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Lipschitzian Error Bounds of Multi-Item Auction Procedures

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Abstract

This paper analyzes the convergence and incentive properties of the multi-item auction procedures constructed by Crawford and Knoer (1981) and Demange, Gale and Sotomayor (1986) when buyers' preferences are Lipschitzian. At first, it is shown that the minimal equilibrium price vector of the auction market is a Lipschitzian function with respect to the buyers' characteristics (preferences and amounts of budgets). Then the error bounds of the procedures are derived depending on the Lipschitzian parameters, and the ε–nonmanipulability of the procedures is proved.

1. Introduction

This paper considers some auctions on a buyer-seller market of heterogeneous indivisible objects such as used cars or housings in a general environment with non-linear preferences and budget constraints. Former researches focus on the direct auction mechanisms making use of the revelation principle. These mechanisms are formulated as the continuum mechanisms neglecting operating costs where all bidders (buyers) report their non-linear demand functions or non-linear preferences. The non-linearity is important, since it reflects income effects of the demand behavior. Hence, in order to reduce the cost, we have to approximate these mechanisms by discrete or finite mechanisms as discussed in Hurwicz and Marschak (1985).

We formalize the approximation problem as an auctioneer's problem: how to set bid increment for a given degree of the approximation, depending on some (publicly known) parameters of the market such as the number of the objects, under the incomplete information assumption: the auctioneer does not know the buyers' individual characteristics. Moreover, the auctioneer may also consider the bid increment to keep the incentive compatibility.

At first, a general property of the market is shown. Fixing the sets of objects and buyers, a market is identified by the buyers' characteristics. For each market, the minimal equilibrium price vector is defined as the minimal vector in the set of equilibrium price vectors of the market. Then it is shown that the minimal equilibrium price vector is a Lipschitzian function with respect to the buyers' characteristics (preferences and amounts of budgets), when the preferences are Lipschitzian.

Second, we apply the result for the auction procedures with bid increment in the market to solve the auctioneer's problem. We show that the error bound of an auction procedure is given by the product of its bid increment and a constant depending on a uniform bound of individual Lipschitzian parameters. We also show that if the bid increment is less than a positive constant, then behaving honestly is an ε-dominant strategy for each bidder.

2. The buyer-seller market and the minimal price equilibrium

In this section, we formulate a buyer-seller market $[\, M, (\succeq_i, e^i, I_i)_{i\in M}\,;\; N, (\succeq_j, I_j)_{j\in N}\,]$, and define the competitive equilibrium of the market. Let $M = \{ i_1, i_2, \cdots, i_m\}$ be the set of sellers and $N = \{j_1, j_2, \cdots, j_n\}$ be the set of buyers.

Initially every seller $i \in M$ owns one unit of i-type object denoted by the i-th unit vector e^i of $\mathbb{R}^M$ which i can sell in the market, and i holds an amount of money I_i (> 0). Seller i's consumption set X_i is $X_i = \{ e^0, e^i \}\times\mathbb{R}$ where e^0 is the origin of $\mathbb{R}^M$, and i's preference ordering $\succeq_i$ is a complete preordering on X_i. The symmetric and asymmetric parts of $\succeq_i$ are denoted by $\sim_i$ and $\succ_i$, respectively. We assume the following conditions for each seller:

S_1 (Monotonicity for money): For all $(t, x) \in X_i$, it holds that

$$(t, x+\Delta x) \succ_i (t, x) \qquad \text{for all } \Delta x > 0.$$

S_2 (Archimedean with desirability): For all $(t, x) \in X_i$, it holds that

$$(t, x) \sim_i (e^0, x+\Delta x) \qquad \text{for some } \Delta x \geq 0.$$

For each $i \in M$, it holds by S_1 and S_2 that there is a unique number $c_i \geq 0$ such that $(e^i, I_i) \sim_i (e^0, I_i + c_i)$. We call the number c_i the *reservation value* of e^i. Moreover it holds by S_1 that $(e^0, I_i + c_i) \succeq_i (e^0, I_i)$. Hence we have that $(e^i, I_i) \succeq_i (e^0, I_i)$, which implies that e^i is desirable. Furthermore, it holds by S_1 that $(e^i, I_i) \succ_i (e^i, 0)$, which implies that money is indispensable.

