数理解析研究所講究録
1013 巻 1997 年 187-193

RIMS Kokyuroku
数理解析研究所講究録
1997
Research Institute for Mathematical Sciences
Kyoto University, Kyoto, Japan

Information erasure without entropy production of $k \ln 2$ per bit by a quasi-static potential change subjected to a Brownian motion

Hideaki Matsueda, Eiichi Goto†, and Kia - Fock Loe‡
Department of Information Science, Kochi University
2-5-1 Akebonocho, Kochi 780, Japan
†Department of Information and Computer Science, Kanagawa University
2946 Tsuchiya, Hiratsuka, Kanagawa 259-12, Japan
‡Department of Information Systems and Computer Science
National University of Singapore, Singapore 0511

Abstract

A possibility of erasing a bit of information without producing the entropy $k \ln 2$ is demonstrated, using a time-dependent potential, changing between double- and single-well forms, subjected to a Brownian motion. Computer simulation was done to estimate the average entropy production in this potential. By this simulation and also from thermodynamical considerations, it is found that the entropy production accompanying the erasure of a bit is far less than $k \ln 2$, when the potential is varied slowly enough.

I. Introduction

The controversy concerning the relation between the amount of information and the thermodynamic entropy may be dated back to 1929 [1]. Although so many documents have been published in this field [2]–[9], no author has ever distinguished clearly the entropy production from the entropy transfer. We wish to make use of the entropy production in the fundamental analysis of the limiting heat dissipation of computing devices.

It is natural to consider that the total entropy change dS of a closed system consists of two parts, as Clausius mentioned [10][11][12], $dS = dS_t + dS_p$ where dS_t and dS_p represent the entropy transfer and the entropy production respectively.

The entropy transfer corresponds to the reversible heat Q_t exchanged between the system and the outside world, devided by the temperature T, as $S_t = \int dQ_t/T$. This could be either positive, zero, or negative depending on how the heat flows. The reversible heat exchange means the conservation of transferred entropy, when we look at the total system including the energy giving and receiving subsystems, according to the first law of thermodynamics.

However, the entropy production dS_p corresponds to the irreversible internal heat generation, being never negative in a closed system, as stated in the second law of thermodynamics. This heat generation is spontaneous and uncontrollable. However, the total entropy S obtained by integrating the two kinds of entropy changes along any path is believed to be a state valuable.

It is a result of the second law of thermodynamics, that any physical process which produces entropy is irreversible. Consequently, information encoded as the initial state of this process will be destroyed or erased by the physical irreversibility. However, the converse may not always be true, having never been proved, to our knowledge.

In this paper, a counter example is given, where the erasure of information is accomplished causing little entropy production which can be as small as one wish, provided that the process is slow enough and the erasure is defined as the reduction to a completely random state. The results of the computer simulation of a one dimensional Brownian motion, under a time-dependent symmetrical potential, together with a viscous resistance, is shown.

Moreover, thermodynamic consideration of possible entropy production processes in a quantum flux parametron (QFP) devise are presented [13]–[16], showing that the entropy production per clock cycle could be far less than $k \ln 2$.

II. Time-dependent Potential

The time-dependent potential is represented as

$$U(\theta,\alpha) = U_0\{\cos^2\theta - \alpha(t)\cos\theta\} \qquad (1)$$

where θ is the one dimensional coordinate, $\alpha(t)$ is the time-varing parameter that controls the shape of the potential, and t is the time. A single particle is assumed to be in the potential to form one bit of information. The similar potentials have been used to describe a spin system [6] and a QFP [13]–[18]. For simplicity we assume a periodic function for $\alpha(t)$ as

$$\alpha(t) = \frac{3}{2}\{1 - \cos(\omega t)\} \qquad (2)$$

where ω is the angular frequency of the modulation. These equations (1) and (2) give a double well potential, having energetically degenerate minima at $\theta = \pm\frac{\pi}{2}$, when $\alpha(t) = 0$ i.e. $\omega t = 0$, as depicted in Fig.1.

