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MAPPING CLASS GROUPS OF 3-DIMENSIONAL HANDLEBODIES AND MERIDIAN DISKS

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1. Introduction

A genus g *handlebody*, H_g, is an oriented 3-manifold, which is constructed from a 3-ball with attaching g 1-handles. Let $Diff^+(H_g)$ (resp. $Diff^+(\partial H_g)$) be the group of orientation preserving diffeomorphisms on H_g (resp. ∂H_g), $\mathcal{H}_g$ (resp. $\mathcal{M}_g$) be a group which consists of isotopy classes of $Diff^+(H_g)$ (resp. $Diff^+(\partial H_g)$) . Generators of $\mathcal{H}_g$ are given in [11] and [7]. Wajnryb gave a presentation for $\mathcal{H}_g$ in [12]. In this note, we give a presentation for $\mathcal{H}_g$ with using other method. When $g \geq 3$, we use a simplicial action of $\mathcal{H}_g$ on simplicial complex (which is a subcomplex of a contractible complex defined by McCullough [10]) defined as follows: its vertices are isotopy classes of *meridian disks* in H_g (essential 2-disks properly embedded in H_g), and its simplex is a system of isotopy classes of meridian disks which are represented by disks, which are disjoint and non-isotopic each other and whose complements is connected. This complex is $(g-2)$-connected, especially, if $g \geq 3$, it is simply connected. (When $g=2$, unfortunatelly, this complex is not simply connected, hence we use a contractible complex defined in [10].) This is subcomplex of a complex X defined by Harer in [5]. Since the orbit space of the former one by $\mathcal{H}_g$ is identical with the latter one by $\mathcal{M}_g$, our method can be applied to giving a presentation for $\mathcal{M}_g$ without using a complex defined by Hatcher and Thurston [4].

This note is a summary of a paper [6].

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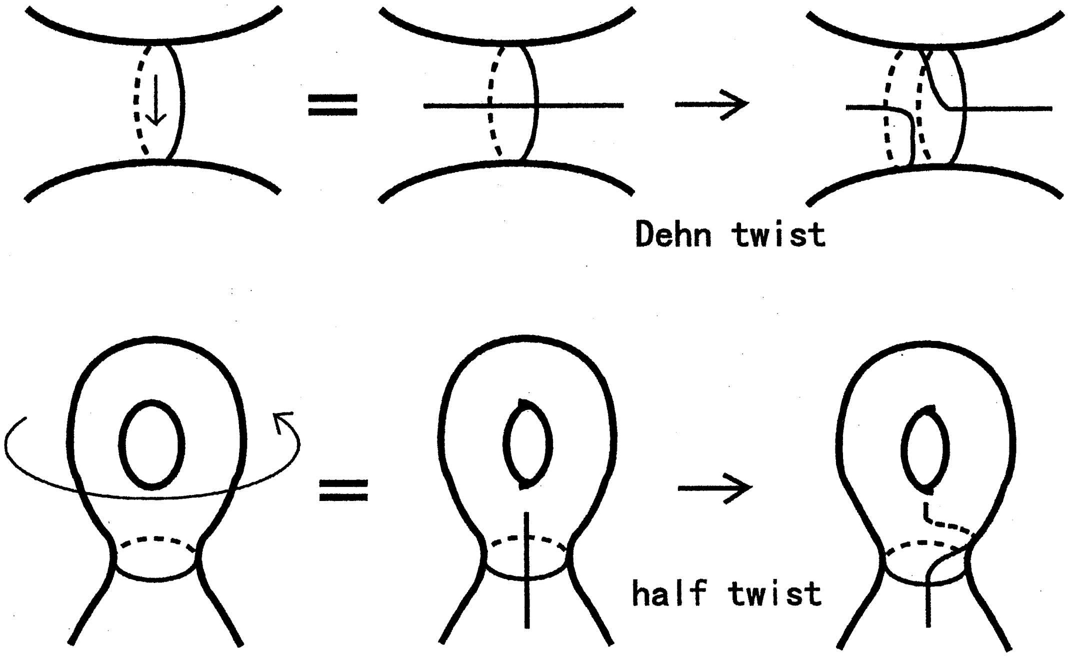

FIGURE 1

2. A PRESENTATION FOR $\mathcal{H}_g$

Before we state a presentation for $\mathcal{H}_g$, we set notations used there. Sometimes, we indicate an element of $\mathcal{H}_g$ by a figure. In Figure 1, the left hand side figure denotes an element given in the right hand side figure. The symbol $\rightleftarrows$ means *commute with.* For example, if L, M, N are any elements of $\mathcal{H}_g$, a relation $L \rightleftarrows M, N$ means that $LM = ML$, $LN = NL$. The commutator of A and B, $A^{-1}B^{-1}AB$, is denoted by $[A, B]$. In this paper, we consider that the group $\mathcal{H}_g$ acts on H_g from the right: for any elements ϕ_1, ϕ_2 of $\mathcal{H}_g$, $\phi_1\phi_2$ means apply ϕ_1 first, then apply ϕ_2.