Every buyer $j \in N$ owns no object initially, but holds an amount of money I_j (> 0) with which j can buy one object in the market. Set $T = \{e^0, e^{i_1}, \cdots, e^{i_m}\}$ and $X = T \times \mathbb{R}$. Buyer *j*'s consumption set is X, and the preference ordering $\succeq_j$ is a complete preordering on X. The symmetric and asymmetric parts of $\succeq_j$ are denoted by $\sim_j$ and $\succ_j$, respectively. The preference $\succeq_j$ is assumed to satisfy the following conditions:

B_1 (Monotonicity for money): For all $(t, x) \in X$, it holds that

$$(t, x+\Delta x) \succ_j (t, x) \qquad \text{for all } \Delta x > 0.$$

$\mathbf{B_2}$ (Archimedean with desirability): For all $(t, x) \in X$, it holds that

$$(t, x) \sim_j (e^0, x+\Delta x) \qquad \text{for some } \Delta x \geq 0.$$

$\mathbf{B_3}$ (Indispensability of money): $(e^0, I_j) \succ_j (e^i, 0)$ for all $i \in M$.

$\mathbf{B_4}$ (Regularity): $(e^i, I_j - c_i) \succ_j (e^0, I_j)$ or $(e^0, I_j) \succ_j (e^i, I_j - c_i)$ for all $i \in M$.

A price vector p is a non-negative vector in $\mathbb{R}^M$. Let P be the set of price vectors. The *supply correspondence*, $S_i(p)$ of $i \in M$ and the *demand correspondence*, $D_j(p)$ of $j \in N$ are defined by

$$S_i(p) = \{ x \in \{ e^0, e^i \} : (e^i - x, I_i + p \cdot x) \succeq_i (e^i - y, I_i + p \cdot y) \text{ for all } y \in \{ e^0, e^i \} \};$$

$$D_j(p) = \{ x \in B_j(p) : (x, I_j - p \cdot x) \succeq_j (y, I_j - p \cdot y) \text{ for all } y \in B_j(p) \},$$

where $B_j(p) = \{ x \in T : p \cdot x \leq I_j \}$. A triple $(p, s, d) \in P \times T^M \times T^N$ is called a *competitive equilibrium* iff

(i) $s_i \in S_i(p)$ for all $i \in M$;

(ii) $d_j \in D_j(p)$ for all $j \in N$;

(iii) $\sum_{i \in M} s_i = \sum_{j \in N} d_j$.

The existence of a competitive equilibrium under the conditions S_1, S_2 and B_1–B_4 is well-known in this market, see Kaneko and Yamamoto (1986). Under these conditions, a sufficient condition for the existence of the active competitive equilibrium is that $(e^i, I_j - c_i) \succ_j (e^0, I_j)$ for some $i \in M$ and some $j \in N$. For a competitive equilibrium (p, s, d), we call p an *equilibrium price vector*. Let P^* be the set of equilibrium price vectors.

We need some definitions for the next proposition: at a competitive equilibrium (p, s, d), we draw a directed graph G whose vertices are $M \cup N$ by the following rules: for any $(i, j) \in M{\times}N$

(Rule 0) Draw an arc from i to i;

(Rule 1) If $d_j = e^i$, then draw an arc from j to i;

(Rule 2) If $e^i \in D_j(p)$ and $d_j \neq e^i$, then draw an arc from i to j.

A subset $\{i_1, i_2, \cdots, i_m\}$ of $M \cup N$ is called a *path* of G iff there is an arc of G from i_k to i_{k+1} for each $k = 1, 2, \cdots, m-1$.

Proposition 1 (Existence and characterization of the minimal equilibrium price vector; Miyake 1994) : Assume S_1, S_2 and B_1–B_4.