The barrier between the minima decreases as $U_o\{1 - \alpha(t)/2\}^2$ when t increases from 0 toward the value for $\alpha(t) = 2$. At $\alpha(t)= 2$, the potential becomes a single well with no harmonic term, and the particle in it loses its memory through Brownian impacts and some viscous resistance. When $2 < \alpha(t)$, the single well potential is dominated by a harmonic term with its minimum at $\theta = 0$. The system follows an inverse path after $\alpha(t) = 3$ or $\omega t = \pi$.

When the potential passes through the point $\alpha(t)= 2$ from the single- well to the double-well phase, a one bit of information is randomly created, reflecting the Brownian motion of the particle, having no correlation with the information in the previous double-well phase. It may be reasonable to consider that one bit of information is either erased irreversibly or created randomly, each time when $\alpha(t)$ passes the value 2 in each period of eq.(2).

III. Brownian Motion

The behaviour of the particle in the time-dependent potential $U(\theta,t)$ is simulated numerically, using the following Langevin-type equation of motion.[1]

$$\ddot{\theta} + \zeta\dot{\theta} + \frac{\partial}{\partial\theta}U(\theta,t) = F(t) \qquad (3)$$

where $U(\theta,t) = U(\theta,\alpha)$, and $F(t)$ is the random Brownian impact force having a Gaussian white-noise distribution. Furthermore, in the time continuum, we postulated that the force $F(t)$ satisfies the condition indicated below.

$$\langle F(t)F(t')\rangle \propto \delta(t - t') \qquad (4)$$

[1]This computer simulation was executed by Dr. Shigemi Ohta at Institute of Physical and Chemical Research (RIKEN).

The term $\zeta\dot{\theta}$ represents the frictional or viscous resistance which leads to a heat dissipation. In the case of QFP, this term corresponds to a damping resistance (order of 10Ω) parallel to a Josephson junction. This resistance is far below the value of the quantum Hall resistance (around $25k\Omega$), so the classical treatment of this paper is adequate.

A Runge-Kutta method was employed to obtain numerical solutions between the Brownian impacts. The Brownian impacts, in turn, were simulated using an integrated impact $B(t)$ at a regular interval ϵ.

$$B^{\epsilon}(t) = \int_t^{t+\epsilon} F(\tau)d\tau \qquad (5)$$

The impacts cause finite but sudden shifts in the velosity $\dot{\theta}$ at the regular intervals.

Using the numerical solutions, the entropy production is estimated as the difference of the heat dissipated through the viscous resistance, and the energy gained from the potential change and the Brownian impact, deviding by the temperature. The entropy production is plotted as a function of computation time t in Fig.2, for different values of the parameter representing different sizes of potential (U_0), with the standard deviation of the random force $\sigma = 0.01 \sim 0.02$, putting the mass equal to 1 for simplicity.

The coordinate θ is also plotted againt time in Fig.3. It is obvious from Fig.3, that the displacement θ approaches its maximum value (i.e. around ±1.7) in either one of the two wells, during the double well periods.

The long-term average of the entropy production is found to decrease with a power of the frequency of the potential modulation ω, seemingly having no constant term which imposes a lower bound [19]. This may be verified by calculating the average entropy production at each typical phase, i.e. at phases of a double well ($\alpha = \alpha_0$), a wide single well ($\alpha = \alpha_2$), and a parabolic single well ($\alpha_2 < \alpha$) phases.

Therefore, it may be concluded that the entropy production associated with the erasure of one bit of information in this scheme could be as small as we wish. It may also be that the entropy production has no lower bound, in the inverse process where one bit of random information is created, in the same scheme. The credibility of the simulation was verified using the Einstein-Perrin diffusion relation [19].

It is needless to mention that in a practical QFP, initial information is encoded through a superregeneration process which overcomes the Brownian disturbances [13][14][20]. So, this scheme may be called NRZ (non return to zero) type encodement scheme, rather than RZ type.

IV. Thermodynamic Consideration

In this section examples of thermodynamic estimation of the entropy production is given for different dissipative mechanisms in QFP.

A. Entropy production by Joule's heat

As is well understood, the generation of Joule's heat by an electric current I is an irreversible process accompanying an entropy production, dS_{pj}. In a QFP, this current is induced by continuous time change of magnetic flux $\frac{d\Phi}{dt}$, as well as by a sudden flux change. However, the latter could be avoided operationally, employing a nonlinear coupling element to isolate a QFP from its adjacent QFPs during its erasure period [14]. This corresponds to prevent a formation of potential well having third order curvature which results in a metastable saddle point shown in Fig.1 (f).