Theorem 2.1. *Let a_1, k_1, d_i $(2 \leq i \leq g), t(2)_{21}, r(2)_{21}$ be the elements of $\mathcal{H}_g$ indicated in Figure 2. The group $\mathcal{H}_g$ admits a presentation with generators a_1,, k_1, d_i $(2 \leq i \leq g)$, $t(2)_{21}$, $r(2)_{21}$ and defining relations:*
if $g \geq 4$,

(A1)
$$d_m^{-1}a_md_m = a_{m-1}, \quad d_m^{-1}a_{m-1}d_m = a_m,$$
$$d_m^{-1}k_md_m = k_{m-1}, \quad d_m^{-1}k_{m-1}d_m = k_m,$$
where $2 \leq m \leq g$,

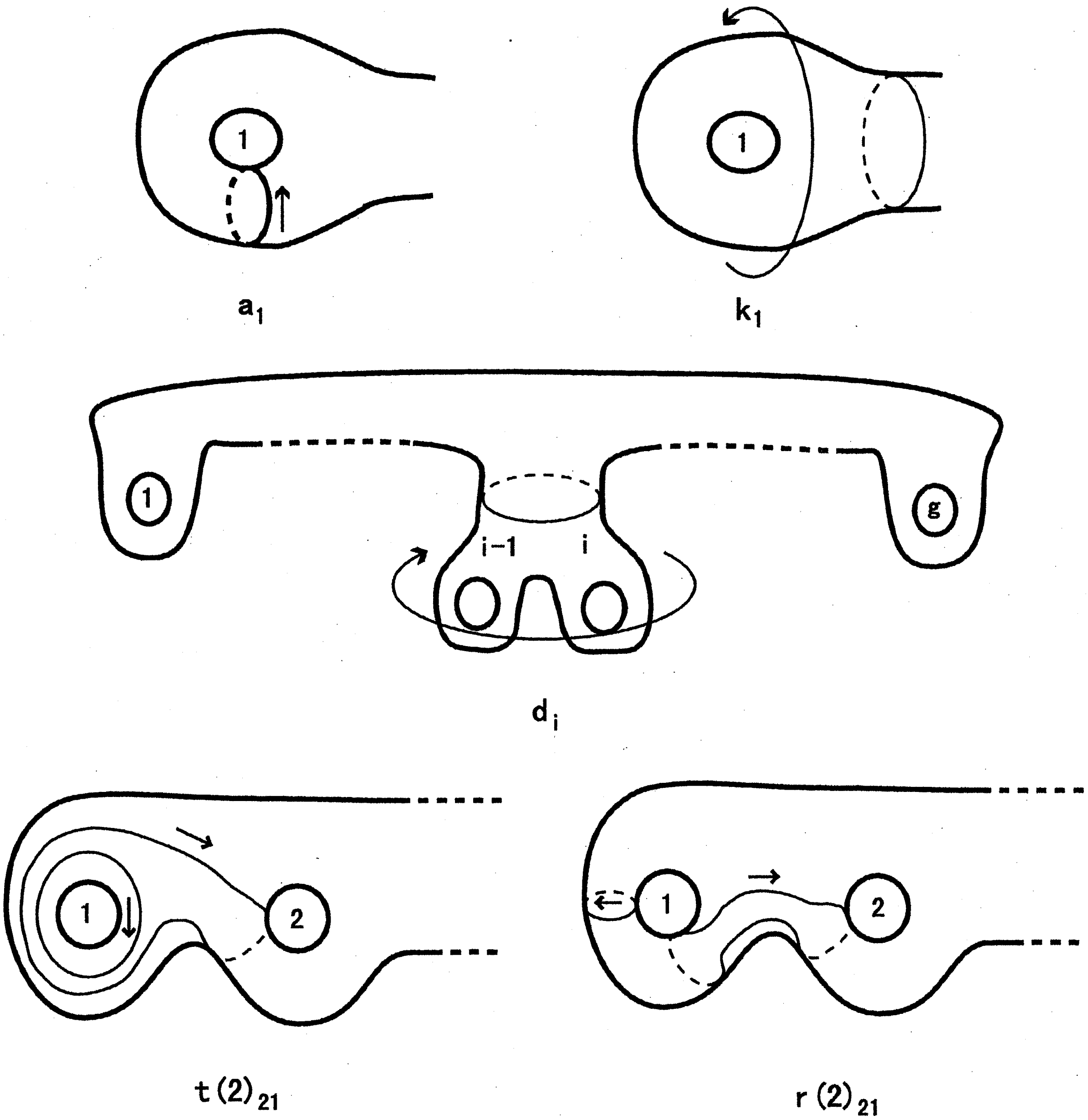

FIGURE 2

(A2)
$$r_g = \begin{cases} d_2d_3\cdots d_gk_1^{-1}k_2^{-1}\cdots k_{g-1}^{-1}, & g \textit{ is odd,} \\ d_2d_3\cdots d_gk_1^{-1}k_2^{-1}\cdots k_g^{-1}, & g \textit{ is even,} \end{cases}$$

