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**Summary**

This paper discusses the application of Newton polyhedrons and formal Gevrey spaces to the study of double indices in linear partial differential operators. The author, Hiroshi Yamazawa, presents new insights into the behavior of solutions in these complex systems, which are fundamental in various fields of mathematics and physics. The research was published in the Journal of the Mathematical Society of Japan, highlighting its importance in the field of partial differential equations.
Newton polyhedrons and a formal Gevrey space of double indices
for linear partial differential operators

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Abstract

In this paper, we shall study a necessary condition to become that the below operator \( L_m(t, x; \partial_t, \partial_x) \) is bijective on \( \mathcal{G}^{\{s_t, s_x\}} \), where it is said that a formal power series \( U(t, x) = \sum_{i\beta} U_{i\beta} t^{i\beta} x^{\beta} \) belongs to \( \mathcal{G}^{\{s_t, s_x\}} \) when \( U(t, x) = \sum_{i\beta} U_{i\beta} t^{i\beta} x^{\beta} / (!)^{s_t} (\beta!)^{s_x} \) is convergent near the origin for \( s_t, s_x \geq 1 \).

1 Introduction

Let us give the operator \( L_m \) that we shall study in this paper.

Let \( t = (t_1, \cdots, t_q) \in \mathbb{C}^q \), \( x = (x_1, \cdots, x_p) \in \mathbb{C}^p \) and \( t \cdot \partial_t = (t_1 \partial_{t_1}, t_2 \partial_{t_2}, \cdots, t_q \partial_{t_q}) \). Set

\[
(t \cdot \partial_t)^j = (t_1 \partial_{t_1})^{j_1} (t_2 \partial_{t_2})^{j_2} \cdots (t_q \partial_{t_q})^{j_q}
\]

for \( j = (j_1, \cdots, j_q) \in \mathbb{N}^q \) and

\[
P_m(t \cdot \partial_t) = \sum_{|j| \leq m} P_j(t \cdot \partial_t)^j,
\]

where \( P_{\sigma} \in \mathbb{C} \), and \( P_m \) is said to be of Fuchs type of order \( m \) in [M]. Then we consider the following operator:

\[
L_m = P_m(t \cdot \partial_t) + A(t, x; \partial_t, \partial_x) + B(t, x; \partial_t, \partial_x)
\]

where

\[
A = \sum_{|\alpha| = 0}^{\text{finite}} a_{\sigma, \sigma'}^\alpha (t, x) t^{\sigma'} \partial_t^\sigma \partial_x^\alpha,
\]

(1.4)

and

\[
B = \sum_{|\alpha'| + |\alpha| > |\sigma| + |\sigma'|}^{\text{finite}} b_{\sigma, \sigma'}^{\alpha, \alpha'} (t, x) t^{\alpha'} x^{\alpha} \partial_t^\sigma \partial_x^\alpha,
\]

(1.5)

where the coefficients \( a_{\sigma, \sigma'}^\alpha (t, x) \) and \( b_{\sigma, \sigma'}^{\alpha, \alpha'} (t, x) \) are holomorphic functions in a neighbourhood of the origin for any \((\sigma, \sigma', \alpha, \alpha') \in \mathbb{Z}^q \times \mathbb{N}^q \times \mathbb{Z}^p \times \mathbb{N}^p \).
Miyake and Hashimoto studied the unique solvability in $G^{\{s_t,1\}}$ for such type operator. They characterized the Gevrey index $s_t$ by Newton polygons in [M] and [MH].

Our motivation comes from the following facts. Put

(1.6) \[ L = (t\partial_t + 1) - 3t^3x\partial_t^2\partial_x - (t\partial_t + 1)x^2\partial_x. \]

This operator is not bijective in $G^{\{s_t,1\}}$ for any $s_t$, but is bijective in $G^{\{s_t,s_x\}}$ for $s_t \geq 3$ and $s_x \geq 2$.

So it is our purpose that we shall consider $G^{\{s_t,s_x\}}$ to obtain the unique solvability for this operator. We shall define a Newton polyhedrons to characterize double Gevrey indices.