In the former, although the continuous current change is inevitable, the heat Q_j generated per computational clock cycle is shown to decrease in proportion to the clock frequency as below.

$$Q_j = \oint T\frac{dS_{pj}}{dt}dt = \oint \frac{\left(\frac{d\Phi}{dt}\right)^2}{Z} = \int_0^{1/f_c} \frac{V^2}{Z}dt = \frac{2\pi^2{\Phi_0}^2}{Z}f_c \tag{6}$$

where T is the temperature, k is Boltzmann constant, Z is the damping resistance, V is the electric voltage induced, and the flux undergoes a cyclic change as $\Phi = \Phi_0 cos(2\pi f_c t)$ with a clock frequency f_c.

A realistic assumption such as $T = 4.2K$, $Z = 10\ \Omega$, and $\Phi_0 = \frac{h}{2e}$ (half Dirac monopole flux) with Planck constant h, eq.(6) gives $kT\ln 2$ at a clock frequency as high as $f_c = 1.33 \times 10^8\ s^{-1}$ or around 100 MH_z [15][16].

Therefore, it is obvious that the entropy production due to the Joul's heat could be made far less than $k \ln 2$ per cycle, by redusing the clock frequency.

By the way, the heat generated at a unit clock rete (i.e. $f_c = 1\ s^{-1}$) is estimated to be on the order of 10^{-29} Js, which is 5 orders of magnitude larger than the Planck constant, showing unapplicability of the quantum mechanical uncertainty [15].

B. Entropy production by heat conduction

As is also well understood, heat condunction between points at different temperatures is irreversible, and produce entropy dS_{pc}. However, in a QFP, the heat Q_c conducted per clock cycle is shown to decrease in proportion to the clock frequency.

$$Q_c = \oint T\frac{dS_{pc}}{dt}dt = T_b \int_0^{1/f_c} \frac{\kappa A\,(T_h - T_c)^2}{lT_hT_c}dt \simeq \frac{k^2 lT_b}{\kappa A}f_c \tag{7}$$

where T is the temperature concerned, T_h and T_c are temperatures of the hot and cold points respectively; at the end, all these temperatures are approximated by the temperature of the heat bath represented by T_b, A is the cross sectional area of the heat path and l is the distance, κ is the heat conductivity, and the irreversible heat flow Q_c per unit area and cycle, is estimated to be far less than $kTln2$.

In fact, most of the heat flow between a QFP and the heat bath is not that of eq.(7), but is reversible heat transfer and of the order of kT per cycle.

C. Entropy production by a gas molecule in an expanding chamber

It is possible to apply a well known scheme of gas expansion to our erasing period of a QFP, if we take into account the velocity of the wall relative to a molecule. The quasi-static process of a quantum flux and a double well potential going into a wide single well state, is modeled by the kinetics of a molecule with mass m and a mean square velocity v at temperature T, in an expanding chamber furnished with a movable wall.

We are considering the case where the wall velocity ω is controllable, and the molecular velocity v is fixed constant by the constant temperature. In other words, the gas molecule of this model never be allowed to acquire uncontrollably high velocity during its expansion. This may be the difference of starting points between our discussion and the other arguments such as in ref.[21].

The kinetic theory of gas gives the molecular velocity as $v = \sqrt{\frac{kT}{m}}$. The pressure of the gas p is also given as

$$p = \frac{m(v-w)^2}{l} = \frac{kT}{l}\left(1 - \frac{w}{v}\right)^2 = (1-r)\frac{kT}{l} \tag{8}$$

where w is the velocity of the wall, l is the width of the chamber, and the factor r is defined as $r = 1 - (1 - \frac{w}{v})^2$, being $r = 0$ for the wall at rest, and $r = 1$ for a wall moving with the same velocity as the molecule.

It may be reasonable to consider that the work done by the molecule to the wall is compensated by a reversible heat flow Q_t from a local heat bath.

$$Q_t = \int_1^2 pdl = (1-r)\,kT\ln 2 \tag{9}$$

where the integration corresponds to doubling the volume of the chamber. This amout of heat will be returned to the heat bath in the next phase of the cycle. In other words, the first law of thermodynamics is applicable to the total system comprising a QFP, a clock circuit, and the local heat bath, and the energy is conserved. Consequently, the entropy difference due to the reversible heat transfer of eq.(9) could be controlled quasi-statically.