(A3)
$$d_1 = r_g^{-1}d_2r_g,$$

(A4)
$$r(m)_{1j} = k_mr(m)_{2j}k_m^{-1}, \quad t(m)_{1j} = k_mt(m)_{2j}k_m^{-1},$$

(A5)
$$r(m)_{ij} = d_jk_{j-1}^{-1}r(m)_{i,j-1}k_{j-1}d_j^{-1},$$
$$t(m)_{ij} = d_jk_{j-1}^{-1}t(m)_{i,j-1}k_{j-1}d_j^{-1},$$
where $m = 1, \cdots, g,\ i = 1, 2,\ j \neq m, m-1,$ *and index* j *is given modulo* g,

(A6)
$$r(m)_{ij} = d_mk_{m-1}^{-1}r(m-1)_{i,j}k_{m-1}d_m^{-1},$$
$$t(m)_{ij} = d_mk_{m-1}^{-1}t(m-1)_{i,j}k_{m-1}d_m^{-1},$$
where $m = 1, \cdots, g,\ i = 1, 2,\ j \neq m, m-1,$ *and index* j *is given modulo* g,

(A7)
$$c(m)_i[j, j+1, \cdots, j+k] = (\prod_{n=0}^{k}[t(m)_{i,j+n}, r(m)_{i,j+n}^{-1}])a_m^{-(2k+1)},$$
where $i = 1, 2,$ *and index* j *is given modulo* g,

(A8)
$$A_m = k_m^{-2}, \text{ where } 1 \leq m \leq g,$$

(A9)
$$t_1 = t(2)_{21}k_1^{-1}a_2,$$

(B1)
$$d_i \rightleftarrows d_j, \text{ where } |i-j| \geq 2,$$
$$d_i^{-1}d_{i-1}^{-1}d_id_{i-1}d_i = d_{i-1} \text{ where } 2 \leq i \leq g,$$

(B2)
$$k_1 \rightleftarrows k_2,$$
$$a_1 \rightleftarrows a_2,$$

(B3)
$$a_1, k_1 \rightleftarrows d_i, \text{ where } 3 \leq i \leq g,$$
$$a_g, k_g \rightleftarrows d_j, \text{ where } 2 \leq j \leq g-1,$$

(B4)
$$k_1 \rightleftarrows a_1, a_2,$$

(B5)
$$d_3 \rightleftarrows a_1,$$

(B6)
$$t(3)_{11},\ r(3)_{11} \rightleftarrows t(3)_{22},\ r(3)_{22},$$

(B7)
$$\begin{aligned} t(2)_{21}A_2t(2)_{11}A_2^{-1}t(2)_{21}^{-1} &= t(2)_{11}, \\ t(2)_{21}A_2r(2)_{11}A_2^{-1}t(2)_{21}^{-1} &= a_2^{-2}A_2r(2)_{11} \\ r(2)_{21}A_2t(2)_{11}r(2)_{21}^{-1} &= a^{-2}t(2)_{11} \\ r(2)_{21}A_2r(2)_{11}A_2^{-1}r(2)_{21}^{-1} &= r(2)_{11} \end{aligned}$$

(B8)
$$\begin{aligned} &k_1^{-1}t(2)_{21}k_1 = t(2)_{21}^{-1}r(2)_{21}t(2)_{21}^{-1}r(2)_{21}^{-1}t(2)_{21}, \\ &k_1^{-1}r(2)_{21}k_1 = t(2)_{21}^{-1}r(2)_{21}^{-1}t(2)_{21}, \end{aligned}$$

(B9)
$$\begin{aligned} a_1 &\rightleftarrows r(2)_{21}, \\ a_1^{-1}t(2)_{21}a_1 &= (r(2)_{21})^{-1}t(2)_{21} \end{aligned}$$

(B10)
$$a_2 \rightleftarrows t(2)_{21}, r(2)_{21},$$

(B11)
$$d_2 \rightleftarrows r(2)_{21}(a_1)^{-1}$$

(B12)
$$a_3, k_3 \rightleftarrows t(2)_{21}, r(2)_{21},$$

(B13)
$$t(2)_{2,1}, r(2)_{2,1} \rightleftarrows t(4)_{2,3}, r(4)_{2,3},$$

(B14)
$$\begin{aligned} t(2)_{2,1}^{-1}t(3)_{2,1}t(2)_{2,1} &= (t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2})^{-1}t(3)_{2,1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2}), \\ t(2)_{2,1}^{-1}t(3)_{2,1}^{-1}r(3)_{2,1}t(2)_{2,1} &= a_3^{-1}t(3)_{2,1}^{-1}r(3)_{2,1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2}), \\ t(2)_{2,1}^{-1}t(3)_{2,2}t(2)_{2,1} &= a_3^{-1}t(3)_{2,2}t(3)_{2,1}^{-1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2}), \\ t(2)_{2,1} &\rightleftarrows r(3)_{2,2}, \end{aligned}$$