In Section 2, we give our results after defining a function space and Newton polyhedrons and listing some notations. In Section 3, we prove our theorems.

2 Statement of results

2.1 Notations.

We denote by $N$, $Z$, $R$ and $C$ the set of non negative integers, integers, real numbers and complex numbers, respectively.

$C[[t,x]]$ denotes the set of formal power series in $t \in C^q$ and $x \in C^q$ with coefficients in $C$.

For multi indices $\sigma = (\sigma_1, \cdots, \sigma_q) \in Z^q$ and $\alpha = (\alpha_1, \cdots, \alpha_p) \in Z^p$, an integro-differential $\partial_t^\sigma\partial_x^\alpha U(t,x)$ of $U(t,x) = \sum_{l,\beta} U_{l\beta} t^{l} x^{\beta} / l! \beta!$ in $C[[t,x]]$ is defined as follows:

(2.1) \[ \partial_t^\sigma\partial_x^\alpha U(t,x) := \sum_{l,\beta \in N^q, \beta \in N^p} U_{l\beta} \frac{t^{l-\sigma} x^{\beta-\alpha}}{(l-\sigma)! (\beta-\alpha)!}. \]

2.2 Formal Gevrey class $G^{\{s_t,s_x\}}_{\tau,\rho}(T,X;m)$.

For $U(t,x) \in C[[t,x]]$, we set $U(t,x) = \sum_{l,\beta} U_{l\beta} t^{l} x^{\beta} / l! \beta!$, where $U_{l\beta} \in C$, $l \in N^q$ and $\beta \in N^p$ and $R_+$ denotes the set of positive real numbers.

Let $s_t, s_x \geq 1, T > 0, X > 0, \tau = (\tau_1, \cdots, \tau_q) \in R_+^q, \rho = (\rho_1, \cdots, \rho_p) \in R_+^p$ and $m \in N$. Then we define a space $G^{\{s_t,s_x\}}_{\tau,\rho}(T,X;m) \subset C[[t,x]]$ as follows.

(2.2) \[ G^{\{s_t,s_x\}}_{\tau,\rho}(T,X;m) := \left\{ U(t,x) \in C[[t,x]]; ||U||_{T,T,X;\rho;m} < \infty \right\}, \]
where

\begin{equation}
||U||_{T/r,x/ρ;m}^{\{s_t,s_x\}} := \sup_{l,β} |U_{lβ}| \frac{|l|!^m T^{|l|} x^{|eta|}}{(s_t + m)|l| + s_x |eta|)! τ^l ρ^β}
\end{equation}

and \(n! := \Gamma(n + 1)\).

Hence there exist positive constants \(R_t, R_x\) and \(C\) such that

\begin{equation}
|U_{lβ}| \leq C \frac{|l|!^{s_t} |eta|!^{s_x}}{R_t^{|l|} R_x^{|\beta|}}
\end{equation}

for any \(l \in \mathbb{N}^q\) and \(β \in \mathbb{N}^p\).

Here we define a formal Gevrey space as follows:

**Definition 2.1**

\begin{equation}
G_{\{s_t,s_x\}} := \cup_{T,X>0} G_{T/ρ,r/m}^{\{s_t,s_x\}}(T,X;m)
\end{equation}

**2.3 Newton polyhedron.**

Here we define Newton polyhedron for a linear partial integro-differential operator and state some remarks \(L_m\).

Let

\begin{equation}
P = \sum_{finite} a_{σ,σ'}^α,α' (t,x) t^α x^α \partial_t^α \partial_x^α
\end{equation}

be a linear partial integro-differential operator of finite order with holomorphic coefficients in a neighbourhood of the origin.

In the space \(\mathbb{R}^3\), we define the following lower half line for \((σ,σ',α,α') \in \mathbb{Z}^q × \mathbb{N}^q × \mathbb{Z}^p × \mathbb{N}^p:\)

\begin{equation}
Q(σ,σ',α,α') := \{(X,Y,Z) \in \mathbb{R}^3; X = |σ'| - |σ|, Y = |α'| - |α|, Z \leq |σ| + |α|}\}
\end{equation}

**Definition 2.2** Newton polyhedron \(N(P)\) of the operator \(P\) is defined by

\begin{equation}
N(P) := Ch\{Q(σ,σ',α,α'); (σ,σ',α,α') \text{ with } a_{σ,σ'}^α,α' (t,x) \neq 0\},
\end{equation}

where \(Ch\{\cdot\}\) denotes the convex hull of sets in \{\cdot\}.