On the contrary, the term $rkT \ln 2$ represents fraction of the molecular energy which was not converted into the mechanical work on the wall. This corresponds to the free expansion of gas, and accompanies an entropy production given below.

$$S_{pg} = rk \ln 2 \qquad (10)$$

As a matter of course, if the wall runs away faster than the molecule, all this process becomes the free expansin to yield the maximum entropy production $S_{pg} = k \ln 2$.

However, if we slow down the wall, i.e. $r \to 0$, the entropy production could be reduced at will. This means that a bit of information could be erased by the slow movemnt of the wall, while the net heat dissipation from the whole computing system is kept as close to zero as we wish.

V. Conclusions

By the computer simulation, the average entropy production is estimated for the proposed scheme of information erasure, involving a periodic modulation of the symmetrical potential and a Brownian motion of a particle, together with the viscous resistance. It is found that the entropy production per clock cycle resulting from the erasure of a bit is less than $k \ln 2$, in a regime where the potential is varied slowly enough.

Thermodynamical consideration of entropy production in QFP, also gave an example of information erasure without producing $k \ln 2$ entropy per bit.

References

[1] VON SZILARD, L., "Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen", *Z. Phys.* **53** (1929), 840-856; . "On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings", *Quantum Theory and Measurement"*, (J. A. Wheeler and W. H. Zurek ed.), Princeton University (1983), 539-548.

[2] SHANNON, C. E., "A Mathematical Theory of Communication", *Bell System Technical J.* **27** (1948), 379-423, 623-656.

[3] BRILLOUIN, Leon, *Science and Information Theory*, Academic Press (1956), Chap.9.

[4] JAYNES, E. T., "Information Theory and Statistical Mechanics", *Phys Rev.* **106** (1957), 620-630.

[5] LANDAUER, Rolf, "Irreversibility and Heat Generation in the Computing Process", *IBM J. Research and Developments* **5** (1961), 183-191.

[6] BENNETT, Charles H., "The Thermodynamics of Computation—a Review", *International J. of Theoretical Phys.* **21** (1982), 905-940.

[7] ZUREK, W. H., "Reversibility and Stability of Information Processing Systems", *Phys. Rev. Lett.* **53** (1984), 391-394.

[8] LEFF, Harvey S., and Andrew F. REX *Maxwell's Demon—Entropy, Information, Computing*, Princeton University Press (1990).

[9] CAVES, Carlton M., "Information, Entropy, and Chaos", *Physical Origines of Time Asymmetry* (J. J. Halliwell, J. Pérez-Mercader, and W. H. Zurek eds.), Cambridge University Press (1994), 47-89.

[10] Clausius introduced the uncompensated entropy (presently called entropy production) for irreversible processes.

[11] NICOLIS, Grégoire, and PRIGOGINE Ilya, *Self-Organization in Nonequilibrium Systems*, John Wiley (1977), Chap.1, Sect.1.2.

[12] KUIKEN, Gerard D. C., *Thermodynamics of Irreversible Processes*, Wiley (1994), Chap.2, Sect.2.3.12.

[13] GOTO, Eiichi, "On the Application of Parametrically Excited Nonlinear Resonators", *J. Elec. Commun. Engrs. Japan* **38** (1955), 770-775.

[14] GOTO, Eiichi, "The Parametron, a Digital Computing Element which Utilizes Parametric Oscillation", *Proc. IRE* **47** (1959), 1304-1316.

[15] GOTO, Eiichi, N. YOSHIDA, K. F. LOE, and Willy HIOE, "A Study on Irreversible Loss of Information without Heat Generation"', *Proc. 3rd Int. Symp. Foundations of Quantum Mechanics* , Tokyo (1989), 412–418.

[16] GOTO, Eiichi, Willy HIOE, and Mutsumi HOSOYA, "Physical Limits to Quantum Flux Parametron Operation", *Physica C* **185-189** (1991), 385-390.

[17] LIKHAREV, Konstantin K., "Classical and Quantum Limitations on Energy Consumption in Computation", *Int. J. Theoretical Phys.* **21** (1982), 311-326.