(A) there exists the *minimal* equilibrium price vector p^* in the sense that

(i) $p^* \in \{p \in P^* : p_i \geq c_i\}$;

(ii) $p^* \leq p$ for all $p \in \{p \in P^* : p_i \geq c_i\}$, where c_i is the reservation value of e^i.

(**B**) Let (p, s, d) be a competitive equilibrium. Then p is the minimal equilibrium price vector iff for every seller $i^* \in M$ there is a path of G starting from i^* to $k \in \{i \in M: p_i = c_i\} \cup \{j \in N: d_j = e^0\}$.

To present Miyake's (1998, Theorem 1) non-manipulability theorem of the minimal-price equilibrium in our market, we need some definitions, fixing sets of agents (M, N) and sellers' characteristics $(\succeq_i, e^i, I_i)_{i \in M}$. Let Φ be the set of buyers' profiles $\varphi = (\succeq_j, I_j)_{j \in N}$ satisfying the conditions $\mathbf{B}_1$ through $\mathbf{B}_4$. Let p^* be the function $p^* : \Phi \to \mathbb{R}^M$ which assigns the minimal equilibrium price vector $p^*(\varphi)$ of the market $[M, (\succeq_i, e^i, I_i)_{i \in M}; N, \varphi]$ for each $\varphi \in \Phi$. Let $\mathcal{D}^*(\varphi)$ be the set of equilibrium demands of buyers corresponding to $p^*(\varphi)$ for each profile $\varphi \in \Phi$, i.e, $\mathcal{D}^*(\varphi) = \{ d = (d_j)_{j \in N}: (p^*(\varphi), s, d)$ is a competitive equilibrium. $\}$.

Proposition 2 (Nonmanipulability of the continuum mechanism; Miyake 1998): Suppose $\varphi^* = (\succeq_j^*, I_j^*)_{j \in N} \in \Phi$ be the true profile of buyers' characteristics. Then it holds that for all $j \in N$, $\varphi = (\succeq_j, I_j)_{j \in N} \in \Phi$, $(d_j)_{j \in N} \in \mathcal{D}^*(\varphi^*_j, \varphi_{-j})$ and all $(f_j)_{j \in N} \in \mathcal{D}^*(\varphi)$

$$(d_j,\ I_j^* - p^*(\varphi^*_j, \varphi_{-j})\cdot d_j) \quad \succeq_j^* \quad (f_j,\ I_j^* - p^*(\varphi)\cdot f_j),$$

where $(\varphi^*_j, \varphi_{-j}) = (\varphi_1, \ldots, \varphi_{j-1}, \varphi^*_j, \varphi_{j+1}, \ldots, \varphi_n)$.

Proposition 2 states that the (continuum) mechanism which selects a minimal-price equilibrium for a reported profile $\varphi \in \Phi$ is non-manipulable for each buyer. Namely, when $\varphi^* = (\succeq_j^*, I_j^*)_{j \in N} \in \Phi$ is the true profile, it is a dominant strategy for each buyer j to report j's true characteristics $\varphi^*_j = (\succeq_j^*, I_j^*)$ to the mechanism.

3. The multi-item auction procedures and Lipschitzian error bounds

This section applies our results to auctions based on Asami (1990), Crawford and Knoer (1981), Demange, Gale and Sotomayor (1986) and Miyake (1998). We describe the auction procedure for $\varphi \in \Phi$, assuming all buyers $j \in N$ play as bidders. Let δ be a positive number. This δ represents the fixed amount of increment of a price in the auction. The δ-auction procedure is defined as follows:

R1: The time structure is given by $r = 1, 2, 3, \cdots$. The prices at time $r \geq 1$ are represented by a price vector $q(r) = (q_i(r))_{i \in M}$ in P. The auctioneer sets the initial price vector $q(1) = (c_i)_{i \in M}$ and announces the bid increment $\delta > 0$. Initially no buyer is committed to an object.