Let \(N(L_m)\) be Newton polyhedron of \(L_m\).

**Remark 2.3** By the form of \(L_m\), the lower half line \{(0,0,Z); Z \leq m\} becomes a side of \(N(L_m)\) and the point \((0,0,m)\) becomes a vertex of \(N(L_m)\).
Next let 

\[(2.9) \quad \mathfrak{A} = \{(X, Y, Z); aX + bY - Z + m \geq 0\}.\]

Then we define the following set of pairing indices:

\[(2.10) \quad \mathcal{S} = \{(s_t, s_x); s_t = a + 1, s_x = b + 1, N(L_m) \subseteq \mathfrak{A}\}.

\textbf{Remark 2.4} Since the boundary of $\mathcal{S}$ is a hyper plane which goes through the point $(0,0,m)$, there exists $(s_t, s_x)$ such that $N(L_m) \subseteq \mathfrak{A}$ by Remark 2.3.

\textbf{Remark 2.5} For any $(s_t, s_x)$ belonging to $\mathcal{S}$, we obtain

\[(2.11) \quad s_t(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\sigma'| + |\alpha'| - m \leq 0\]

for $(\sigma, \sigma', \alpha, \alpha')$ with $Q(\sigma, \sigma', \alpha, \alpha') \in N(L_m)$.

\section*{2.4 Main results.}

We assume the following additional condition.

(A.1) If $m(|\sigma'| - |\sigma|) + |\sigma'| < m$ for $(\sigma, \sigma', \alpha, \alpha')$ with $b^\alpha(\sigma', t, x) \neq 0$, then

\[(2.12) \quad (m + s_t)(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\alpha'| \leq 0,\]

for $(s_t, s_x)$ belonging to $\mathcal{S}$.

(S.C) For any $\epsilon > 0$, there exist $\tau \in \mathbb{R}^q_+$ and $\rho \in \mathbb{R}^p_+$ such that

\[(2.13) \quad \sum_{|\alpha| = 0}^{\text{finite}} \sum_{|\sigma'| = |\sigma| \leq m} |a^\alpha_{\sigma, \sigma'}| \tau^{\sigma - \sigma'} \rho^\alpha < \epsilon,

\text{where } a^\alpha_{\sigma, \sigma'} = a^\alpha_{\sigma, \sigma'}(0,0).

It is said that the condition (S.C) is Spectral condition in [M].

Then we obtain the following results.

\textbf{Theorem 2.6} Assume that $L_m$ satisfies the condition (A.1) and (S.C) and further assume that there exists a positive constant $C$ such that

\[(2.14) \quad |P_m(l)| \geq C(|l| + 1)^m \quad \text{for all } l \in \mathbb{N}^q.\]

Then the mapping

\[(2.15) \quad L_m : G^{\{s_t, s_x\}} \rightarrow G^{\{s_t, s_x\}}\]

is bijective for any $(s_t, s_x)$ belonging to $\mathcal{S}$. 
Next set

\[(2.16)\]
\[
\delta = \max \left\{ \frac{\left| \alpha' \right| + \max \left\{ \left| \sigma' \right| - m, m \left( \left| \sigma \right| - \left| \sigma' \right| \right) \right\}}{\left| \alpha \right| + \left| \sigma \right| - \left| \alpha' \right| - \left| \sigma' \right|}; b_{\sigma' \alpha'}(t, x) \neq 0 \right\}, 1 \right\}.
\]

