ARGUMENT ESTIMATES OF MEROMORPHICALLY MULTIVALENT FUNCTIONS

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MEROMORPHICALLY MULTIVALENT FUNCTIONS

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ABSTRACT. The object of the present paper is to obtain some argument properties of meromorphically multivalent functions in the punctured open unit disk. We also derive the integral preserving properties in a sector.

1. Introduction

Let \(\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}\). For \(f\) and \(g\) which are analytic in \(\mathcal{U}\), we say that \(f\) is subordinate to \(g\), written \(f \prec g\) or \(f(z) \prec g(z)\), if there exists a Schwarz function \(w\) in \(\mathcal{U}\) such that \(f(z) = g(w(z))\).

Let \(\Sigma_p\) denote the class of all meromorphic functions of the form

\[ f(z) = \frac{1}{z^p} + \frac{a_0}{z^{p-1}} + \cdots + a_{k+p-1}z^k + \cdots \quad (p \in \mathbb{N} = \{1, 2, \cdots\}) \]

which are analytic in the annulus \(\mathcal{D} = \{z : 0 < |z| < 1\}\). We denote by \(\Sigma_p^*(\beta)\) the subclass of \(\Sigma_p\) consisting of all functions which is meromorphically starlike of order \(\beta\) in \(\mathcal{U}\).

The Hadamard product or convolution of two functions \(f\) and \(g\) in \(\Sigma_p\) will be denoted by \(f \ast g\).

Let

\[ D^{n+p-1}f(z) = \frac{1}{z^p(1-z)^{n+p}} \ast f(z) \quad (z \in \mathcal{D}) \quad (1.1) \]

or, equivalently,

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\[ D^{n+p-1}f(z) = \frac{1}{z^{n+2p-1}} \left( \frac{z^{n+2p-1}f(z)}{(n+p-1)!} \right)^{(n+p-1)} \]

\[ = \frac{1}{zp} + (n+p)a_0 \frac{1}{zp-1} + \frac{(n+p+1)(n+p)}{2!} a_1 \frac{1}{zp-2} + \ldots \]

\[ \ldots + \frac{(n+k+2p-1)\ldots(n+p)}{(k+p)!} a_{k+p-1} z^k + \ldots \quad (z \in D), \]

where \( n \) is any integer greater than \(-p\).

For various interesting developments involving the operators \( D^{n+p-1} \) for functions belonging to \( \Sigma_p \), the reader may be referred to the recent works of author [1], Uralegaddi and Path [7], and others [8,9].

Let

\[ \Sigma^*_p[n; A, B] = \left\{ f \in \Sigma_p : \frac{z(D^{n+p-1}f(z))'(z)}{D^{n+p-1}f(z)} < p \frac{1 + Az}{1 + Bz}, \quad z \in \mathcal{U} \right\}, \quad (1.2) \]

where \(-1 \leq B < A \leq 1\). In particular, we note that \( \Sigma^*_p[-p+1; 1, -1] \) is the well known class of meromorphically \( p \)-valent starlike functions. From (1.2), we observe [6] that a function \( f \) is in \( \Sigma^*_p[n; A, B] \) if and only if

\[ \left| \frac{z(D^{n+p-1}f(z))'(z)}{D^{n+p-1}f(z)} + p(1-AB) \frac{1}{1-B^2} \right| < \frac{p(A-B)}{1-B^2} \quad (-1 < B < A \leq 1; \quad z \in \mathcal{U}). \quad (1.3) \]

The object of the present paper is to give some argument estimates of meromorphically multivalent functions belonging to \( \Sigma_p \) and the integral preserving properties in connection with the differential operators \( D^{n+p-1} \) defined by (1.1).

### 2. Main results

To establish our main results, we need the following lemmas.