(B15)
$$\begin{aligned} r(2)_{2,1}^{-1}t(3)_{2,1}r(2)_{2,1} &= a_3^{-1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2})^{-1}t(3)_{2,1}, \\ r(2)_{2,1}^{-1}r(3)_{2,1}r(2)_{2,1} &= (t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2})^{-1}r(3)_{2,1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2}), \\ r(2)_{2,1}^{-1}t(3)_{2,2}r(2)_{2,1} &= a_3^{-1}t(3)_{2,2}r(3)_{2,1}^{-1}(t(3)_{2,2}^{-1}r(3)_{2,2}t(3)_{2,2}), \\ r(2)_{2,1} &\rightleftarrows r(3)_{2,2}, \end{aligned}$$

(B16)
$$d_2^2 = a_2^{-4}\{(t(2)_{11})^{-1}r(2)_{11}t(2)_{11}(r(2)_{11})^{-1}\}k_2^2\{(t(2)_{21})^{-1}r(2)_{21}t(2)_{21}(r(2)_{21})^{-1}\}k_1^2,$$

(B17)
$$\begin{aligned} r(g)_{2,g-1}, t(g)_{2,g-1} &\rightleftarrows d_i \textit{ where } 2 \leq i \leq g-2,\\ r(g)_{2,1}, t(g)_{2,1} &\rightleftarrows d_i \textit{ where } 3 \leq i \leq g-1,\\ t(2)_{2,1}, t(2)_{2,1} &\rightleftarrows d_i \textit{ where } 4 \leq i \leq g, \end{aligned}$$

(B18)
$$\begin{aligned} &d_2^{-1} r(3)_{2,1} d_l\\ &\quad = t(3)_{2,1}^{-1} r(3)_{2,1} t(3)_{2,1} r(3)_{2,1}^{-1} \times t(3)_{2,2}^{-1} r(3)_{2,2}^{-1} t(3)_{2,2}\\ &\qquad \times r(3)_{2,1} t(3)_{2,1}^{-1} r(3)_{2,1}^{-1} t(3)_{2,1},\\ &d_2^{-1} t(3)_{2,1} d_2\\ &\quad = t(3)_{2,1}^{-1} r(3)_{2,1} t(3)_{2,1} r(3)_{2,1}^{-1} \times t(3)_{2,2}^{-1} r(3)_{2,2} t(3)_{2,2}^{-1} r(3)_{2,2}^{-1} t(3)_{2,2}\\ &\qquad \times r(3)_{2,1} t(3)_{2,1}^{-1} r(3)_{2,1}^{-1} t(3)_{2,1}, \end{aligned}$$

(B19)
$$(d_2^{-1} t_1 d_2 t_1)^3 = d_2^2,$$

(B20)
$$A_g = \prod_{i=1}^{g-1} [t(g)_{2,i}, r(g)_{2,i}^{-1}] a_g^{-2(g-1)},$$

(B21)
$$r(2)_{11} = r(1)_{12} a_1 a_2^{-1}$$

(B22)
$$t(2)_{11} t(2)_{21} A_2 a_2^2 = \prod_{i=3}^{g} \{ t(i)_{2,1}^{-1} t(i)_{1,1}^{-1} c(i)_2[2, \cdots, i-1] c(i)_1[2, \cdots, i-1] A_i^{-1} a_i^{-4} \},$$

(B23)
$$k_1^2 A_2 = \prod_{i=3}^{g} \{ c(i)_2[2, \cdots, i-1] c(i)_1[2, \cdots, i-1] A_i^{-1} a_i^{-2} \},$$

and if $g=3$, the above relations except (B13) and (B17) are satisfied and sufficient, and if $g=2$, (A1), (A4), (A8), (A9), (B2), (B4), (B7), (B8), (B9), (B10), (B11), (B20) and

(B16')
$$d_2^2 = 1,$$

(B19')
$$(d_2^{-1} t_1 d_2 t_1)^3 = 1,$$

(B21')
$$r(2)_{11} r(2)_{21} A_2 a_2^2 = 1,$$

(B22')
$$t(2)_{11} t(2)_{21} A_2 a_2^2 = 1,$$

(B23')
$$k_1^2 A_2 = 1,$$

are satisfied and sufficient.

In this presentation, (A*)'s are the relations which define some generators from a_1, k_1, d_i ($2 \leq i \leq g$), $t(2)_{2,1}$, and $r(2)_{2,1}$ (these are indicated in the sequal of this paper by Figures). (B*)'s are easily checked by drawing some figures. From here to the end of this paper, we will show sufficiency of these relations.

