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Two properties added to M_3-spaces

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1. Introduction.

In this article, we talk about what we recently obtained on the outstanding open problem whether M_3-spaces are M_1. Assume here that all spaces are regular T_2.

As well-known, M_i-spaces(i=1,2,3) are defined by Ceder in terms of special bases as follows:

Definition 1.1. (1) A space X is M_1 if and only if there exists a σ-CP (=closure-preserving) base for X.
(2) A space X is M_2 if and only if there exists a σ-CP quasi-base for X.
(3) A space X is M_3 if and only if there exists a σ-cushioned pair-base.

The equality $M_2 = M_3$ is obtained independently by Gruenhage and Junnila, and M_3-spaces are studied by Borges under the name of stratifiable spaces. But the implication $M_3 \Rightarrow M_1$ is not answered yet.

The partial answers to this problem have been obtained. The first is due to Slaughter that every Lasnev space is M_1. The sequence of positive answers which insists that every M_3-space with some special property is M_1 is shown by the diagram below. Among them, we note that the results od Ito and Tamano are applicabe to our cases here.

Ito showed that every Nagata space, more widely every M_3-space whose every point has a CP open neighborhood base is M_1. Choosing the essential part from his discussion, Tamano showd that every Baire, Frechet M_3-space is M_1. The difference between them is one little thing, but acute. Tamano proposed ther the following question: Is every Frechet M_3-space an M_1-space? We clear it by adding some property to M_3-spaces.

2. Two properties.

To solve his question positively, we introduce the following property (*), which is weaker than that of Frechet spaces:

Definition 2.1([1, Definition 2.3]). A space X has property (*) if for any open subset O of X and any point of B(O), the boundary of O in X, there exists a CP closed network $\mathcal{B}$ at p in X such that for each B $\in$ $\mathcal{B}$,

$B \subset \overline{O}, B \cap B(O) = \{p\}$.

With the aid of property (*), we can show the following:

Theorem 2.2([1, Theorem 3.1]). Let X be an M_3-space with property (*). Then every closed subset of X has a CP open neighborhood base in X, and hence X is an M_1-space.

Corollary 2.3. Every Frechet M_3-space is M_1.

Does every M_3-space have property (*) ? But unfortunately this is not the case , because there exists an M_1-space which does not satisfy property (*) ([1, Example 1]). This example leads us to define another property added to M_3-spaces that is weaker than the former one.

Definition 2.4([2, Definition 8]). A space X satisfies property (P) if for each open subset O of X and each point p of B(O), there exists a CP closed local network $\mathcal{B}$ at p in X such that for each $B \in \mathcal{B}$, $\overline{B \cap O} = B$.

In this case again, we can show the following:

Theorem 2.5([2, Theorem 15]). If X is an M_3-space with property (P), then every closed subset of X has a CP open neighborhood base in X.

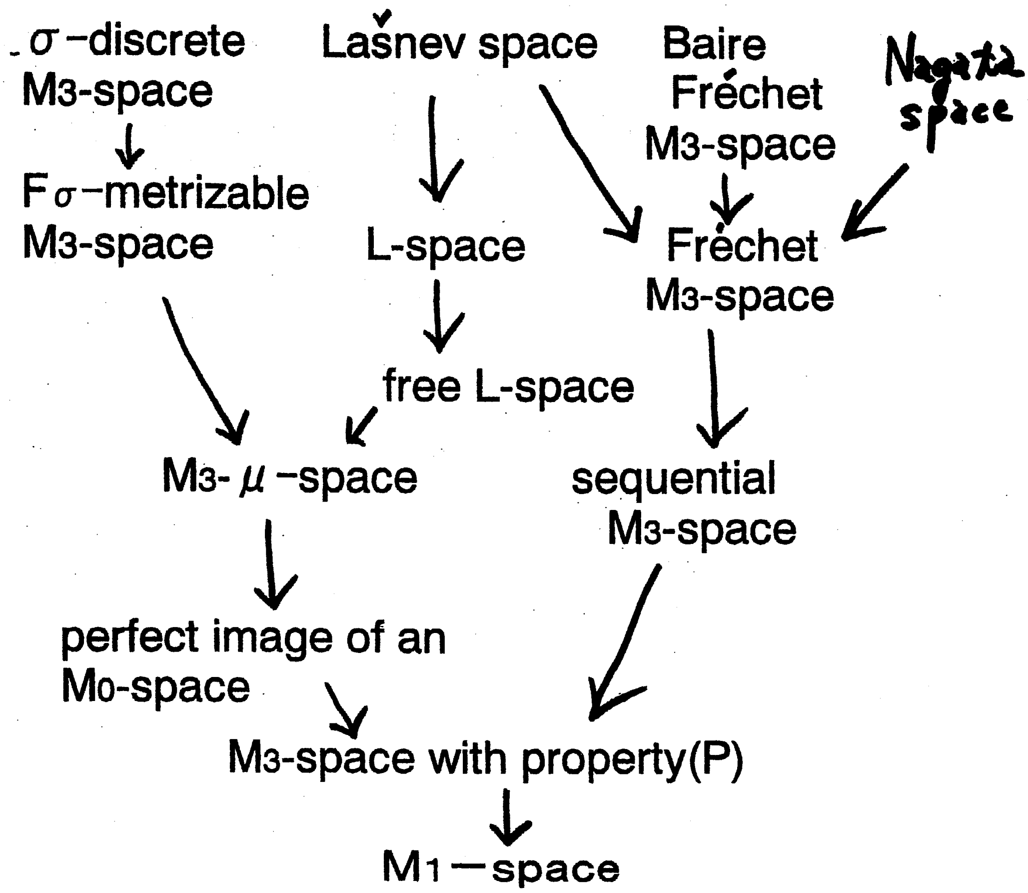
What kind of spaces except for Frechet M_3-spaces have this property (P) ? As for this question we can show the following:

Theorem 2.6([2, Theorem 16]). Every sequential M_3-space have property (P), and hence is an M_1-space.

But we do not know whether every M_3-space have property (P). Maybe this is a question equivalent to the well-known M_3.M_1problem.

References

1. T. Mizokami and N. Shimane: On Frechet M_3-spaces, to appear in Math. Japonica.
2. __________________ : On the M_3 vs. M_1 problem, to appear in Top. Appl.


σ-discrete M3-space
Lašnev space
Baire Fréchet M3-space
Nagata space
Fσ-metrizable M3-space
L-space
Fréchet M3-space
free L-space
M3-μ-space
sequential M3-space
perfect image of an M0-space
M3-space with property(P)
M1-space