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Finite completely 0-simple semigroups and amalgamation bases for finite semigroups

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According to [2], we recall the definitions concerned with amalgam. Let $\mathcal{A}$ be a class of semigroups. A triple of semigroups S, T, U with $U = S \cap T$ being a subsemigroup of S and T is called an *amalgam* of S and T with a *core* U in $\mathcal{A}$ and denoted by $[S, T; U]$. An amalgama $[S, T; U]$ of $\mathcal{A}$ is *weakly embedable* in $\mathcal{A}$ if there exist a semigroup K belonging to $\mathcal{A}$ and monomorphisms $\xi_1 : S \to K$, $\xi_1 : T \to K$ such that the restrictions to U of ξ_1 and ξ_2 are equal to each other (that is, $\xi_1(S) \cap \xi_2(T) \supseteq \xi_1(U)$). In the case that $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$, we say that an amalgama $[S, T; U]$ of $\mathcal{A}$ is *strongly embeddable* in $\mathcal{A}$. A semigroup U in $\mathcal{A}$ is *amalgamation base* [resp. *weak amalgamation base*] if any amalgam with a core U in $\mathcal{A}$ is strongly embeddable [resp. weakly embeddable] in $\mathcal{A}$. In this paper, we restrict ourselves to the cases that $\mathcal{A}$ is the class of all semigroups or the class of all finite semigroups. We will use the terms "*amalgamation base for semigroups*" or "*weak amalgamation base for finite semigroups*" in the former case or the latter.

Okuniñki and Putcha [7] proved that any finite semigroup U is an amalgamation base for all finite semigroups if the $\mathcal{J}$-classes of U is linearly ordered and the semigroup algebra $\mathbb{C}[U]$ over $\mathbb{C}$ has a zero Jacobson radical.

Result (Hall [2]). *A finite semigroup U is an amalgamation base for finite semigroups if and only if U is a weak amalgamation base for those.*

In the paper [5] Hall and Shoji proved that any semigroup which is an amalgamation base for finite semigorups has (REP) and $(REP)^{op}$.

Let U be a semigroup with zero, 0, and $a, b \in S$.

The set $\{s \in U \mid sa = 0\}$ is called the *left annihilator* of a in S and is denoted by $ann_l(a)$.

In this case, we say that U satisfies the condition Ann_l if $ann_l(a) = ann_l(b)$ implies $aU = bU$.

The *right annihilator* and the condition Ann_r are defined by left-right duality.

The main theorem. *Let U be a finite competely 0-simple semigroup. Then the following are equivalent :*

(1) *U is an amalgamation base for semigorups.*
(2) *U is an amalgamation base for finite semigorups.*
(3) *U satisfies the conditions Ann_l and Ann_r.*

By the main theorem, there exists a finite completely 0-simple semigroup S such that (1) S is an amalgamation base for finite semigroups but (2) the semigroup algebra $\mathbb{C}[S]$ over the complex number field $\mathbb{C}$ has nonzero Jacobson radical. Actually, we have

Example. Let $S = M(3,2;\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix})$. Then we take the element $e = (1,1) - (2,1) - (3,1) \in Q[S]$. Then $Se = 0$ and so $(e\mathbb{C}[S])^2 = 0$. Hence $\mathbb{C}[S]$ has the nonzero radical. On the other hand S is an amalgamation base for finite semigroups.

References

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