3. Disk complices

Let H_g be a three dimensional handlebody of genus g, $E_1, \dots, E_l$ be mutually disjoint 2-disks embedded in ∂H_g. By a *disc* in $(H_g, \{E_1, \dots, E_l\})$ we mean a properly imbedded 2-disc $(D, \partial D) \subseteq (H_g, \partial H_g)$ which is disjoint from $E_1 \cup \cdots \cup E_l$. The disc D is called *meridian disc* in $(H_g, \{E_1, \dots, E_l\})$ when $H_g - D$ is connected. Define the *nonseparating disc complex* of H_g to be the simplicial complex $L'(H_g, \{E_1, \dots, E_l\})$ whose vertices(0-simplices) are the isotopy classes of meridian discs in $(H_g, \{E_1, \dots, E_l\})$, and whose simplices are determined by the rule that a collection of $n+1$ distinct vertices spans an n-simplex if and only if it admits a collection of representative which are pairwise disjoint. Define the comlex $Y(H_g, \{E_1, \dots, E_l\})$ to be the subcomplex of $L'(H_g, \{E_1, \dots, E_l\})$ whose n-simplex is determined by $n+1$ distinct vertices represented by pairwise disjoint discs $D_0, D_1, \dots, D_n$ such that $H_g - D_0 \cup D_1 \cup \cdots \cup D_n$ is connected. If there is no distinguished discs $\{E_1, \dots, E_l\}$ on ∂H_g, we denote these complices by the notation $L'(H_g)$ and $Y(H_g)$. We call a *system of meridian discs* the set of mutually disjoint and nonisotopic meridian discs in $(H_g, \{E_1, \dots, E_l\})$. Each simplex of $L'(H_g, \{E_1, \dots, E_l\})$ is represented by a system of meridian disks. The definition of $L'(H_g, \{E_1, \dots, E_l\})$ is a modification of the *disc complex* defined in section 5 of [10]. The following theorem is proved by a slight modification of the proof for Theorem 5.2 of [10].

Theorem 3.1. *$L'(H_g, \{E_1, \dots, E_l\})$ is contractible.* □

We will show the following theorem.

Theorem 3.2. *$Y(H_g, \{E_1, \dots, E_l\})$ is $(g-2)$-connected.*

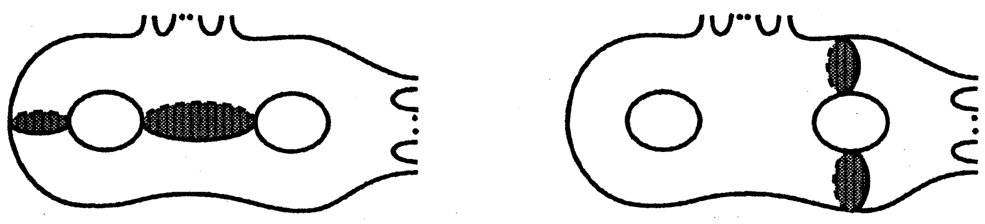

FIGURE 3

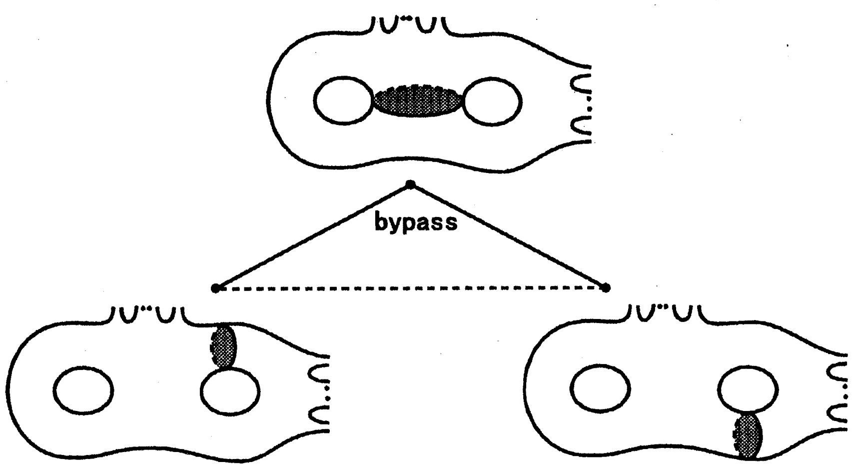

FIGURE 4

This comlex $Y(H_g, \{E_1, \dots, E_l\})$ resembles complices X defined in [5] and Y defined in [3]. We prove the above theorem as in the proof of Theorem 1.1 of [5] and the proof of Propositon of [3].

Proof. We prove this theorem by the induction of genus g.

At first, we prove $Y(H_2, \{E_1, \dots, E_l\})$ is connected(0-connected). By 3.1, $L'(H_2, \{E_1, \dots, E_l\})$ is connected. As is indicated in Figure 3, there is two types of edges in $L'(H_2, \{E_1, \dots, E_l\})$. The first one is in $Y(H_2, \{E_1, \dots, E_l\})$. The second one is not in $Y(H_2, \{E_1, \dots, E_l\})$, but there is a bypass in $Y(H_2, \{E_1, \dots, E_l\})$ given in Figure 4. Hence, $Y(H_2, \{E_1, \dots, E_l\})$ is connected.