Then for \( s_x \geq \delta \), there exist indices \( s_t \) with \( s_t \geq 1 \) such that if \( m(\left| \sigma' \right| - \left| \sigma \right|) + \left| \sigma' \right| - m \geq 0 \), then

\[(2.17)\]
\[
s_t(\left| \sigma \right| - \left| \sigma' \right|) + s_x(\left| \alpha \right| - \left| \alpha' \right|) + \left| \sigma' \right| + \left| \alpha' \right| - m \leq 0,
\]

and if \( m(\left| \sigma' \right| - \left| \sigma \right|) + \left| \sigma' \right| - m < 0 \), then

\[(2.18)\]
\[
(s_t + m)(\left| \sigma \right| - \left| \sigma' \right|) + s_x(\left| \alpha \right| - \left| \alpha' \right|) + \left| \alpha' \right| \leq 0.
\]

For example if \( s_x = s_t \geq \delta \), then the above formulas are satisfied.

So we obtain the following Corollary.

**Corollary 2.7** Assume that \( L_m \) satisfies the condition \((S.C)\) and further assume the inequality \((2.14)\). Then for \( s_x \geq \delta \), there exist indices \( s_t \) with \( s_t \geq 1 \) such that the mapping \((2.15)\) is bijective.

### 3 Proof of Theorem.

In this section first we estimate an operator of form \( t^{x^{' \alpha'}} \partial^{x^{' \alpha'}} \partial^{x^{' \alpha'}} P^{-1}_m \) on \( G_{\tau \rho}^{\{s, \epsilon\}}(T, X; m) \), next by using the estimate we show \( L_m P^{-1}_m \) is bijective on same space, at last we give a proof of Theorem 2.6.

#### 3.1 The estimate of operator \( t^{x^{' \alpha'}} \partial^{x^{' \alpha'}} \partial^{x^{' \alpha'}} P^{-1}_m \).

Here we study an estimate of the operator \( t^{x^{' \alpha'}} \partial^{x^{' \alpha'}} \partial^{x^{' \alpha'}} P^{-1}_m \) of the mapping

\[(3.1)\]
\[
t^{x^{' \alpha'}} \partial^{x^{' \alpha'}} \partial^{x^{' \alpha'}} P^{-1}_m : G_{\tau \rho}^{\{s_t, s_x\}}(T, X; m) \to G_{\tau \rho}^{\{s_t, s_x\}}(T, X; m),
\]

where

\[(3.2)\]
\[
P^{-1}_m : \sum U_{l \beta} t^{l \beta} \frac{\xi}{l! \beta!} \mapsto \sum P_m(l)^{-1} U_{l \beta} t^{l \beta} \frac{\xi}{l! \beta!}.
\]

**Lemma 3.1** Assume that the conditions in Theorem 2.6 are satisfied. Then for any \((s_t, s_x)\) belonging to \( S \), there exist a positive constant \( C \) such that the operator norm of the mapping \((3.1)\) is estimated as follows:

\[(3.3)\]
\[
\| t^{x^{' \alpha'}} \partial^{x^{' \alpha'}} \partial^{x^{' \alpha'}} P^{-1}_m \| \leq C T^{|\alpha'| - |\alpha|} X^{|\sigma'| - |\sigma|} \tau^{|\sigma'| - |\sigma|} \rho^{|\sigma' - \alpha'|},
\]

where the constant \( C \) depends only on \( m, s_t, s_x, \sigma, \sigma', \alpha \) and \( \alpha' \), and \( \| \cdot \| \) denotes the operator norm on \( G_{\tau \rho}^{\{s_t, s_x\}}(T, X; m) \).
Proof. Let \( t^\sigma x^\alpha \partial_t^\sigma \partial_x^\alpha P_{m}^{-1} U(t, x) = \sum_{\beta} V_{l\beta} t^l x^\beta / l! \beta! \). Then we obtain

\[
(3.4) \quad V_{l\beta} = \frac{l!}{(l - \sigma')! (\beta - \alpha')!} P_{m}(l + \sigma - \sigma')^{-1} U_{l\beta\alpha-} + \sigma - \sigma' + \alpha',
\]

where \( l + \sigma - \sigma' \in \mathbb{N}^q \) and \( \beta + \alpha - \alpha' \in \mathbb{N}^p \).