[18] VON NEUMANN, John, "Non-linear Capacitance or Inductance Switching, Amplifying, and Memory Organs", *U.S. Patent* **No.2,815,488** (1957).

[19] GOTO, Eiichi, and Shigemi OHOTA, in preparation.

[20] ARMSTRONG, Edwin H., " Some Recent Developments of Regenerative Circuits", *Proc.I.R.E.* **10** (1922), 244-260.

[21] LANDAUER, Rolf, "Comment on "Physical limits to quantum flux parametron"", *Physica C* **208** (1993), 205-207.

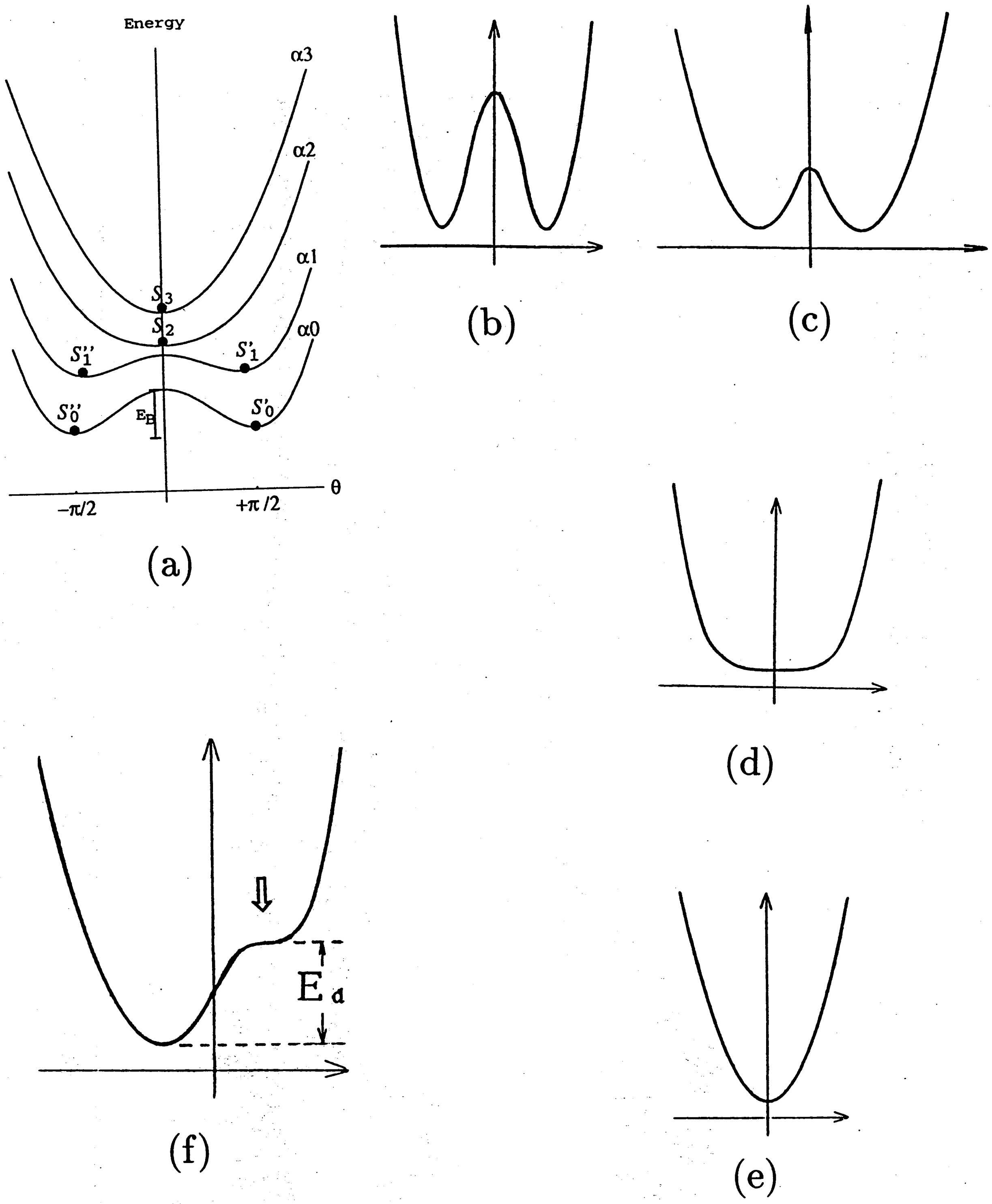

Fig.1 Different potential shapes.
(a) periodic change of the potential shape,
(b) independent double well, (c) double well,
(d) wide single well, (e) parabolic single well,
(f) undesirable third order metastability.