R2: At time $r \geq 1$ all uncommitted bidders bid simultaneously. The bid $b_j(r)$ of uncommitted bidder j is an element in $D_j(q(r))$. (Bidder j may choose any element in $D_j(q(r))$.) Let $M_r = \{ i \in M : b_j(r) = e^i$ for some uncommitted bidder j $\}$. Then the transition and stopping rules are given by the following:

(1) If a bidder j has been committed to some i in M_r, j becomes uncommitted;

(2) Every seller i in M_r selects (arbitrarily) one, k_i, of the bidders who bid for i, and then k_i is committed to i and the other bidders who bid for i are still uncommitted;

(3) If a bidder j bids for the null item e^0, i.e., $b_j(r) = e^0$, then j goes out from the auction;

(4) If there is no uncommitted bidder in the auction, the auction stops; otherwise the auction proceeds to the next round r+1, setting q(r+1) by

$$q_i(r+1) = \begin{cases} q_i(r) + \delta & \text{if } i \in M_r \\ q_i(r) & \text{otherwise,} \end{cases} \qquad \text{for all } i \in M.$$

R3: If the auction stops at time r*, a bidder j who is committed to i buys e^i at the effective price when j bids for e^i.

Fixing a δ-auction procedure (history), the resultant trade of the δ-auction is represented by (q, s, d) defined by

$$q_i = \begin{cases} \text{the price paid by } j & \text{if some buyer } j \text{ buys } e^i \\ q_i(1) & \text{otherwise} \end{cases} \qquad \text{for all } i \in M;$$

$$s_i = \begin{cases} e^i & \text{if some buyer } j \text{ buys } e^i \\ e^0 & \text{otherwise} \end{cases} \qquad \text{for all } i \in M;$$

$$d_j = \begin{cases} e^i & \text{if } j \text{ buys } e^i \\ e^0 & \text{otherwise} \end{cases} \qquad \text{for all } j \in N.$$

By R2(3,4) and the budget constraint the δ-auction terminates in a finite time. Since a bidder's selection of the bid from his demand set in R2 and the selection of a bidder in R2(2) may not be unique, the resultant trade (p,s,d) of δ-auction also may not be unique. We write the set of resultant trades of δ-auction for $\varphi \in \Phi$ as $T(\varphi; \delta)$.

In order to derive the properties of $T(\varphi; \delta)$, we assume an additional condition:

$\mathbf{B_5}$ (Lipschitzian condition): There exist two positive numbers $\alpha_j, \beta_j > 0$ such that:

if $(t, x) \sim_j (e^0, y)$ and

$$(t, x+\Delta x) \sim_j (e^0, y+\Delta y) \quad \text{for } (t, x) \in X, y \in \mathbb{R}, \Delta x > 0, \ \Delta y > 0,$$

then $\alpha_j \geq \Delta y/\Delta x \geq \beta_j$.

We call the number $a_j \equiv \alpha_j/\beta_j$ *Lipschitzian coefficient*, since the following proposition holds:

Proposition 3 (Existence of a nicely Lipschitzian utility function): If $(\succeq_j, I_j)$ satisfies $\mathbf{B_1}$-$\mathbf{B_5}$, then there exists a real-valued function U_j on X such that:

(i) U_j is a utility function of $\succeq_j$, i.e.,

$$U_j(t, x) \geq U_j(t^*, y) \quad \text{iff} \quad (t, x) \succeq_j (t^*, y) \qquad \text{for all } (t, x), (t^*, y) \in X.$$

(ii) U_j is *nicely Lipschitzian* in the sense that for each $(t, x) \in X$

$$a_j \geq [U_j(t, x+\Delta x) - U_j(t, x)]/\Delta x \geq 1 \qquad \text{for all } \Delta x > 0.$$

For any $A \geq 1$, let

$$\Phi_A = \{ (\succeq_j, I_j)_{j \in N} \in \Phi : (\succeq_j, I_j) \text{ satisfies } \mathbf{B_5} \text{ and } A \geq a_j \text{ for each } j \in N \}.$$

The main results of this paper are the following two theorems:

Theorem 1 (Lipschitzian error bounds): For all $\varphi = (\succeq_j, I_j)_{j \in N} \in \Phi_A$ and all $\delta > 0$, it holds that

$$\max_{(q,s,d) \in T(\varphi;\delta)} \max_{i \in M} \| p^*_i(\varphi) - q_i \| < \delta \cdot (A+1)^{\gamma},$$

where $\gamma \equiv 4 + 2 \cdot \min[|M|, |N|]$.

