce the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specification of  $Q_1$  and  $Q_2$ . In fact,  $\mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2)$  can be efficiently produced from  $Q_1$  and  $Q_2$ , if  $Q_1$  and  $Q_2$  have finite states.

**Proposition 4.3**  *$\mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2)$  is defined as follows:*

$$\begin{aligned} \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2) \equiv & \sum \{ \mathcal{V} \{ \alpha . \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q'_1, Q'_2) : (\alpha, Q'_1, Q'_2) \in E_{\Omega_2}^{\Omega_1}(\nu_1, N_1, Q_2) \} \\ & : (\nu_1, N_1) \in D(Q_1) \} \\ & + \sum \{ \mathcal{V} \{ \alpha . \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q'_1, Q'_2) : (\alpha, Q'_2, Q'_1) \in E_{\Omega_2}^{\Omega_1}(\nu_2, N_2, Q_1) \} \\ & : (\nu_2, N_2) \in D(Q_2) \} \end{aligned}$$

where

$$\begin{aligned} D(P) &= \{(\mu, M) : P \xrightarrow{\mu} M\} \\ E_{\Omega_2}^{\Omega_1}(\mu, M, Q) &= \{(\alpha, P', Q') : \exists (\nu, N, i, j), Q \xrightarrow{\nu} N, (\mu \triangleleft i, \Omega_1) \bullet (\nu \triangleleft j, \Omega_2) = \alpha, \\ & \quad M \triangleleft i \Omega_1 \sim_{\Omega_2} N \triangleleft j, P' \equiv M \triangleleft i, Q' \equiv N \triangleleft j\} \end{aligned}$$

Then,  $\mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2) \simeq (Q_1 \Omega_1 \sqcap \Omega_2 Q_2)$ .

**Proof** We show that the following  $\mathcal{T}$  is a principal strong  $(\Omega_1, \Omega_2)$ -synthetic relation.

$$\mathcal{T} = \{(\mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2), Q_1, Q_2) : Q_1, Q_2 \in \mathcal{P}\}$$

Let  $(P, Q_1, Q_2) \in \mathcal{T}$ . Thus  $P \equiv \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2)$ .

(i) Let  $\mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2) \xrightarrow{\mu} M$ . By **Choice**<sub>1,2</sub>, it implies the following either case (1) or (2).

(1) For some  $(\nu_1, N_1) \in D(Q_1)$ ,

$$\mathcal{V} \{ \alpha . \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q'_1, Q'_2) : (\alpha, Q'_1, Q'_2) \in E_{\Omega_2}^{\Omega_1}(\nu_1, N_1, Q_2) \} \xrightarrow{\mu} M.$$

We have  $Q_1 \xrightarrow{\nu_1} N_1$ , because  $(\nu_1, N_1) \in D(Q_1)$ .

- (a) Let  $N'_2, \nu'_2, j'$ , and  $k'$  such that  $Q_2 \xrightarrow{\nu'_2} N'_2, \nu_1 \triangleleft j' \Omega_1 \doteq_{\Omega_2} \nu'_2 \triangleleft k'$ , and  $N_1 \triangleleft j' \Omega_1 \sim_{\Omega_2} N'_2 \triangleleft k'$ . In this case,  $(\alpha, N_1 \triangleleft j', N'_2 \triangleleft k') \in E_{\Omega_2}^{\Omega_1}(\nu_1, N_1, Q_2)$ , where  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \alpha$ . Thus, by **Andor** and **Act**, for some  $i \in [1, \#\mu]$ , we have  $\mu \triangleleft i = \alpha$  and  $M \triangleleft i \equiv \mathcal{SPS}_{\Omega_2}^{\Omega_1}(N_1 \triangleleft j', N'_2 \triangleleft k')$ . Hence,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \alpha = \mu \triangleleft i$  and  $(M \triangleleft i, N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ .
- (b) Let  $i \in [1, \#\mu]$ . By **Andor** and **Act**, for some  $(\alpha, Q'_1, Q'_2) \in E_{\Omega_2}^{\Omega_1}(\nu_1, N_1, Q_2)$ , we have  $\mu \triangleleft i = \alpha$  and  $M \triangleleft i \equiv \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q'_1, Q'_2)$ . Thus, for some  $N'_2, \nu'_2, j'$ , and  $k'$ , we have  $Q_2 \xrightarrow{\nu'_2} N'_2, (\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \alpha, N_1 \triangleleft j' \Omega_1 \sim_{\Omega_2} N'_2 \triangleleft k', Q'_1 \equiv N_1 \triangleleft j'$ , and  $Q'_2 \equiv N'_2 \triangleleft k'$ , because  $(\alpha, Q'_1, Q'_2) \in E_{\Omega_2}^{\Omega_1}(\nu_1, N_1, Q_2)$ . Hence,  $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu'_2 \triangleleft k', \Omega_2) = \alpha = \mu \triangleleft i$  and  $(M \triangleleft i, N_1 \triangleleft j', N'_2 \triangleleft k') \in \mathcal{T}$ .