**Lemma 2.1 [2].** Let \( h \) be convex univalent in \( \mathcal{U} \) with \( h(0) = 1 \) and \( \text{Re} (\beta h(z) + \gamma) > 0 (\beta, \gamma \in \mathbb{C}) \). If \( q \) is analytic in \( \mathcal{U} \) with \( q(0) = 1 \), then

\[ q(z) + \frac{zq'(z)}{\beta q(z) + \gamma} \prec h(z) \quad (z \in \mathcal{U}) \]

implies
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\[ q(z) \prec h(z) \quad (z \in \mathcal{U}). \]

**Lemma 2.2** [4]. Let \( h \) be convex univalent in \( \mathcal{U} \) and \( \lambda(z) \) be analytic in \( \mathcal{U} \) with \( \Re \lambda(z) \geq 0 \). If \( q \) is analytic in \( \mathcal{U} \) and \( q(0) = h(0) \), then

\[ q(z) + \lambda(z) z q'(z) < h(z) \quad (z \in \mathcal{U}) \]

implies

\[ q(z) < h(z) \quad (z \in \mathcal{U}). \]

**Lemma 2.3** [5]. Let \( q \) be analytic in \( \mathcal{U} \) with \( q(0) = 1 \) and \( q(z) \neq 0 \) in \( \mathcal{U} \). Suppose that there exists a point \( z_0 \in \mathcal{U} \) such that

\[ |\arg q(z)| < \frac{\pi}{2} \alpha \quad \text{for} \quad |z| < |z_0| \]

and

\[ |\arg q(z_0)| = \frac{\pi}{2} \alpha \quad (0 < \alpha \leq 1). \]

Then we have

\[ \frac{z_0 q'(z_0)}{q(z_0)} = i k \alpha, \]

where

\[ k \geq \frac{1}{2} \left( a + \frac{1}{a} \right) \quad \text{when} \quad \arg q(z_0) = \frac{\pi}{2} \alpha \]  \hspace{1cm} (2.4)

\[ k \leq \frac{1}{2} \left( a + \frac{1}{a} \right) \quad \text{when} \quad \arg q(z_0) = -\frac{\pi}{2} \alpha \]  \hspace{1cm} (2.5)

and

\[ q(z_0)^{\frac{1}{\alpha}} = \pm i a \quad (a > 0). \]  \hspace{1cm} (2.6)

At first, with the help of Lemma 2.1, we obtain the following

**Proposition 2.1.** Let \( h \) be convex univalent in \( \mathcal{U} \) with \( h(0) = 1 \) and \( \Re h \) be bounded in \( \mathcal{U} \). If \( f \in \Sigma_p \) satisfies the condition

\[ -z(D^{n+p}f(z))' \prec \frac{h(z)}{p D^{n+p}f(z)} \quad (z \in \mathcal{U}), \]
then
\[-\frac{z(D^{n+p-1}f(z))'}{pD^{n+p-1}f(z)} \prec h(z) \quad (z \in U)\]
for \(\max_{z \in U} \Re h(z) < \frac{n+2p}{p}\) (provided \(D^{n+p-1}f(z) \neq 0\) in \(U\)).

Proof. Let
\[\begin{align*}
q(z) &= -\frac{z(D^{n+p-1}f(z))'}{pD^{n+p-1}f(z)}.
\end{align*}\]

By using the equation
\[z(D^{n+p-1}f(z))' = (n+p)D^{n+p}f(z) - (n+2p)D^{n+p-1}f(z),\] (2.7)
we get
\[q(z) - \frac{n + 2p}{p} = -\frac{(n+p)D^{n+1}f(z)}{pD^{n+p-1}f(z)}.\] (2.8)

Taking logarithmic derivatives in both sides of (2.8) and multiplying by \(z\), we have
\[-\frac{zq'(z)}{-pq(z) + n + 2p} + q(z) = -\frac{z(D^{n+p}f(z))'}{pD^{n+p}f(z)} \prec h(z) \quad (z \in U).\]

From Lemma 2.1, it follows that \(q(z) \prec h(z)\) for \(\Re (-h(z) + \frac{n+2p}{p}) > 0 \quad (z \in U)\), which means
\[-\frac{z(D^{n+p-1}f(z))'}{pD^{n+p-1}f(z)} \prec h(z) \quad (z \in U)\]
for \(\max_{z \in U} \Re h(z) < \frac{n+2p}{p}\).