We assume, for any integer k smaller than g, $Y(H_k, \{E_1, \dots, E_l'\})$ is $(k-2)$-connected. Let i be an integer smaller than or equal to $g-2$, and $f : S^i \longrightarrow Y(H_g, \{E_1, \dots, E_l\})$ be a continuous map. Since $L'(H_g, \{E_1, \dots, E_l\})$ is contractible, there is a continuous map $\tilde{f} : D^{i+1} \longrightarrow L'(H_g, \{E_1, \dots, E_l\})$ such that $\tilde{f}|_{\partial D^{i+1}} = f$.

We may assume $\tilde{f}$ is piecewise linear with respect to some triangulation of D^{i+1}. If $\tilde{f}(D^{i+1})$ is contained in $Y(H_g, \{E_1, \dots, E_l\})$, this means that f is mull-homotpic. We assume $\tilde{f}(D^{i+1})$ is not contained in $Y(H_g, \{E_1, \dots, E_l\})$. Then, there is a simplex σ of D^{i+1} such that $\tilde{f}(\sigma)$ is not contained in $Y(H_g, \{E_1, \dots, E_l\})$, and this means that there is a representative $\{D_0 \cup \dots \cup D_j\}$ of $\tilde{f}(\sigma)$ with disconnected complement. Let G_σ be a graph defined as follows. Each vertex corresponds to components of $H_g - D_0 \cup \dots \cup D_j$. Each edge corresponds to one of $D_0, \dots, D_j$ and connecting vertices corresponding to the components containing this disc. Since $D_0 \cup \dots \cup D_j$ has disconnected complement, G_σ has at least two components. There is a nonempty subgraph of G_σ whose edges connects distinct vertices. Let $\mathcal{D}$ be the subsystem of $\{D_0, \dots, D_j\}$ which corresponds to the edges of this graph. This system $\mathcal{D}$ satisfies the following property:

(*) Let $\mathcal{D} = \{D'_0, \dots, D'_m\}$. Each $D'_a \in \mathcal{D}$ separetes $H_g - \cup_{n \neq a} D'_n$.

A system of meridian discs which satisfies property(*) is called *purely separating* system of meridian discs, and a simplex of $L'(H_g, \{E_1, \dots, E_l\})$ is called *purely separating* if it is represented by a purely separating system of meridian discs. $\mathcal{D}$ represent a face of $\tilde{f}\sigma$, so there is a simplex τ of D^{i+1} such that $\tilde{f}(\tau)$ is purely separating. In general, we have shown the following lemma.

Lemma 3.3. *Each simplex in $L'(H_g, \{E_1, \dots, E_l\})$ has a face which is purely separating.*

Let σ be a simplex of D^{i+1} of maximal dimension p such that $\tilde{f}(\sigma)$ is purely separating. This simplex σ is not contained in ∂D^{i+1}, hence, *Link* σ is homeomophic to S^{i-p}. Let $\{D_0, \dots, D_p\}$ be a system of meridian discs which represents $\tilde{f}(\sigma)$, and let $M = H_g - D_0 \cup \dots \cup D_p$. We define the complex $Y(M, *)$ for $(M, \{E_1, \dots, E_l,$ discs which are emerged as a cut end along $D_0 \cup \dots \cup D_p\})$ in the same manner as $Y(H_g, \{E_1, \dots, E_l\})$. Then, each disc which represents each vertex of *Link* σ does not separate M. In fact, if some vertex v of *Link* σ separates M, then

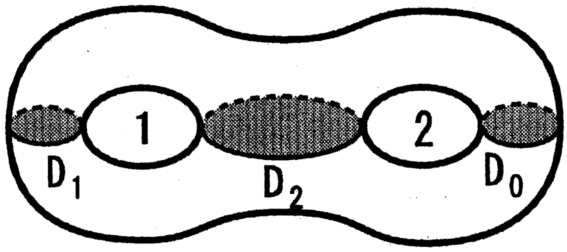

FIGURE 5

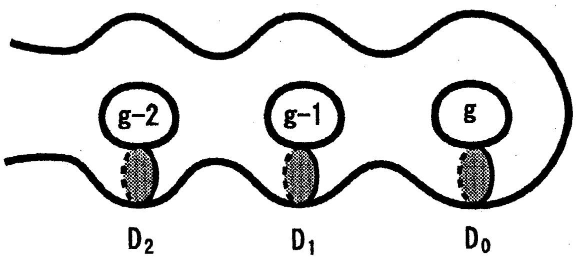

FIGURE 6

$\tilde{f}(v * \sigma)$ is purely separating, and this fact contradicts the maximality assumption of p. Hence, each vertex of *Link* σ is mapped into $Y(M, *)$ by $\tilde{f}$. If there is a simplex ρ of *Link* σ such that $\tilde{f}(\rho)$ separetes M, then, by 3.3 there is a face τ of ρ such that $\tilde{f}(\tau)$ is purely separating. Then $\tilde{f}(\tau * \sigma)$ is purely separating, and $\tau * \sigma \subset Link\ \sigma \subset D^{i+1}$. This fact contradicts the maximality assumption of p. Hence, $\tilde{f}(Link\ \sigma) \subset Y(M, *)$. By the way, M is a disjoint union of handle bodies, and the total number m of these genera is at least $g-p$. $Y(M)$ is a join of Y's of each components of M. Hence, by the hypothesis for the induction, $Y(M)$ is $(m-2)$-connected. Here, we remember that $i \leq g-2$, then we can see $i-p \leq g-2-p \leq m-2$. This shows that $\tilde{f}|_{Link\ \sigma}$ is null homotopic in $Y(M)$, hence, in $Y(H_g, \{E_1, \cdots, E_l\})$. Therefore, we can homotope $\tilde{f}$ such that $\tilde{f}(Link\ \sigma) \subset Y(H_g, \{E_1, \cdots, E_l\})$. We do the same way for other simplices whose images are purely separating, then $\tilde{f}$ is homotoped to a continupus map whose image is in $Y(H_g, \{E_1, \cdots, E_l\})$. □