(2) For some  $(\nu_2, N_2) \in D(Q_2)$ ,

$$\bigvee \{ \alpha. \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q'_1, Q'_2) : (\alpha, Q'_2, Q'_1) \in E_{\Omega_2}^{\Omega_1}(\nu_2, N_2, Q_1) \} \xrightarrow{\mu} M.$$

This case is symmetric with the case (i)(1).

(ii) This case can be shown by a similar argument to the case (i). ■

We can check that  $P \simeq (Q_1 \Omega_1 \sqcap_{\Omega_2} Q_2)$  by finding a principal strong  $(\Omega_1, \Omega_2)$ -synthetic relation  $\mathcal{T}$  such that  $(P, Q_1, Q_2) \in \mathcal{T}$ , but Definition 4.3 is not easy. Proposition 4.2 and 4.3 imply another method for the check. Thus, we can check that  $P \simeq (Q_1 \Omega_1 \sqcap_{\Omega_2} Q_2)$  by checking that  $P \mathcal{N} \simeq_{\mathcal{N}} \mathcal{SPS}_{\Omega_2}^{\Omega_1}(Q_1, Q_2)$ .

Finally a theorem for stepwise synthesizing  $n$  specifications is given.

**Theorem 4.4** Assume that  $n \geq 1$ , for any ground specification  $P_0 \in \mathcal{P}_0$ ,

$$P_0 \Omega \sim_{\Omega} P_n \quad \text{iff} \quad \text{for any } i \in [1, n], P_0 \Omega \sim_{\Omega_i} Q_i,$$

$$P_n \Omega \sim_{\Omega_{n+1}} Q_{n+1}, P_{n+1} \simeq (P_n \Omega \sqcap_{\Omega_{n+1}} Q_{n+1}), \text{ and } \Omega_i \subseteq \Omega \quad (i \in [1, n]).$$

Then, for any ground specification  $P_0 \in \mathcal{P}_0$ ,

$$P_0 \Omega' \sim_{\Omega'} P_{n+1} \quad \text{iff} \quad \text{for any } i \in [1, n+1], P_0 \Omega' \sim_{\Omega_i} Q_i$$

where  $\Omega' = \Omega \cup \Omega_{n+1}$ .

**Proof** This result can be easily shown by Proposition 4.1. ■

In Theorem 4.4, if  $P_n \Omega \not\sim_{\Omega_{n+1}} Q_{n+1}$ , then there is no ground specification  $P_0$  such that  $P_0 \Omega' \sim_{\Omega_i} Q_i$  for any  $i \in [1, n+1]$  by Proposition 3.2(3). Therefore  $P_n \Omega \sim_{\Omega_{n+1}} Q_{n+1}$  is needed.

For example, Theorem 4.4 is used for the following situations.

1. Reconstruction of the complete ground specification  $P_0$  from many partial specifications  $Q_i$  ( $i \in I$ ), such that  $P_0 \Omega \sim_{\Omega_i} Q_i$  and  $\bigcup_{i \in I} \Omega_i = \Omega$ . We produce a specification  $P_I$  by recursively using Theorem 4.4 and Proposition 3.2(3) like  $P_I \Omega \sim_{\Omega} P_0$  and  $P_I \Omega \sim_{\Omega_i} Q_i$ .
2. Refinement of a specification  $P_n$ , by synthesizing a new requirement  $Q_{n+1}$ . Theorem 4.4 guarantees that the refined specification  $P_{n+1}$  satisfies all the requirements  $Q_i$  which  $P_n$  satisfies, thus preservation of the requirements.

Finally we apply our synthesis method to the example (\*<sub>2</sub>). By Proposition 4.3, the principal strong  $(ab, ac)$ -synthetic specification  $A_{12}^\square$  of  $B_1$  and  $B_2$  is produced as follows:

$$A_{12}^\square \equiv \mathcal{SPS}_{ac}^{ab}(B_1, B_2) \equiv a.\tau.\mathbf{0} + (b.\tau.\mathbf{0} \vee b.c.\mathbf{0}) + (\tau.c.\mathbf{0} \vee b.c.\mathbf{0}) \\ + (b.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (\tau.c.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

Proposition 4.1 guarantees that  $A_c abc \sim_{abc} A_{12}^\square$ , because  $A_c abc \sim_{ab} B_1$  and  $A_c abc \sim_{ac} B_2$ . Furthermore by Proposition 4.3, the principal strong  $(abc, bc)$ -synthetic specification  $A_{123}^\square$  of  $A_{12}^\square$  and  $B_3$  is produced as follows:

$$A_{123}^\square \equiv \mathcal{SPS}_{ac}^{abc}(A_{12}^\square, B_3) \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} \\ + (a.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (b.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (\tau.c.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

By Proposition 3.2(3), we have that  $A_{12}^\square abc \sim_{bc} B_3$ . Thus, by Theorem 4.4, for any ground specification  $A_0$ ,

$$A_0 abc \sim_{abc} A_{123}^\square \quad \text{iff} \quad \text{for any } i \in [1, 3], A_0 abc \sim_{\Omega_i} B_i. \quad (*_5)$$

The following two specifications are all the ground specifications  $A_0$  up to  $abc \sim_{abc}$  such that  $A_0 abc \sim_{abc} A_{123}^\square$ .

