数理解析研究所講究録
1014 巻 1997 年 133-133

Exact WKB analysis of anharmonic oscillators.

KOIKE, Tatsuya (RIMS) (小池達也)

Anharmonic oscillators have attracted much attention of physists, particularly because of their relevance to the ϕ^4-model in quantum field theory.

Here we show how to apply exact WKB analysis to the analysis of their eigenvalue problems, namely,

$$(-\frac{d^2}{dx^2}+\frac{1}{4}x^2(1+\lambda x^{2N}))\psi = E(\lambda)\psi \qquad (\lambda > 0),$$

$$\lim_{|x|\to\infty} \psi(x) = 0.$$

For $\lambda \ll 1$, an eigenvalue $E(\lambda)$ has a formal expansion with respect to λ (the so-called Rayleigh-Shrödinger perturbation series) ;

$$E^K(\lambda) = K + \frac{1}{2} + \sum_{n=1}^{\infty} A_n^K \lambda^n \qquad \text{where} \quad K = 0, 1, 2, \cdots .$$

Our purpose is to determine the asymptotic behavior of A_n^K for arbitrary N. The result is as follows :

$$A_n^K = \frac{(-1)^{n+1}N}{K!(2\pi^3)^{1/2}}\, 4^{(K+\frac{1}{2})/N} \left(\frac{B(\frac{3}{2},\frac{1}{N})}{2N}\right)^{-K-\frac{1}{2}-nN} \Gamma(K+\frac{1}{2}+nN)\left(1+O(\frac{1}{n})\right), \ (n\to\infty).$$

Here $B(x,y)$ denotes the Beta function.

For $N=1$, this result was shown by Bender and Wu (Phys.Rev.D,**7**(1973)) in a heuristic manner. By making use of exact WKB analysis we verify the result rigorously.