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Introduction to the Method of Non-Equilibrium Thermo Field Dynamics

— A Unified System of Stochastic Differential Equations —*

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1 Introduction

In order to treat dissipative systems dynamically, we constructed the method of Non-Equilibrium Thermo Field Dynamics (NETFD) [1]-[5]. It is a *canonical operator formalism* of quantum systems in far-from-equilibrium state which enables us to treat dissipative quantum systems by a method similar to the usual quantum field theory that accommodates the concept of the dual structure in the interpretation of nature, i.e. in terms of the *operator algebra* and the *representation space.* In NETFD, the time evolution of the vacuum is realized by a condensation of $\gamma^{\natural}\tilde{\gamma}^{\natural}$-pairs into vacuum, and that the amount how many pairs are condensed is described by the one-particle distribution function $n(t)$ whose time-dependence is given by a kinetic equation (see appendix A).

Recently we succeeded to construct a unified framework of the *canonical operator formalism* for quantum stochastic differential equations with the help of NETFD [1]-[5]. To the author's knowledge, it was not realized, until the formalism of NETFD had been constructed, to put all the stochastic differential equations for quantum systems into a unified method of canonical operator formalism; the stochastic Liouville equation [6] and the Langevin equation within NETFD are, respectively, equivalent to the Schrödinger equation and the Heisenberg equation in quantum mechanics. These stochastic equations are consistent with the quantum master equation which can be derived by taking random average of the stochastic Liouville equation.

In this paper, we will investigate the structures of the stochastic differential equations in a systematic manner by means of martingale operator by paying attention to the *non-commutativity* between the annihilation and the creation random force operators.

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2 System of Stochastic Differential Equations

2.1 Stochastic Liouville Equation

Let us start the consideration with the stochastic Liouville equation of the Ito type:

$$d|0_f(t)\rangle = -i\hat{\mathcal{H}}_{f,t}dt\ |0_f(t)\rangle. \tag{1}$$

The generator $\hat{V}_f(t)$, defined by $|0_f(t)\rangle = \hat{V}_f(t)|0\rangle$ satisfies

$$d\hat{V}_f(t) = -i\hat{\mathcal{H}}_{f,t}dt\ \hat{V}_f(t), \tag{2}$$

with $\hat{V}_f(0) = 1$. The stochastic hat-Hamiltonian $\hat{\mathcal{H}}_{f,t}dt$ is a tildian operator satisfying $\left(i\hat{\mathcal{H}}_{f,t}dt\right)^{\sim} = i\hat{\mathcal{H}}_{f,t}dt$. Any operator A of NETFD is accompanied by its partner (tilde) operator $\tilde{A}$, which enables us treat non-equilibrium and dissipative systems by the method similar to usual quantum mechanics and/or quantum field theory as was pointed out before. Here, the *tilde conjugation* $\sim$ is defined by $(A_1A_2)^{\sim} = \tilde{A}_1\tilde{A}_2$, $(c_1A_1 + c_2A_2)^{\sim} = c_1^*\tilde{A}_1 + c_2^*\tilde{A}_2$, $(\tilde{A})^{\sim} = A$, $(A^\dagger)^{\sim} = \tilde{A}^\dagger$, with A's and c's being operators and c-numbers, respectively. The thermal ker-vacuum is tilde invariant: $|0_f(t)\rangle^{\sim} = |0_f(t)\rangle$.

From the knowledge of the stochastic integral, we know that the required form of the hat-Hamiltonian should be

$$\hat{\mathcal{H}}_{f,t}dt = \hat{H}dt + :d\hat{M}_t:, \tag{3}$$

where $\hat{H}$ is given by

$$\hat{H} = \hat{H}_S + i\hat{\Pi}, \qquad \text{with} \qquad \hat{H}_S = H_S - \tilde{H}_S, \qquad \hat{\Pi} = \hat{\Pi}_R + \hat{\Pi}_D, \tag{4}$$

where $\hat{\Pi}_R$ and $\hat{\Pi}_D$ are, respectively, the ***relaxational*** and the ***diffusive*** parts of the damping operator $\hat{\Pi}$. The martingale $d\hat{M}_t$ is the term containing the operators representing the quantum Brownian motion dB_t, $d\tilde{B}_t^\dagger$ and their tilde conjugates, and satisfies

$$\langle|d\hat{M}_t|\rangle = 0. \tag{5}$$

The symbol $:d\hat{M}_t:$ indicates to take the normal ordering with respect to the annihilation and the creation operators both in the relevant and the irrelevant systems (see (23)).