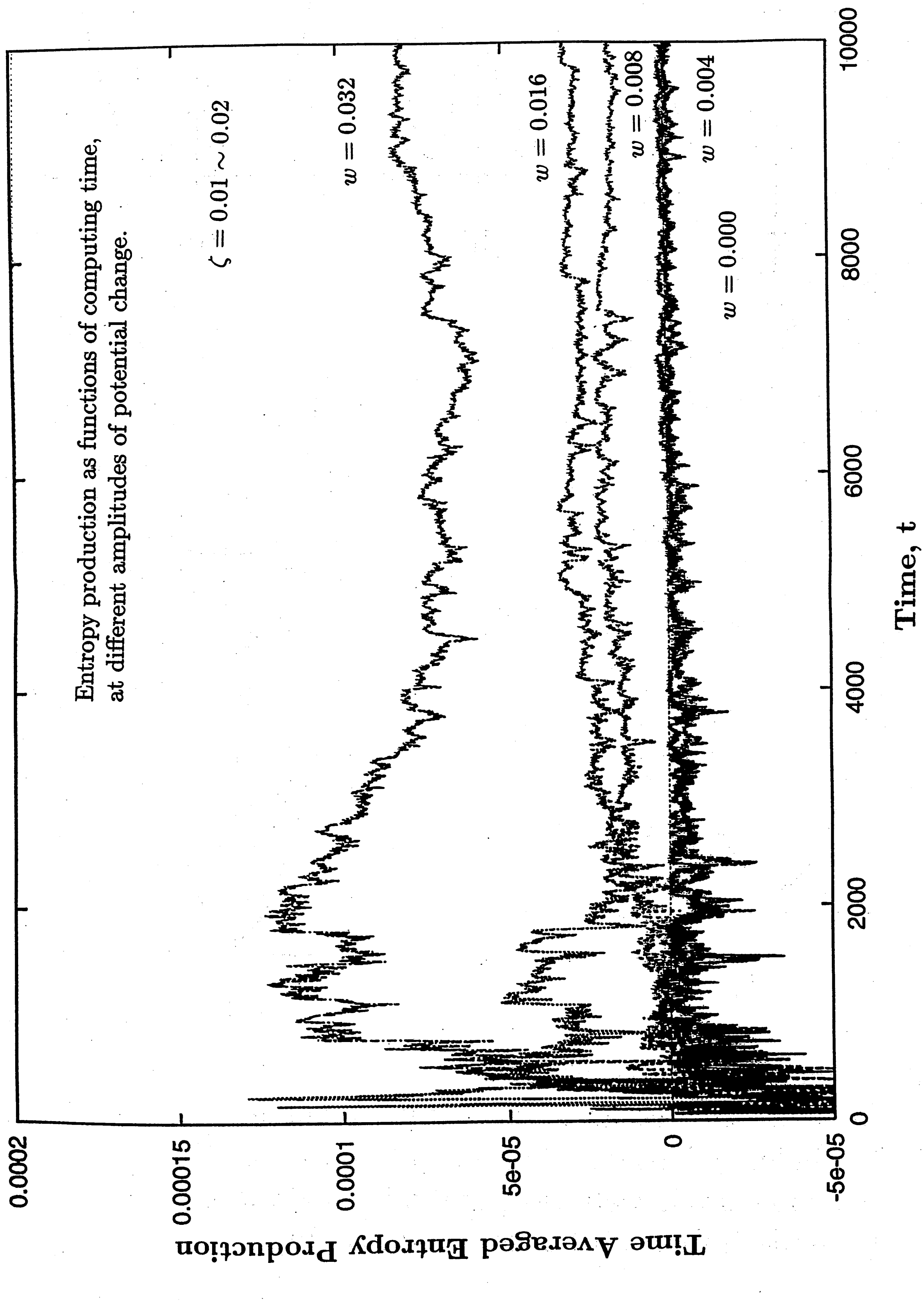

Fig.2 Simulated entropy production as functions of computing time, at different amplitudes ω of potential change.