Theorem 2 (ε–nonmanipulability of the discrete mechanism): Suppose $\varphi^* = (\succeq_j^*, I_j^*)_{j \in N} \in \Phi_A$ be the true characteristics of buyers. For any $\varepsilon > 0$, set the bid increment δ as

$$\frac{\varepsilon}{(A+1)^{\gamma+3}} > \delta > 0.$$

Then it holds that for all $j \in N$, $\varphi = (\succeq_j, I_j)_{j \in N} \in \Phi_A$, $(q, s, d) \in T((\varphi^*_j, \varphi_{-j}); \delta)$ and all $(r, t, f) \in T(\varphi; \delta)$

$$(d_j, I_j^* - q \cdot d_j + \varepsilon) \succeq_j^* (f_j, I_j^* - r \cdot f_j).$$

4. Proof of theorems

We need a Lipschitzian property of the minimal price equilibrium. For any $I > 0$, define a subset $\Phi(A, I)$ of Φ_A by

$$\Phi(A, I) = \{ (\succeq_j, I_j)_{j\in N} \in \Phi_A : I > I_j \text{ for all } j \}.$$

Note that $\Phi(A, I^1) \subset \Phi(A, I^2)$ whenever $I^1 \leq I^2$. In order to state the Lipschitzian property of $p^*(\varphi)$ on $\Phi(A, I)$ we have to introduce a pseudo-metric on $\Phi(A, I)$. For any two real-valued continuous functions f_1 and f_2 on X, define a pseudo-metric h_I for $I > 0$ by

$$h_I(f_1, f_2) = \max_{(t,x)\in T\times[-2I, 2I]} \| f_1(t, x) - f_2(t, x) \|.$$

Since nicely Lipschitzian functions are continuous, we define a pseudo-metric μ_I on $\Phi(A, I)$ by

$$\text{for all } \varphi = (\succeq_j, I_j)_{j\in N},\ \varphi^* = (\succeq_j^*, I_j^*)_{j\in N} \in \Phi(A, I),$$

$$\mu_I(\varphi, \varphi^*) = \max\, [\, \max_{j\in N} h_I(U_j, U_j^*),\ \max_{j\in N} \| I_j - I_j^* \| \,],$$

where $(U_j)_{j\in N}$ and $(U^*_j)_{j\in N}$ are the utility functions for $(\succeq_j)_{j\in N}$ and $(\succeq_j^*)_{j\in N}$ defined in Proposition 3, respectively. Then we have the following propositions:

Proposition 4 (Lipschitzian continuity of p^*): For any $A \geq 1$, $I > 0$, the following inequality holds:

$$\max_{i \in M} \| p^*_i(\varphi^1) - p^*_i(\varphi^2) \| \leq \mu_I(\varphi^1, \varphi^2) \cdot (A+1)^{\xi} \quad \text{for all } \varphi^1, \varphi^2 \in \Phi(A, I),$$

where $\xi \equiv 2 + 2 \cdot \min[\,|M|, |N|\,]$.

Proposition 5 (Embedding $T(\varphi; \delta)$ into Φ): For all $\varphi \in \Phi(A, I)$, $\delta > 0$, and all $(q, s, d) \in T(\varphi; \delta)$, there exists some $\varphi^* \in \Phi(A, I+\delta)$ satisfying:

(i) $q = p^*(\varphi^*)$ and $d \in \mathcal{D}^*(\varphi^*)$;

(ii) $\mu_{I+\delta}(\varphi, \varphi^*) < \delta\,(A+1)^2$.