The operators of the quantum Brownian motion are introduced in appendix B, and satisfy the ***weak*** relations:

$$dB_t^\dagger\ dB_t = \bar{n}dt, \qquad dB_t\ dB_t^\dagger = (\bar{n}+1)\,dt, \tag{6}$$

$$d\tilde{B}_t\ dB_t = \bar{n}dt, \qquad d\tilde{B}_t^\dagger\ dB_t^\dagger = (\bar{n}+1)\,dt, \tag{7}$$

and their tilde conjugates with $\bar{n}$ being the Planck distribution function defined by (40). $\langle|$ and $|\rangle$ are the vacuum states representing the quantum Brownian motion. they are tlde invariant: $\langle|^{\sim} = \langle|$, $|\rangle \sim = |\rangle$. It is assumed that, at $t = 0$, a relevant system starts to contact with the irrelevant system representing the stochastic process included in the martingale $d\hat{M}_t$.†

2.2 Quantum Langevin Equations

The dynamical quantity $A(t)$ of the relevant system is defined by

$$A(t) = \hat{V}_f^{-1}(t)\ A\ \hat{V}_f(t), \tag{8}$$

where $\hat{V}_f^{-1}(t)$ satisfies

$$d\hat{V}_f^{-1}(t) = \hat{V}_f^{-1}(t)\ i\hat{\mathcal{H}}_{f,t}^{-}dt, \tag{9}$$

with

$$\hat{\mathcal{H}}_{f,t}^{-}dt = \hat{\mathcal{H}}_{f,t}dt + id\hat{M}_t\ d\hat{M}_t. \tag{10}$$

In NETFD, the Heisenberg equation for $A(t)$ within the Ito calculus is the quantum Langevin equation of the form

$$dA(t) = i[\hat{\mathcal{H}}_f(t)dt,\ A(t)] - d'\hat{M}(t)\ [d'\hat{M}(t),\ A(t)], \tag{11}$$

with

$$\hat{\mathcal{H}}_f(t)dt = \hat{V}_f^{-1}(t)\ \hat{\mathcal{H}}_{f,t}dt\ \hat{V}_f(t), \qquad d'\hat{M}(t) = \hat{V}_f^{-1}(t)\ d\hat{M}_t\ \hat{V}_f(t). \tag{12}$$

Since $A(t)$ is an arbitrary observable operator in the relevant system, (11) can be the Ito's formula generalized to quantum systems.

2.3 Langevin Equation for the Bra-Vector

Applying the bra-vacuum $\langle\langle 1| = \langle|\langle 1|$ to (11) from the left, we obtain the Langevin equation for the bra-vector $\langle\langle 1|A(t)$ in the form

$$d\langle\langle 1|A(t) = i\langle\langle 1|[H_S(t),\ A(t)]dt + \langle\langle 1|A(t)\hat{\Pi}(t)dt - i\langle\langle 1|A(t)\ d'\hat{M}(t). \tag{13}$$

In the derivation, use had been made of the properties

$$\langle 1|\tilde{A}^{\dagger}(t) = \langle 1|A(t), \qquad \langle|d'\tilde{B}^{\dagger}(t) = \langle|d'B(t), \qquad \langle\langle 1|d'\hat{M}(t) = 0. \tag{14}$$

Here, the thermal bra-vacuum $\langle|$ of the relevant system is tilde invariant: $\langle|^{\sim} = \langle|$.

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†Within the formalism, the random force operators dB_t and $dB_t^{\dagger}$ are assumed to commute with any relevant system operator A in the Schrödinger representation: $[A,\ dB_t] = [A,\ dB_t^{\dagger}] = 0$ for $t \geq 0$.