Therefore we have

\[
|V_{l\beta}| \leq C_0 l! \frac{|l|!}{(l - \sigma)! (\beta - \alpha')!} P_{m}(l + \sigma - \sigma')^{-1} U_{l\beta\alpha-} + \sigma - \sigma' + \alpha',
\]

By Remark 2.5 and the condition (A.1), we obtain the estimation (3.3). Q.E.D.

3.2 The estimate of operator \( L_{m}P_{m}^{-1} \).

For \( x \in \mathbb{C}^p \), we set \( |x| = x_1 + \cdots + x_p \) and \( ||x|| = |x_1| + \cdots + |x_p| \). For a domain \( \Omega \subset \mathbb{C}^p \), \( O(\Omega) \) denotes the set of holomorphic functions in \( \Omega \), \( O(\Omega) := O(\Omega) \cap \mathbb{C}(\overline{\Omega}) \). Similar notations will be used frequently for functions defined in a domain \( \mathbb{C}_{t_1x}^{q+p} \).

Let \( a(t, x) = \sum a_{l\beta} t^l x^\beta / l! \beta! \in O(||t|| \leq \kappa T) \times ||x|| \leq \kappa X) \) (\( \kappa > 0 \)) and put

\[
(3.6) \quad ||a||_{\kappa T, \kappa X} := \max_{||t|| \leq \kappa T, ||x|| \leq \kappa X} |a(t, x)|.
\]

By Cauchy's integral formula on a polycircle \( \prod_{j=1}^q \{|t_j| = \eta_j \kappa T\} \times \prod_{i=1}^p \{|x_i| = \xi_i \kappa X\} \) (\( \eta_j > 0, \eta_1 + \cdots + \eta_q = 1 \)) and \( \xi_i > 0, \xi_1 + \cdots + \xi_p = 1 \), we have

\[
(3.7) \quad |a_{l\beta}| \leq C \frac{l! \beta!}{(\kappa T)^{|l| (\kappa X)^{|\beta|}}} \frac{1}{\eta^l \xi^\beta}.
\]

Since \( \eta^l \) and \( \xi^\beta \) take its maximum on the above mentioned domain at a point \( \eta = (\eta_1/|l|, \cdots, \eta_q/|l|) \) and \( \xi = (\beta_1/|\beta|, \cdots, \beta_p/|\beta|) \), we have

\[
(3.8) \quad |a_{l\beta}| \leq ||a||_{\kappa T, \kappa X} \frac{1}{(\kappa T)^{|l| (\kappa X)^{|\beta|}}} \frac{|l|! |\beta|! |\beta|! |\beta|!}{l! |\beta|! |\beta|!}.
\]

Hence by Stirling's formula, we have

\[
(3.9) \quad |a_{l\beta}| \leq C(q, p) ||a||_{\kappa T, \kappa X} \frac{(|l| + [q/2])! (|\beta| + [p/2])!}{(\kappa T)^{|l| (\kappa X)^{|\beta|}}},
\]
for some positive constant $C(q, p)$ depending only on the dimension $q$ of $t$ and the dimension $p$ of $x$. Here $[q/2]$ (resp.$[p/2]$) denotes the integral part of $q/2$ (resp.$p/2$).

Then we have the following lemma.

**Lemma 3.2** Let $U(t, x) \in G_{\tau \rho}^{\{s, \delta_{s}\}}(T, X; m)$ and $a(t, x) \in O(\{|\tau t| \leq \kappa T\} \times \{|\rho x| \leq \kappa X\})$ ($\kappa > 0$). Then $a(t, x)U(t, x) \in G_{\tau \rho}^{\{s, \delta_{s}\}}(T, X; m)$ for any $\kappa > 1$ and it holds

$$
||aU||_{T/\tau, X/\rho; m}^{\{s, \delta_{s}\}} \leq C(q, p) \frac{[q/2]!}{(1 - 1/\kappa)^{\frac{q}{2}} + 1} \frac{[p/2]!}{(1 - 1/\kappa)^{\frac{p}{2}} + 1} 
\times ||a||_{\kappa T, \kappa X} ||U||_{T/\tau, X/\rho; m}^{\{s, \delta_{s}\}}.
$$

(3.10)