- (1)  $A_{01} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0}$
- (2)  $A_{02} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + \tau.\tau.\mathbf{0}$

The first specification  $A_{01}$  corresponds to the complete specification  $A_c$ . The second specification  $A_{02}$  is also a candidate satisfying  $B_1$ ,  $B_2$ , and  $B_3$ .

## 5 Related work

Decomposition and refinement methods of specifications have been proposed, for example in [2, 3]. In [2], algorithms to decompose a specification into two parallel processes are presented in LOTOS. The algorithms are useful for implementation. Our strong  $(\Omega_1, \Omega_2)$ -equivalence is used for extracting partial specification about interesting actions, thus the purpose is different from one of [2].

In [3], a stepwise refinement of specifications is proposed based on a new parallel combinator in LOTOS. The synchronous rule of the combinator is

$$\mathbf{Sync} \frac{Q_1 \xrightarrow{\mu_1} Q'_1 \quad Q_2 \xrightarrow{\mu_2} Q'_2}{Q_1|_A Q_2 \xrightarrow{\mu_1 \cup \mu_2} Q'_1|_A Q'_2} \left( \mu_1 \cap \mu_2 = \mu_1 \cap A = \mu_2 \cap A \right).$$

The labels  $\mu$  on the transitions are sets of actions  $\{a_1, \dots, a_n\}$ . It increases greatly the simplicity and modularity of refined specifications. Our refinement intuitively bases on **GLB** rule introduced in Section 4. The most important difference between **GLB** and **Sync** is that **GLB** has the condition  $Q'_1 \Omega_1 \sim_{\Omega_2} Q'_2$ . The condition is available for specifications with indeterminate choices and produces a refined specification  $P$  such that  $P \Omega_1 \cup \Omega_2 \sim_{\Omega_i} Q_i$ .

In order to partially analyze processes, we already proposed a process algebra *CCSG*[4]. In *CCSG* we approximately analyze processes by neglecting unimportant distant actions. A disadvantage of *CCSG* is used for only specific systems, where actions has information about the importance and the position. *SPEC<sup>V</sup>* presented in this paper is widely applicable to models based on LTS.

## 6 Conclusion

In this paper we have presented strong  $(\Omega_1, \Omega_2)$ -equivalence  $\Omega_1 \sim_{\Omega_2}$  to relate partial specifications and have presented principal strong  $(\Omega_1, \Omega_2)$ -synthetic specifications to synthesize partial specifications preserving  $\Omega_1 \sim_{\Omega_2}$ . Theorem 4.4 is used for stepwise refinement of specifications and it guarantees that a new requirement does not break previous requirements. If  $P$  is the principal strong  $(\Omega_1, \Omega_2)$ -synthetic specifications of  $Q_1$  and  $Q_2$ , then all the ground specifications included in  $P$  satisfy both  $Q_1$  and  $Q_2$ , and they are all the ground specifications satisfying both  $Q_1$  and  $Q_2$ .

We have future works as follows:

1. The most important future work is to develop a *weak* version of principal *strong*  $(\Omega_1, \Omega_2)$ -synthetic specifications, by neglecting internal actions as far as possible. Interesting examples will be shown by using the weak version.
2. We should clarify more properties of principal *strong*  $(\Omega_1, \Omega_2)$ -synthetic specifications. For example, it is expected that Definition 4.3 of principal strong  $(\Omega_1, \Omega_2)$ -synthetic specifications is slightly weakened, because there are synthetic specifications  $P$  such that, for any ground specifications  $P_0$ ,

$P_0 \Omega_1 \cup \Omega_2 \sim_{\Omega_1 \cup \Omega_2} P$  iff  $P_0 \Omega_1 \cup \Omega_2 \sim_{\Omega_1} Q_1$  and  $P_0 \Omega_1 \cup \Omega_2 \sim_{\Omega_2} Q_2$   
and  $P \not\leq (Q_1 \Omega_1 \sqcap_{\Omega_2} Q_2)$ . The example  $(*_2)$  in Section 4 is used here again. The following specification  $A_{123}^{\square'}$  satisfies  $(*_5)$ , but  $A_{123}^{\square'} \not\leq (A_{12}^{\square} \text{ abc } \sqcap_{bc} B_3)$ .

$$A_{123}^{\square'} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + (a.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

3. We will develop a verification tool for strong  $(\Omega_1, \Omega_2)$ -equivalence such as the concurrency workbench [5] and a synthesis tool for producing principal strong  $(\Omega_1, \Omega_2)$ -synthesis specifications.

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