2.4 Quantum Master Equation

Taking the random average by applying the bra-vacuum $\langle|$ of the irrelevant sub-system to the stochastic Liouville equation (1), we can obtain the quantum master equation as

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}|0(t)\rangle, \tag{15}$$

with $\hat{H}dt = \langle|\hat{\mathcal{H}}_{f,t}dt|\rangle$ and $|0(t)\rangle = \langle|0_f(t)\rangle$.

3 An Example

3.1 Model

We will apply the above formalism to the model of a harmonic oscillator embedded in an environment with temperature T. The Hamiltonian H_S of the relevant system in (4) is given by

$$H_S = \omega a^\dagger a, \tag{16}$$

where a, $a^\dagger$ and their tilde conjugates are stochastic operators of the relevant system satisfying the canonical commutation relation

$$[a,\ a^\dagger] = 1, \qquad [\tilde{a},\ \tilde{a}^\dagger] = 1. \tag{17}$$

The tilde and non-tilde operators are related with each other by the relation

$$\langle 1|a^\dagger = \langle 1|\tilde{a}, \tag{18}$$

where $\langle 1|$ is the thermal bra-vacuum of the relevant system.

We are now confining ourselves to the case where the stochastic hat-Hamiltonian $\hat{\mathcal{H}}_t$ is bi-linear in a, $a^\dagger$, dB_t, $dB_t^\dagger$ and their tilde conjugates, and is invariant under the phase transformation $a \longrightarrow a\mathrm{e}^{i\theta}$, and $dB_t \longrightarrow dB_t\ \mathrm{e}^{i\theta}$.

Then, $\hat{\Pi}_R$ and $\hat{\Pi}_D$ consisting of $\hat{\Pi}$ introduced in (4) become

$$\hat{\Pi}_R = -\kappa\left(\gamma^{\ddagger}\gamma_\nu + \tilde{\gamma}^{\ddagger}\tilde{\gamma}_\nu\right), \qquad \hat{\Pi}_D = 2\kappa\left(\bar{n}+\nu\right)\gamma^{\ddagger}\tilde{\gamma}^{\ddagger}, \tag{19}$$

respectively, where we introduced a set of canonical stochastic operators

$$\gamma_\nu = \mu a + \nu\tilde{a}^\dagger, \qquad \gamma^{\ddagger} = a^\dagger - \tilde{a}, \tag{20}$$

with $\mu + \nu = 1$, which satisfy the commutation relation

$$[\gamma_\nu,\ \gamma^{\ddagger}] = 1. \tag{21}$$

The new operators $\gamma^{\natural}$ and $\tilde{\gamma}^{\natural}$ annihilate the relevant bra-vacuum:

$$\langle 1|\gamma^{\natural} = 0, \qquad \langle 1|\tilde{\gamma}^{\natural} = 0. \tag{22}$$

3.2 Martingale Operator

Let us adopt the martingale operator:

$$: d\hat{M}_t : = i\left[\gamma^{\natural} dW_t + \tilde{\gamma}^{\natural} d\tilde{W}_t\right] - i\lambda\left[dW_t^{\natural}\gamma_\nu + d\tilde{W}_t^{\natural}\tilde{\gamma}_\nu\right]. \tag{23}$$

Here, the annihilation and the creation random force operators dW_t and $dW_t^{\natural}$ are defined, respectively, by

$$dW_t = \sqrt{2\kappa}\left(\mu dB_t + \nu d\tilde{B}_t^{\dagger}\right), \qquad dW_t^{\natural} = \sqrt{2\kappa}\left(dB_t^{\dagger} - d\tilde{B}_t\right). \tag{24}$$

The latter annihilates the bra-vacuum $\langle|$ of the irrelevant system:

$$\langle|dW_t^{\natural} = 0, \qquad \langle|d\tilde{W}_t^{\natural} = 0. \tag{25}$$

The real parameter λ measures the degree of non-commutativity among the random force operators. There exist at least two physically attractive cases [5], i.e., one is the case for $\lambda = 0$ giving non-Hermitian martingale, and the other for $\lambda = 1$ giving Hermitian martingale. The former follows the characteristics of the classical stochastic Liouville equation where the stochastic distribution function satisfies the conservation of probability within the phase-space of a relevant system (see [6] for the system of classical stochastic differential equations). Whereas the latter employed the characteristics of the Schrödinger equation where the norm of the stochastic wave function preserves itself. In this case, the consistency with the structure of classical system is destroyed.