**Proposition 2.2.** Let \(h\) be convex univalent in \(U\) with \(h(0) = 1\) and \(\Re h\) be bounded in \(U\). Let \(F\) be the integral operator defined by
\[F(z) = \frac{c}{z^{c+p}} \int_{0}^{z} t^{c+p-1}f(t)dt \quad (c > 0).\] (2.9)

If \(f \in \Sigma_p\) satisfies the condition
\[-\frac{z(D^{n+p-1}f(z))'}{pD^{n+p-1}f(z)} \prec h(z) \quad (z \in U),\]
then
\[-\frac{z(D^{n+p-1}F(z))'}{pD^{n+p-1}F(z)} \prec h(z) \quad (z \in U)\]
for \(\max_{z \in U} \Re h(z) < \frac{c+p}{p}\) (provided \(D^{n+p-1}F(z) \neq 0\) in \(U\)).
Proof. From (2.9), we have
\[ z(D^{n+p-1}F(z))' = cD^{n+p-1}f(z) - (c + p)D^{n+p-1}F(z). \] (2.10)

Let
\[ p(z) = \frac{z(D^{n+p-1}F(z))'}{pD^{n+p-1}F(z)}. \]

Then, by using (2.10), we get
\[ q(z) - (c + p) = -c \frac{D^{n+p-1}f(z)}{D^{n+p-1}F(z)}. \] (2.11)

Taking logarithmic derivatives in both sides of (2.11) and multiplying by \( z \), we have
\[ \frac{zq'(z)}{pq(z) + (c + p)} + q(z) = -\frac{z(D^{n+p-1}f(z))'}{pD^{n+p-1}f(z)} < h(z) \ (z \in \mathcal{U}). \]

Therefore, by Lemma 2.1, we have
\[ -\frac{z(D^{n+p-1}F(z))'}{pD^{n+p-1}F(z)} < h(z) \ (z \in \mathcal{U}) \]
for \( \max_{z \in \mathcal{U}} \Re h(z) < \frac{c+p}{p} \) (provided \( D^{n+p-1}F(z) \neq 0 \) in \( \mathcal{U} \)).

Remark. Taking \( p = 1 \) and \( h(z) = \frac{1+z}{1-z} \) in Proposition 2.1 and Proposition 2.2, we have the results obtained by Ganigi and Urlegaddi[3].

Applying Lemma 2.2, Lemma 2.3 and Proposition 2.1, we now derive

**Theorem 2.1.** Let \( f \in \Sigma_p \). Choose an integer \( n \) such that
\[ n \geq \frac{p(1 + A)}{1 + B} - 2p, \]
where \(-1 < B < A \leq 1\) and \( p \in \mathbb{N} \). If
\[ \left| \arg \left( -\frac{z(D^{n+p}f(z))'}{D^{n+p}g(z)} - \gamma \right) \right| < \frac{\pi}{2} \delta \ (0 \leq \gamma < p; \ 0 < \delta \leq 1) \]
for some \( g \in \Sigma_p^*[n + 1; A, B] \), then
\[ \left| \arg \left( -\frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} - \gamma \right) \right| < \frac{\pi}{2} \alpha, \]
where \( \alpha \ (0 < \alpha \leq 1) \) is the solution of the equation
\[ \delta = \alpha + \frac{2}{\pi} \tan^{-1} \left( \frac{\alpha \sin \frac{\pi}{2}(1 - t(A, B))}{(n+2p)/(1-B)+A-1 + \alpha \cos \frac{\pi}{2}(1 - t(A, B))} \right) \]  

(2.12)

when

\[ t(A, B) = \frac{2}{\pi} \sin^{-1} \left( \frac{p(A-B)}{(n+2p)(1-B^2) - p(1-AB)} \right). \]  

(2.13)

**Proof.** Let

\[ q(z) = -\frac{1}{p - \gamma} \left( \frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} + \gamma \right). \]

By (2.7), we have

\[ (p - \gamma)zq'(z)D^{n+p-1}g(z) + (1 - \gamma)q(z)z(D^{n+p-1}g(z))' \]

\[ - (n + 2p)z(D^{n+p-1}f(z))' = -(n + p)z(D^{n+p}f(z))' - \gamma z(D^{n+p-1}g(z))' (z). \]  

(2.14)

Dividing (2.14) by \( D^{n+p-1}g(z) \) and simplifying, we get

\[ q(z) + \frac{zq'(z)}{-r(z) + n + 2p} = -\frac{1}{p - \gamma} \left( \frac{z(D^{n+p}f(z))'}{D^{n+p}g(z)} + \gamma \right), \]  