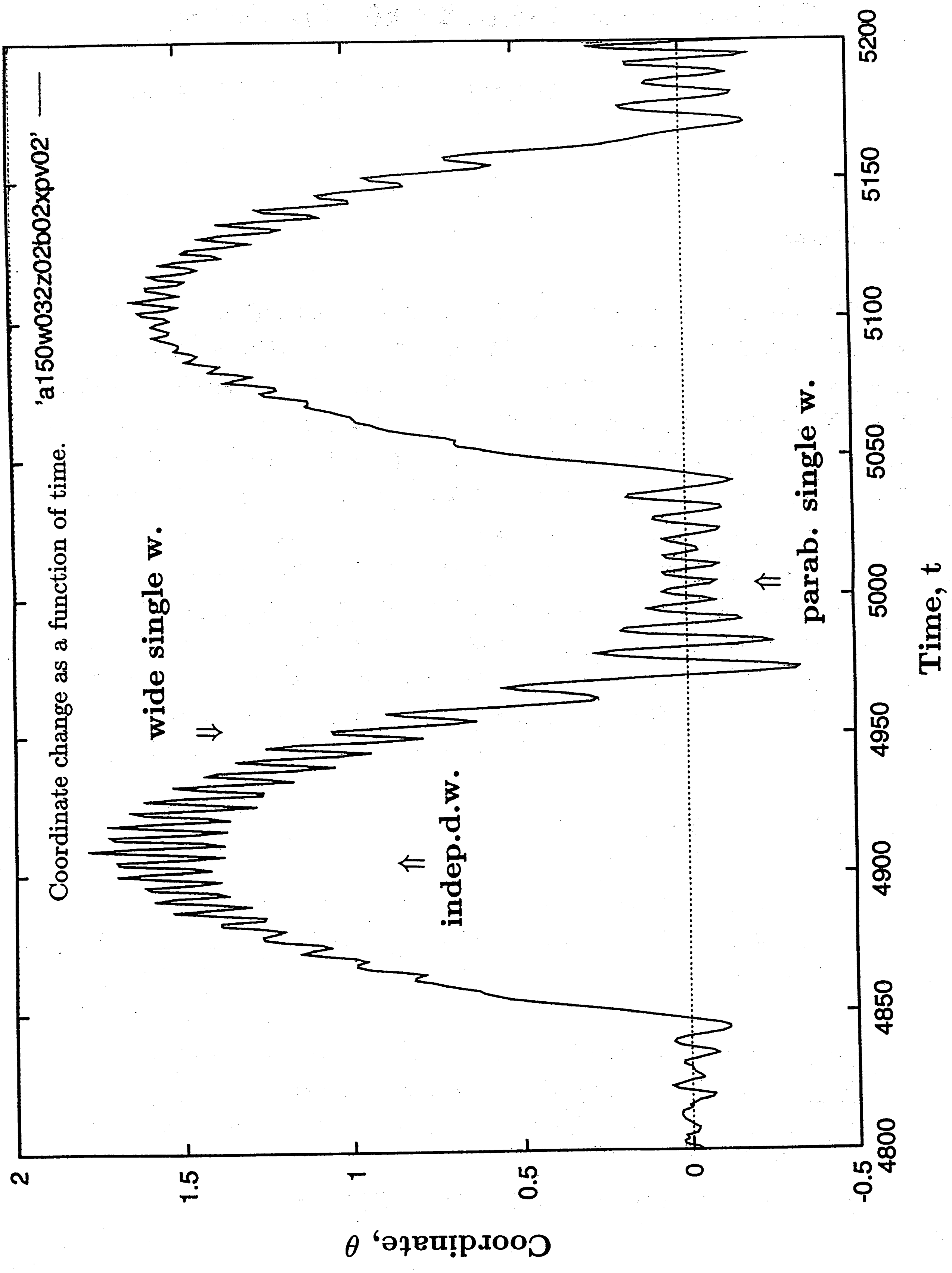

Fig.3 Simulated coordinate change as a function of time.