3.3 Fluctuation-Dissipation Theorem of the Second Kind

In order to specify the martingale, we need another condition which gives us the relation between multiple of the martingale and the damping operator:

$$d\hat{M}_t\, d\hat{M}_t = -2\left(\hat{\Pi}_R + \lambda\hat{\Pi}_D\right) dt. \tag{26}$$

This operator relation may be called a generalized fluctuation dissipation theorem of the second kind, which should be interpreted within the weak relation.

3.4 Heisenberg Operators of the Quantum Brownian Motion

The Heisenberg operators of the Quantum Brownian motion are defined by

$$B(t) = \hat{V}_f^{-1}(t)\; B_t\; \hat{V}_f(t), \qquad B^\dagger(t) = \hat{V}_f^{-1}(t)\; B_t^\dagger\; \hat{V}_f(t), \tag{27}$$

and their tilde conjugates. Their derivatives

$$dB_t^{\#} = d\left(\hat{V}_f^{-1}(t)\; B_t^{\#}\; \hat{V}_f(t)\right), \qquad (\#:\text{ nul, dagger and/or tilde}) \tag{28}$$

with respect to time in the Ito calculus are given, respectively, by

$$dB(t) = dB_t + \sqrt{2\kappa}\left[(1-\lambda)\,\nu\left(\tilde{a}^\dagger(t) - a(t)\right) - \lambda a(t)\right] dt, \tag{29}$$

$$dB^\dagger(t) = dB_t^\dagger - \sqrt{2\kappa}\left[(1-\lambda)\,\mu\left(a^\dagger(t) - \tilde{a}(t)\right) + \lambda a^\dagger(t)\right] dt, \tag{30}$$

and their tilde conjugates. Then, we have

$$dW(t) = dW_t - \lambda 2\kappa\gamma_\nu(t)dt, \qquad dW^{\text{♀}}(t) = dW_t^{\text{♀}} - 2\kappa\gamma^{\text{♀}}(t)dt. \tag{31}$$

Since, by making use of (31), we see that

$$d\hat{M}(t) = d'\hat{M}(t) = i\left[\gamma^{\text{♀}}(t)dW_t + \tilde{\gamma}^{\text{♀}}(t)d\tilde{W}_t\right] - i\lambda\left[dW_t^{\text{♀}}\gamma_\nu(t) + d\tilde{W}_t^{\text{♀}}\tilde{\gamma}_\nu(t)\right], \tag{32}$$

we know that the martingale operator in the Heisenberg representation keeps the property:

$$\langle | d\hat{M}(t) | \rangle = 0. \tag{33}$$

3.5 Explicit Forms of the Quantum Langevin Equation

The quantum Langevin equation is given by

$$\begin{aligned}
dA(t) &= i[\hat{H}_S(t),\ A(t)]dt \\
&\quad +\kappa\Big\{(1-2\lambda)\left(\gamma^{\text{♀}}(t)[\gamma_\nu(t),\ A(t)] + \tilde{\gamma}^{\text{♀}}(t)[\tilde{\gamma}_\nu(t),\ A(t)]\right) \\
&\qquad +[\gamma^{\text{♀}}(t),\ A(t)]\gamma_\nu(t) + [\tilde{\gamma}^{\text{♀}}(t),\ A(t)]\tilde{\gamma}_\nu(t)\Big\}\,dt \\
&\quad +2\kappa\,(\bar{n}+\nu)\,[\tilde{\gamma}^{\text{♀}}(t),\ [\gamma^{\text{♀}}(t),\ A(t)]]dt \\
&\quad -\left\{[\gamma^{\text{♀}}(t),\ A(t)]dW_t + [\tilde{\gamma}^{\text{♀}}(t),\ A(t)]d\tilde{W}_t\right\} \\
&\qquad +\lambda\left\{dW_t^{\text{♀}}[\gamma_\nu(t),\ A(t)] + d\tilde{W}_t^{\text{♀}}[\tilde{\gamma}_\nu(t),\ A(t)]\right\} \\
&= i[\hat{H}_S(t),\ A(t)]dt \\
&\quad +\kappa\Big\{\gamma^{\text{♀}}(t)[\gamma_\nu(t),\ A(t)] + \tilde{\gamma}^{\text{♀}}(t)[\tilde{\gamma}_\nu(t),\ A(t)]
\end{aligned} \tag{34}$$