(2.15)

where

\[ r(z) = -\frac{z(D^{n+p-1}g(z))'}{D^{n+p-1}g(z)}. \]

Since \( g \in \Sigma_p^{*}[n+1; A, B] \), from Proposition 2.1, we have

\[ r(z) < p \frac{1+Az}{1+Bz}. \]

Using (1.3), we have

\[ -r(z) + n + 2p = \rho e^{i\frac{\pi}{2}} \phi, \]

where

\[ \left\{ \begin{array}{l}
\frac{(n+2p)(1+B)-(1+A)}{1+B} < \rho < \frac{(n+2p)(1-B)+A-1}{1-B} \\
-t(A, B) < \phi < t(A, B)
\end{array} \right. \]
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when \( t(A, B) \) is given by (2.13). Let \( h \) be a function which maps \( \mathcal{U} \) onto the angular domain \( \{ w : |\arg w| < \frac{\pi}{2}\delta \} \) with \( h(0) = 1 \). Applying Lemma 2.2 for this \( h \) with \( \lambda(z) = \frac{1}{-r(z)+n+2p} \), we see that \( \text{Re} q(z) > 0 \) in \( \mathcal{U} \) and hence \( q(z) \neq 0 \) in \( \mathcal{U} \).

If there exists a point \( z_0 \in \mathcal{U} \) such that the conditions (2.1) and (2.2) are satisfied, then (by Lemma 2.3) we obtain (2.3) under the restrictions (2.4), (2.5) and (2.6).

At first, suppose that \( q(z_0)^{\frac{1}{a}} = ia \) \((a > 0)\). Then we obtain

\[
\arg \left[ -\frac{1}{p-\gamma} \left( \frac{z_0(D^{n+p}f(z_0)'}{D^{n+p}g(z_0)} + \gamma \right) \right] = \arg \left( q(z_0) + \frac{z_0q'(z_0)}{-r(z_0) + n + 2p} \right)
\]

\[
= \frac{\pi}{2} \alpha + \arg \left( 1 + i\alpha k(\rho e^{i\frac{\pi}{2}\phi})^{-1} \right)
\]

\[
= \frac{\pi}{2} \alpha + \tan^{-1} \left( \frac{\eta k \sin \frac{\pi}{2}(1-\phi)}{\rho + \alpha k \cos \frac{\pi}{2}(1-\phi)} \right)
\]

\[
\geq \frac{\pi}{2} \alpha + \tan^{-1} \left( \frac{\alpha \sin \frac{\pi}{2}(1-t(A,B))}{\frac{(n+2p)(1-B)+A-1}{1-B} + \alpha \cos \frac{\pi}{2}(1-t(A,B))} \right)
\]

\[
= \frac{\pi}{2} \delta,
\]

where \( \delta \) and \( t(A, B) \) are given by (2.12) and (2.13), respectively. This is a contradiction to the assumption of our theorem.

Next, suppose that \( p(z_0)^{\frac{1}{a}} = -ia \) \((a > 0)\). Applying the same method as the above, we have

\[
\arg \left[ -\frac{1}{p-\gamma} \left( \frac{z_0(D^{n+p}f(z_0)'}{D^{n+p}g(z_0)} + \gamma \right) \right] \leq -\frac{\pi}{2} \alpha - \tan^{-1} \left( \frac{\alpha \sin \frac{\pi}{2}(1-t(A,B))}{\frac{(n+2p)(1-B)+A-1}{1-B} + \alpha \cos \frac{\pi}{2}(1-t(A,B))} \right)
\]

\[
= -\frac{\pi}{2} \delta,
\]

where \( \delta \) and \( t(A, B) \) are given by (2.12) and (2.13), respectively, which contradicts the assumption. Therefore we complete the proof of our theorem.

