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Almost \( n \)-dimensional spaces

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We consider only separable metric spaces. A space \( X \) is said to be almost \( n \)-dimensional if it has a basis \( \{ U_i \} \) such that if \( \text{cl}U_i \cap \text{cl}U_j = \emptyset \) then \( X = G \cup H \) where \( G \) and \( H \) are closed sets, \( U_i \subset G \setminus H, U_j \subset H \setminus G \) and \( \dim G \cap H \leq n-1 \) and \( n \) is the smallest natural number such that such a basis exists for \( n \). It is clear that \( n \)-dimensional spaces are at most almost \( n \)-dimensional.

Oversteegen and Tymchatyn [9] proved that almost 0-dimensional spaces are at most 1-dimensional. The Erdős space of irrational sequences in Hilbert space is known to be a universal almost 0-dimensional space [5]. Erdős space is 1-dimensional. Homeomorphism groups of positive dimensional Menger compacta are almost 0-dimensional [9] and at least 1-dimensional by classical results of Brechner [2] and Bestvina [1].

Almost 0-dimensional spaces are at most 1-dimensional and the 1-dimensionality cannot be improved. Our first result shows that this interesting behaviour does not occur in higher dimensions and the following one points out an interesting property of almost 0-dimensional spaces.

**Theorem 1** (Levin-Tymchatyn [7]) If \( X \) is almost \( n \)-dimensional, \( n \geq 1 \) then \( X \) is \( n \)-dimensional.

**Theorem 2** (Levin-Tymchatyn [7]) Let \( X = X_1 \cup X_2 \) where \( X_1 \) is almost 0-dimensional and \( X_2 \) is 0-dimensional. Then \( \dim X \leq 1 \).

The proof of these theorems employs so-called \( L \)-embeddings. A subset \( X \) of a compactum \( K \) is \( L \)-embedded in \( K \) if for every open cover \( \mathcal{U} \) of \( K \) there is a neighbourhood \( U \) of \( X \) in \( K \) such that the continua in \( U \) refine \( \mathcal{U} \). An almost 0-dimensional space is \( L \)-embeddable in a compactum [6] and

**Theorem 3** (Levin-Pol [6]) If a space \( X \) is \( L \)-embeddable in a compactum \( K \) then \( \dim X \leq 1 \).

As an application of almost 1-dimensional spaces we will consider an old question of R. Duda about the dimension of a hereditarily locally connected,
non-degenerate space $X$. Nishiura and Tymchatyn [8] showed that each pair of disjoint, closed, connected subsets of $X$ can be separated by a closed countable subset of $X$. Hence each basis for $X$ of open connected sets witnesses the almost 1-dimensionality of $X$. Then Theorem 1 implies:

**Theorem 4** (Levin-Tymchatyn [7]) *If $X$ is a hereditarily locally connected, non-degenerate space then $\dim X = 1$.*

A partial solution to the question of R. Duda was given in [9] where it was proved that hereditarily locally connected spaces are at most 2-dimensional.

Finally let us note that Theorem 2 does not hold if $X_2$ is almost 0-dimensional. Indeed, let $Y$ be 1-dimensional and almost 0-dimensional, let $M$ be a 1-dimensional compactum and let $M = M_1 \cup M_2$, $\dim M_1 = \dim M_2 = 0$. Then $X_1 = Y \times M_1$ and $X_2 = Y \times M_2$ are almost 0-dimensional, and by a theorem of Hurewicz [4] (see also [3], p. 78, 1.9.E(b)) $X = X_1 \cup X_2 = Y \times M$ is 2-dimensional.

**References**