$$
\begin{aligned}
&\quad +(1-2\lambda)\left([\gamma^{\natural}(t),\ A(t)]\gamma_\nu(t)+[\tilde{\gamma}^{\natural}(t),\ A(t)]\tilde{\gamma}_\nu(t)\right)\Big\}\,dt\\
&+2\kappa\,(\bar{n}+\nu)\,[\tilde{\gamma}^{\natural}(t),\ [\gamma^{\natural}(t),\ A(t)]]dt\\
&-\Big\{[\gamma^{\natural}(t),\ A(t)]dW(t)+[\tilde{\gamma}^{\natural}(t),\ A(t)]d\tilde{W}(t)\Big\}\\
&\quad +\lambda\Big\{dW^{\natural}(t)[\gamma_\nu(t),\ A(t)]+d\tilde{W}^{\natural}(t)[\tilde{\gamma}_\nu(t),\ A(t)]\Big\},
\end{aligned}
\tag{35}
$$

with $\hat{H}_S(t)=\hat{V}_f^{-1}(t)\hat{H}_S\hat{V}_f(t)=H_S(t)-\tilde{H}_S(t)$. Note that the Langevin equation is written by means of the quantum Brownian motion in the Schrödinger (the interaction) representation (the input field [7]) in (34), and by means of that in the Heisenberg representation (the output field [7]) in (35).

The Langevin equation for the bra-vector state, $\langle\!\langle 1|A(t)$, reduces to

$$
\begin{aligned}
d\langle\!\langle 1|A(t) &= i\langle\!\langle 1|[H_S(t),\ A(t)]dt\\
&\quad -\kappa\Big\{\langle\!\langle 1|[A(t),\ a^\dagger(t)]a(t)+\langle\!\langle 1|a^\dagger(t)[a(t),\ A(t)]\Big\}\,dt\\
&\quad +2\kappa\bar{n}\langle\!\langle 1|[a(t),\ [A(t),\ a^\dagger(t)]]dt\\
&\quad +\langle\!\langle 1|[A(t),\ a^\dagger(t)]\sqrt{2\kappa}\,dB_t+\langle\!\langle 1|\sqrt{2\kappa}\,dB_t^\dagger[a(t),\ A(t)] \qquad (36)\\
&= i\langle\!\langle 1|[H_S(t),\ A(t)]dt\\
&\quad -\kappa(1-2\lambda)\Big\{\langle\!\langle 1|[A(t),\ a^\dagger(t)]a(t)+\langle\!\langle 1|a^\dagger(t)[a(t),\ A(t)]\Big\}\,dt\\
&\quad +2\kappa\bar{n}\langle\!\langle 1|[a(t),\ [A(t),\ a^\dagger(t)]]dt\\
&\quad +\langle\!\langle 1|[A(t),\ a^\dagger(t)]\sqrt{2\kappa}\,dB(t)+\langle\!\langle 1|\sqrt{2\kappa}\,dB^\dagger(t)[a(t),\ A(t)]. \qquad (37)
\end{aligned}
$$

The relation between the expression (36) and (37) can be interpreted as follows. Substituting the *solution* of the Heisenberg random force operators (29) and (30) for $dB(t)$ and $dB^\dagger(t)$, respectively, into (37), we obtain the quantum Langevin equation (36) which does not depend on the non-commutativity parameter λ.