Proof of Theorem 1: Fix any $\varphi = (\succeq_j, I_j)_{j \in N} \in \Phi_A$ and fix $I > \max_j I_j$. It holds that $\varphi \in \Phi(A, I) \subset \Phi(A, I+\delta)$. Set $\Phi^* = \{\, \varphi^* \in \Phi(A, I+\delta),\ \mu_{I+\delta}(\varphi, \varphi^*) < \delta \cdot (A+1)^2 \,\}$. Then it holds by Propositions 4 and 5 that:

$$\max_{(q,s,d) \in T(\varphi; \delta)} \ \max_{i \in M} \| p^*_i(\varphi) - q_i \| \leq \max_{\varphi^* \in \Phi^*} \ \max_{i \in M} \| p^*_i(\varphi) - p^*_i(\varphi^*) \|$$

$$< \delta\,(A+1)^2 \cdot (A+1)^{\xi} < \delta\,(A+1)^{\xi+2}.$$

QED

Proof of Theorem 2: Suppose $\varphi^* = (\succeq_j^*, I_j^*)_{j\in N} \in \Phi_A$ be the true characteristics of buyers. For all $j \in N$, $\varphi \in \Phi_A$, $\delta > 0$, $(q, s, d) \in T((\varphi^*_j, \varphi_{-j}); \delta)$, $(r, t, f) \in T(\varphi; \delta)$, and all $(d^*_j)_{j\in N} \in \mathcal{D}^*(\varphi^*_j, \varphi_{-j})$, the following lemma holds:

Lemma 1: **(i)** $U^*_j(d_j, I^*_j - q\cdot d_j) + \delta\cdot(A+1)^{\xi+4} > U^*_j(d^*_j, I^*_j - p^*(\varphi^*_j, \varphi_{-j})\cdot d^*_j)$, where ξ is the number defined in Proposition 4 and U^*_j is the utility function for $\succeq^*_j$. **(ii)** There exists some $\varphi^0 \in \Phi_A$ satisfying

(a) $r = p^*(\varphi^0)$ and $f \in \mathcal{D}^*(\varphi^0)$;

(b) $U^*_j(d^*_j, I^*_j - p^*(\varphi^*_j, \varphi_{-j})\cdot d^*_j) >$

$$U^*_j(d^0_j, I^*_j - p^*(\varphi^*_j, \varphi^0_{-j})\cdot d^0_j) - \delta\cdot(A+1)^{\xi+4} \text{ for all } (d^0_j)_{j\in N} \in \mathcal{D}(\varphi^*_j, \varphi^0_{-j}).$$

For any $\varepsilon > 0$, set the bid increment δ as $\varepsilon\cdot(A+1)^{-\gamma-3} = \varepsilon\cdot(A+1)^{-\xi-5} > \delta > 0$, which implies that

$$\varepsilon/2 \geq \varepsilon/(A+1) > \delta\cdot(A+1)^{\xi+4}.$$

Then Lemma 1 and Proposition 2 together imply that

$$\begin{aligned} U^*_j(d_j, I^*_j - q\cdot d_j) + \varepsilon/2 &> U^*_j(d^*_j, I^*_j - p^*(\varphi^*_j, \varphi_{-j})\cdot d^*_j) \\ &> U^*_j(d^0_j, I^*_j - p^*(\varphi^*_j, \varphi^0_{-j})\cdot d^0_j) - \varepsilon/2 \\ &\geq U^*_j(f_j, I^*_j - p^*(\varphi^0)\cdot f_j) - \varepsilon/2 = U^*_j(f_j, I^*_j - r\cdot f_j) - \varepsilon/2. \end{aligned}$$

Thus we have by Proposition 3 that

$$U^*_j(d_j, I^*_j - q\cdot d_j + \varepsilon) \geq U^*_j(d_j, I^*_j - q\cdot d_j) + \varepsilon \geq U^*_j(f_j, I^*_j - r\cdot f_j)$$

$$\text{and } (d_j, I_j^* - q\cdot d_j + \varepsilon) \succeq_j^* (f_j, I_j^* - r\cdot f_j).$$ **QED**

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