4. Obtaining a presentation from the action of $\mathcal{H}_g$ on $Y(H_g)$

For each element ϕ of $\mathcal{H}_g$ and simplex $([D_0], \cdots, [D_n])$ of $Y(H_g)$ (resp.$\Delta'(H_g)$), $([\phi(D_0)], \cdots, [\phi(D_n)])$ is also a simplex of $Y(H_g)$ (resp.$\Delta'(H_g)$). Hence, we can define a right action of $\mathcal{H}_g$ on $Y(H_g)$ (resp.$\Delta'(H_g)$) by $([D_0],\cdots,[D_n])\phi = ([\phi(D_0)],\cdots,[\phi(D_n)])$. We can see that, if $g=2$, each of { 2-simplices of $\Delta'(H_2)\}/\mathcal{H}_2$, { 1-simplices of $\Delta'(H_2)\}/\mathcal{H}_2$ and { vertices of $\Delta'(H_2)\}/\mathcal{H}_2$ consists of one element, each of which is represented by $([D_0],[D_1],[D_2])$, $([D_0],[D_1])$, and $([D_0])$, where D_0, D_1, D_2 are indicated in Figure 5, and if $g \geq 3$, each of { 2-simplices of $Y(H_g)\}/\mathcal{H}_g$, { 1-simplices of $Y(H_g)\}/\mathcal{H}_g$ and { vertices of $Y(H_g)\}/\mathcal{H}_g$ consists of one element, each of which is represented by $([D_0],[D_1],[D_2])$, $([D_0],[D_1])$, and $([D_0])$, where D_0, D_1, D_2 are indicated in Figure 6. If stabilizer of each vertex is finitely presented, and if that of each 1-simplex is finitely generated, we can obtain a presentation for $\mathcal{H}_g$ as in the way of [9], [12]. Here, we will mension these method. The action of $\mathcal{H}_2$ on $\Delta'(H_2)$ is similar to the action of $\mathcal{H}_g$ on 2-skelton of $Y(H_g)$ when $g \geq 3$. Hence, we mension just on the case of $g \geq 3$.

We fix a vertex v_0 of $Y(H_g)$, fix an edge (= a 1-simplex with orientation) e_0 of $Y(H_g)$ which emanates from v_0 and fix a 2-simplex f_0 of $Y(H_g)$ which contains v_0. Let D_0, D_1 and D_2 be meridian disks indicated in Figure 6, we set $v_0 = [D_0]$, $e_0 = ([D_0],[D_1])$ and $f_0 = ([D_0],[D_1],[D_2])$. We choose an element r_0 of $\mathcal{H}_g$ which switches the vertices of e_0, in our situation, we set $r_0 = d_g$. By this notation, we see $e_0 = (v_0,(v_0)d_g)$ We denote the stabilizer of v_0 by $(\mathcal{H}_g)_{v_0}$, that of e_0 by $(\mathcal{H}_g)_{e_0}$ and an infinite cyclic group generated by d_g by $< d_g >$. The free product $(\mathcal{H}_g)_{v_0} * < d_g >$ with the following three types of relation define a presentation for $\mathcal{H}_g$.

(Y1) $d_g^2 =$ a presentation of d_g^2 as an element of $(\mathcal{H}_g)_{v_0}$.

(Y2) For each element t of $(\mathcal{H}_g)_{e_1}$,

$$\begin{aligned}(d_g)^{-1}(&\text{ a presentation of } t \text{ as an element of } (\mathcal{H}_g)_{v_0})d_g \\ &= \text{ a presentation of } (d_g)^{-1}td_g \text{ as an element of } (\mathcal{H}_g)_{v_0}.\end{aligned}$$