4 Concluding Remarks

We enumerate here the steps how to derive the quantum Langevin equation from the microscopic point of view with the help of the field theoretical formalism, NETFD, in order to show what was revealed and what is to be solved. We interpret that the process in deriving the quantum Langevin equation starting with the Heisenberg equation, whose time evolution generator is unitary, is realized by changing representation spaces, i.e., Changing the representation space to the one representing the quantum Brownian motion from the ordinary one [8], the term given by the interaction Hamiltonian reduces to the martingale term (23) with $\lambda = 1$ [5]. Then, the Heisenberg equation should be interpreted as the stochastic differential equation of the Stratonovich type. Note that the introduction of the stochastic calculus is nothing but the introduction of coarse graining [9]. In rewriting the Langevin equation of the Stratonovich type into that of the Ito type, we see that there appear the terms taking care of relaxation and diffusion as can be shown in (35).

Introducing the parameter λ in the martingale term as given by (23), we can transform the equation to the non-Hermitian version by shifting $\lambda \to 0$ (see (35)). In other words, it seems that the non-commutativity is renormalized into the relaxational and diffusive terms. Substituting the solution of the random force operators in the Heisenberg representation (the output field), we have the Langevin equation expressed by means of those in the Schrödinger (or, more properly, the interaction) representation (the input field). Note that the Langevin equation for the bra-vector state $\langle\!\langle 1|A(t)$ does not depend on λ when it is represented by the random force operator in the Schrödinger representation (the input field). We are intensively investigating what is the physical meaning of the renormalization of non-commutativity by changing the parameter λ.

We would like to close the paper by quoting several comments. An extension of NETFD to the hydrodynamical stage is one of the challenging future problem related to the dynamical mapping [3, 10]. An interpretation of the stochastic calculuses in terms of the projection operator method will be published elsewhere [9]. The system of the stochastic differential equations within NETFD will be applied to the problem of the well-localized paths of ionized molecules in the cloud chamber as an example of the non-demolition continuous measurements, i.e., the quantum Zeno effects [11].

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A Condensation of Thermal Pair

The time-evolution of the thermal vacuum $|0(t)\rangle$, satisfying the quantum master equation (15) with the hat-Hamiltonian (4) for the semi-free system specified by (16) and (19), is given by

$$|0(t)\rangle\!\rangle = \exp\left\{[n(t) - n(0)]\,\gamma^{\ddagger}\tilde{\gamma}^{\ddagger}\right\}|0\rangle, \tag{38}$$

where the one-particle distribution function, $n(t) = \langle\!\langle 1|a^{\dagger}(t)a(t)|0\rangle\!\rangle$, satisfies the kinetic (Boltzmann) equation of the model:

$$\frac{d}{dt}n(t) = -2\kappa\,[n(t) - \bar{n}]\,, \tag{39}$$

with the Planck distribution function

$$\bar{n} = \left(e^{\omega/T} - 1\right). \tag{40}$$

Here, T is the temperature of environment system, and ω represents the frequency of the harmonic oscillator under consideration.

B Quantum Brownian Motion

Let us introduce the annihilation and creation operators b_t, $b_t^{\dagger}$ and their tilde conjugates satisfying the canonical commutation relation:

$$[b_t,\ b_{t'}^{\dagger}] = \delta(t - t'), \qquad [\tilde{b}_t,\ \tilde{b}_{t'}^{\dagger}] = \delta(t - t'). \tag{41}$$

The vacuums $(0|$ and $|0)$ are defined by

$$b_t|0) = 0, \quad \tilde{b}_t|0) = 0, \quad (0|b_t^{\dagger} = 0, \quad (0|\tilde{b}_t^{\dagger} = 0. \tag{42}$$

The argument t represents time.