**Proof.** We may be $\rho = (1, \cdots, 1) \in \mathbb{R}_{+}^{p}$ and $\tau = (1, \cdots, 1) \in \mathbb{R}_{+}^{q}$. Set $a(t, x)U(t, x) = \sum V_{l\beta} t^{l} x^{\beta} / l! \beta!,$ where

$$
V_{l\beta} = \sum_{\substack{0 \leq n \leq l \n 0 \leq \gamma \leq \beta \n 0 \leq \gamma \leq \beta}} a_{n\gamma l \beta} U_{n \gamma} l! \beta! / n! (l - n)! \gamma! (\beta - \gamma)!.
$$

(3.11)

Then we have

$$
|V_{l\beta}| \leq C(q, p) ||a||_{\kappa T, \kappa X} ||U||_{T/\tau, X/\rho; m}^{\{s, \delta_{s}\}} \sum_{\substack{0 \leq n \leq l \n 0 \leq \gamma \leq \beta \n 0 \leq \gamma \leq \beta}} \frac{(|n| + [q/2])(|\gamma| + [p/2])!}{(\kappa T)^{|n|}(\kappa X)^{|\gamma|}} 
\times \frac{l!}{n! (l - n)!} \frac{\beta!}{\gamma! (\beta - \gamma)!}.
$$

(3.12)

Hence we have

$$
||aU||_{T/\tau, X/\rho; m}^{\{s, \delta_{s}\}} \leq C(q, p) ||a||_{\kappa T, \kappa X} ||U||_{T/\tau, X/\rho; m}^{\{s, \delta_{s}\}} \frac{[q/2]!}{(1 - 1/\kappa)^{\frac{q}{2}} + 1} \frac{[p/2]!}{(1 - 1/\kappa)^{\frac{p}{2}} + 1}.
$$

(3.13)

Q.E.D

By Lemma 3.1 and Lemma 3.2, we obtain the following essential proposition.

**Proposition 3.3** Assume that the conditions of Theorem 2.6 are satisfied. Then for any $(s_{t}, s_{x})$ belonging to $S$, there exist a positive constant $R_{0}, \tau \in \mathbb{R}_{+}^{q}$ and $\rho \in \mathbb{R}_{+}^{p}$ such that the mapping

$$
L_{m} P_{m}^{-1} : G_{\tau \rho}^{\{s_{t}, s_{x}\}}(R, R; m) \rightarrow G_{\tau \rho}^{\{s_{t}, s_{x}\}}(R, R; m)
$$

(3.14)

is bijective for any $R$ with $0 < R < R_{0}$. 

Proof. Since \( P_m P_m^{-1} = I \) on \( G_{\tau\rho}^{s_1, s_s}(R, R; m) \), it is sufficiently that we show that \( Q = (A + B)P_m^{-1} \) is a contraction mapping. By Lemma 3.1, Lemma 3.2 and the condition (A.1), we obtain an estimation \( ||AP_m^{-1}|| = O(\epsilon) + O(R) \), and by Lemma 3.1, Lemma 3.2 and the condition of the operator \( B (|\sigma'| + |\alpha'| > |\sigma| + |\alpha|) \), we obtain an estimation \( ||BP_m^{-1}|| = O(R) \). Hence the operator \( Q \) is a construction mapping on \( G_{\tau\rho}^{s_1, s_s}(R, R; m) \) for sufficiently small \( \epsilon \) and \( R \). Q.E.D.

Proof of Theorem 2.6.

Let \( P_m^{-1}U(t, x) = \sum P_m(l)^{-1}U_{l\beta}t^l x^\beta / l! \beta! \) for \( U(t, x) = \sum U_{l\beta}t^l x^\beta / l! \beta! \). By Proposition 3.3, \( L_m P_m^{-1} \) is bijective on \( G_{\tau\rho}^{s_1, s_s}(R, R; m) \), and since \( P_m^{-1}P_m = P_m P_m^{-1} = I \) (identity) holds on \( G_{\tau\rho}^{s_1, s_s}(R, R; m) \), \( L_m \) is bijective on \( G_{\tau\rho}^{s_1, s_s}(R, R; m) \). This completes the proof.

References