(Y3) For the loop ∂f_0 in $Y(H_g)$, we define an element W_{f_0} in the follwing manner. The loop ∂f_0 consists of three vertices v_0, v_1, v_2 and three edges e_1, e_2, e_3 such that $e_1 = (v_0, v_1)$, $e_2 = (v_1, v_2)$, $e_3 = (v_2, v_0)$. There is an element h_1 of $(\mathcal{H}_g)_{v_0}$ such that $e_0 h_1 = e_1$ i.e. $e_1 = (v_0, (v_0)d_g h_1)$, then $e_2(d_g h_1)^{-1}$ is an edge emanating from v_0. Hence, there is an element h_2 of $(\mathcal{H}_g)_{v_0}$ such that $e_0h_2 = e_2(d_gh_1)^{-1}$ i.e. $e_2 = ((v_0)d_gh_1, (v_0)d_gh_2d_gh_1)$, then $e_3(d_gh_2d_gh_1)^{-1}$ is an edge emanating from v_0. So, there is an element h_3 of $(\mathcal{H}_g)_{v_0}$ such that $e_0h_3 = e_3(d_gh_2d_gh_1)^{-1}$ i.e. $e_3 = ((v_0)d_gh_2d_gh_1, (v_0)d_gh_3d_gh_2d_gh_1)$. We define $W_{f_0} = d_gh_3d_gh_2d_gh_1$. This element W_{f_0} fixes v_0, so the following is a relation for $\mathcal{H}_g$. W_{f_0} = a presentation of W_{f_0} as an element of $(\mathcal{H}_g)_{v_0}$. If $g \geq 3$, then this type of relation is $d_gd_{g-1}d_g = d_{g-1}d_gd_{g-1}$, and if $g = 2$, then this type of relation is $(d_2^{-1}t_1d_2t_1)^3 = 1$, where $t_1 = t(2)_{2,1}k_1^{-1}a_2$.

For subsets $A_1, \dots, A_n$ of H_g, we define $Diff^+(H_g, A_1, \dots, A_n) = \{\phi \in Diff^+(H_g) | \phi(A_1) = A_1, \cdots, \phi(A_n) = A_n\}$. We can see $(\mathcal{H}_g)_{v_0} = \pi_0(Diff^+(H_g, D_0))$, and $(\mathcal{H}_g)_{e_0} = \pi_0(Diff^+(H_g, D_0, D_1))$ Let $\Sigma_g = \partial H_g$ and let $F_n\Sigma_g$ be the space of all ordered n-tuples of distinct points of Σ_g: $F_n\Sigma_g = \{(p_1, \cdots, p_n) |$ each $p_i \in \Sigma_g$, and $p_i \neq p_j$ if $i \neq j\}$. We define the n-string pure braid group $P_n(\Sigma_g)$ to be the fundamental group of $F_n\Sigma_g$. Let p_1 and p_2 be points on ∂H_{g-1}, and $p_{0,1}$, $p_{0,2}$, $p_{1,1}$, and $p_{1,2}$ be points on ∂H_{g-2}.

We can get a presentation for $(\mathcal{H}_g)_{v_0}$ by investigating the following three exact sequaences:

$$(1)\quad P_2\Sigma_{g-1} \xrightarrow{\beta} \pi_0(Diff^+(H_{g-1}, p_1, p_2)) \xrightarrow{\alpha\#} \mathcal{H}_{g-1}(= \pi_0(Diff^+(H_{g-1}))) \longrightarrow 0,$$

$$(2)\quad 0 \longrightarrow \pi_0(Diff^+(H_{g-1}, p_1, p_2)) \xrightarrow{\delta} \pi_0(Diff^+(H_{g-1}, \{p_1, p_2\})) \xrightarrow{\gamma} \mathbb{Z}_2 \longrightarrow 0,$$

$$(3)\quad 0 \longrightarrow \mathbb{Z} \longrightarrow \pi_0(Diff^+(H_g, D_0)) \xrightarrow{\lambda} \pi_0(Diff^+(H_{g-1}, \{p_1, p_2\})) \longrightarrow 0.$$

We can get a set of generators of $(\mathcal{H}_g)_{e_0}$ by investigating the following three exact sequences:

(4)

$$P_4\Sigma_{g-2} \xrightarrow{\beta} \pi_0(Diff^+(H_{g-2}, p_{0,1}, p_{0,2}, p_{1,1}, p_{1,2})) \xrightarrow{\alpha\#} \mathcal{H}_{g-2}(= \pi_0(Diff^+(H_{g-2}))) \longrightarrow 0,$$

$$(5)\quad \begin{array}{l} 0 \longrightarrow \pi_0(Diff^+(H_{g-2}, p_{0,1}, p_{0,2}, p_{1,1}, p_{1,2})) \xrightarrow{\delta} \\ \qquad \pi_0(Diff^+(H_{g-2}, \{p_{0,1}, p_{0,2}\}, \{p_{1,1}, p_{1,2}\})) \xrightarrow{\gamma} \mathbb{Z}_2 \times \mathbb{Z}_2 \longrightarrow 0 \end{array}$$

$$(6) \quad \begin{aligned} 0 \longrightarrow \mathbb{Z} \times \mathbb{Z} \longrightarrow & \pi_0(Diff^+(H_g, D_0, D_1)) \\ & \xrightarrow{\lambda} \pi_0(Diff^+(H_{g-2}, \{p_{0,1}, p_{0,2}\}, \{p_{1,1}, p_{1,2}\})) \longrightarrow 0. \end{aligned}$$

We can give relations of type (Y1) and (Y2) by drawing some figures.

For details please see [6].

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