Introducing the operators

$$B_t = \int_0^{t-dt} dB_{t'} = \int_0^t dt'\ b_{t'}, \qquad B_t^{\dagger} = \int_0^{t-dt} dB_{t'}^{\dagger} = \int_0^t dt'\ b_{t'}^{\dagger}, \tag{43}$$

and their tilde conjugates for $t \geq 0$, we see that they satisfy $B(0) = 0$, $B^{\dagger}(0) = 0$,

$$[B_s,\ B_t^{\dagger}] = \min(s, t), \tag{44}$$

and their tilde conjugates, and that they annihilate the vacuums $|0)$ and $(0|$:

$$dB_t|0) = 0, \quad d\tilde{B}_t|0) = 0, \quad (0|dB_t^{\dagger} = 0, \quad (0|d\tilde{B}_t^{\dagger} = 0. \tag{45}$$

These operators represent the quantum Brownian motion.

Let us introduce a set of new operators by the relation

$$dC_t^{\mu} = \mathcal{B}^{\mu\nu} dB_t^{\nu}, \tag{46}$$

with the Bogoliubov transformation defined by

$$\mathcal{B}^{\mu\nu} = \begin{pmatrix} 1+\bar{n} & -\bar{n} \\ -1 & 1 \end{pmatrix}, \tag{47}$$

where $\bar{n}$ is the Planck distribution function. We introduced the thermal doublet:

$$dB_t^{\mu=1} = dB_t, \quad dB_t^{\mu=2} = d\tilde{B}_t^{\dagger}, \quad d\bar{B}_t^{\mu=1} = dB_t^{\dagger}, \quad d\bar{B}_t^{\mu=2} = -d\tilde{B}_t, \tag{48}$$

and the similar doublet notations for dC_t^{μ} and $d\bar{C}_t^{\mu}$. The new operators annihilate the new vacuum $\langle|$ and $|\rangle$:

$$dC_t|\rangle = 0, \quad d\tilde{C}_t|\rangle = 0, \quad \langle|dC_t^{\dagger} = 0, \quad \langle|d\tilde{C}_t^{\dagger} = 0. \tag{49}$$

We will use the representation space constructed on the vacuums $\langle|$ and $|\rangle$. Then, we have, for example,

$$\langle|dB_t|\rangle = \langle|dB_t^{\dagger}|\rangle = 0, \tag{50}$$

$$\langle|dB_t^{\dagger}dB_t|\rangle = \bar{n}dt, \qquad \langle|dB_t dB_t^{\dagger}|\rangle = (\bar{n}+1)\,dt. \tag{51}$$

C Stratonovich-Type Stochastic Equations

By making use of the relation between the Ito and Stratonovich stochastic calculuses, we can rewrite the Ito stochastic Liouville equation (1) and the Ito Langevin equation (11) into the Stratonovich ones, respectively, i.e.,

$$d|0(t)\rangle = -i\hat{H}_{f,t} \circ |0(t)\rangle, \tag{52}$$

$$\hat{H}_{f,t}dt = \hat{H}_S dt + i\left(\hat{\Pi} dt + \frac{1}{2} d\hat{M}_t d\hat{M}_t\right) + d\hat{M}_t, \tag{53}$$

and

$$dA(t) = i[\hat{H}_f(t)dt \circ\!\!,\, A(t)], \tag{54}$$

with

$$\hat{H}_f(t)dt = \hat{H}_S(t)dt + i\left(\hat{\Pi}(t)dt + \frac{1}{2} d'\hat{M}(t)d'\hat{M}(t)\right) + :d'\hat{M}(t):. \tag{55}$$

The symbol $\circ$ indicates the Stratonovich multiplication in the stochastic calculus.

D Hat-Hamiltonians of the Model

The hat-Hamiltonians (53), (10) and (55) of the model are, respectively, given by

$$\hat{H}_{f,t}dt = \hat{H}_S dt + i(1-\lambda)\hat{\Pi}_R dt + d\hat{M}_t, \tag{56}$$

$$\hat{\mathcal{H}}^{-}_{f,t}dt = \hat{H}_S dt - i\left(\hat{\Pi}_D + (2\lambda-1)\hat{\Pi}_R\right)dt + d\hat{M}_t, \tag{57}$$

$$\hat{H}_f(t)dt = \hat{H}_S(t)dt + i(1-\lambda)\hat{\Pi}_R(t)dt + :d'\hat{M}(t):. \tag{58}$$

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