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On the cohomology of finite Chevalley groups and free loop spaces of classifying spaces

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Abstract

1 notations

Let p be a prime and \mathbb{F}_q be the finite field with q elements. Let $G_{\mathbb{Z}}$ be a Chevalley group scheme such that \mathbb{C} -rational points $G_{\mathbb{Z}}(\mathbb{C})$ is a simply connected complex Lie group when we change its topology. Hereafter we denote $G_{\mathbb{Z}}(K)$ (resp. $G_{\mathbb{Z}}(\mathbb{C})$) by $G(K)$ (resp. G) for a field K . We also denote its classifying space by BG and define the free loop space $\mathcal{L}BG$ of BG and the loop space ΩBG of BG by

$$\mathcal{L}BG = \{l \mid l: S^1 \rightarrow BG\} \quad \text{and} \quad \Omega BG = \{l \mid l(1) = *, l \in \mathcal{L}BG\},$$

where S^1 is the unit circle on the complex number \mathbb{C} and $*$ is a base point of BG . It is well known that ΩBG is weakly homotopy equivalent to G .

2 results and comments

Theorem . *Let \mathbb{F}_q be a finite field with $q = p^n$ elements and l be a prime number that divides $q - 1$ but does not divide the order of the Weyl group of G . Then we have an ring isomorphism*

$$H^*(\mathcal{L}BG, \mathbb{Z}/l) \cong H^*(G(\mathbb{F}_q), \mathbb{Z}/l) \cong H^*(BG, \mathbb{Z}/l) \otimes H^*(G, \mathbb{Z}/l).$$

We can prove the theorem immediately from Kleinerman [3] and Kono-Kozima [4].

Remark . Our theorem is partial. Here we indicate an example.

Theorem (Fong-Milgram [1], Kono-Kozima [4]). *Let G_2 be an exceptional Lie type G_2 . Then we have a ring isomorphism*

$$H^*(\mathcal{L}BG_2, \mathbb{Z}/2) \cong H^*(G_2(\mathbb{F}_q), \mathbb{Z}/2)$$

for $4|q - 1$.

We propose a question : Let l be a prime number such that l (resp. 4) divides $q - 1$ if l is odd (resp. even). Then we have a ring isomorphism

$$H^*(\mathcal{L}BG, \mathbb{Z}/l) \cong H^*(G(\mathbb{F}_q), \mathbb{Z}/l)$$

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