Letting \( A = 1, B = 0 \) and \( \delta = 1 \) in Theorem 2.1, we have

**Corollary 2.1.** Let \( f \in \Sigma \). If

\[
-\text{Re} \left\{ \frac{z(D^{n+p}f(z)')}{D^{n+p}g(z)} \right\} > \gamma \quad (0 \leq \gamma < p)
\]
for some $g \in \Sigma_p$ satisfying the condition

$$\left| \frac{z(D^{n+p}g(z))'}{D^{n+p}g(z)} + p \right| < p,$$

then

$$-\text{Re} \left\{ \frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} \right\} > \gamma.$$

Taking $A = 1$, $B = 0$ and $g(z) = \frac{1}{z^p}$ in Theorem 2.1, we have

**Corollary 2.2.** Let $f \in \Sigma_p$. If

$$\left| \arg \left[ -z^{p+1}(D^{n+p}f(z))' - \gamma \right] \right| < \frac{\pi}{2} \delta \quad (0 \leq \gamma < p; \ 0 < \delta \leq 1),$$

then

$$\left| \arg \left[ -z^{p+1}(D^{n+p-1}f(z))' - \gamma \right] \right| < \frac{\pi}{2} \delta.$$
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for some \( g \in \Sigma_p^*[n+1; A, B] \), then

\[
\left| \arg \left( \frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} + \gamma \right) \right| < \frac{\pi}{2} \alpha,
\]

where \( \alpha (0 < \alpha \leq 1) \) is the solution of the equation given by (2.12).

Next, we prove

**Theorem 2.3.** Let \( f \in \Sigma_p \) and choose a positive number \( c \) such that

\[
c \geq \frac{1+A}{1+B} - p,
\]

where \(-1 < B < A \leq 1\) and \( p \in \mathbb{N} \). If

\[
\left| \arg \left( -\frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} - \gamma \right) \right| < \frac{\pi}{2} \delta (0 \leq \gamma < p ; 0 < \delta \leq 1)
\]

for some \( g \in \Sigma_p^*[n; A, B] \), then

\[
\left| \arg \left( -\frac{z(D^{n+p-1}F(z))'}{D^{n+p-1}G(z)} - \gamma \right) \right| < \frac{\pi}{2} \alpha,
\]

where \( F \) is the integral operator given by (2.9),

\[
G(z) = \frac{c}{z^{c+p}} \int_{0}^{z} t^{c+p-1}g(t)dt, \ (c > 0), \quad (2.16)
\]

and \( \alpha(0 < \alpha \leq 1) \) is the solution of the equation

\[
\delta = \alpha + \frac{2}{\pi} \tan^{-1} \left( \frac{\alpha \sin \frac{\pi}{2}(1-t(A, B, c))}{\frac{(c+p)(1-B)+A-1}{1-B} + \alpha \cos \frac{\pi}{2}(1-t(A, B, c))} \right) \quad (2.17)
\]

when

\[
t(A, B, c) = \frac{2}{\pi} \sin^{-1} \left( \frac{p(A-B)}{(c+p)(1-B^2) - p(1-AB)} \right).
\]

**Proof.** Let

\[
q(z) = -\frac{1}{p-\gamma} \left( \frac{z(D^nF(z))'}{D^nG(z)} + \gamma \right).
\]

Since \( g \in \Sigma_p^*[n; A, B] \), from Proposition 2.2, \( g \in \Sigma_p^*[n; A, B] \).
Using (2.10), we have

\[(p - \gamma)q(z)D^{n+p-1}G(z) - (c + p)D^{n+p-1}F(z) = -cD^{n+p-1}f(z) - \gamma D^{n+p-1}G(z).\]

Then, by a simple calculation, we get

\[(p - \gamma)(zq'(z) + q(z)(-r(z) + c + p)) + \gamma(-r(z) + c + p) = -\frac{cz(D^{n+p-1}f(z))'}{D^{n+p-1}G(z)},\]

where

\[r(z) = -\frac{z(D^{n+p-1}G(z))'}{D^{n+p-1}G(z)}.\]

Hence we have

\[q(z) + \frac{zq'(z)}{-r(z) + c + p} = -\frac{1}{p - \gamma} \left( \frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}G(z)} + \gamma \right).\]

The remaining part of the proof is similar to that of Theorem 2.1 and so we omit it.

Letting \(n = -p + 1, A = 1, B = 0\) and \(\delta = 1\) in Theorem 2.3, we have

**Corollary 2.4.** Let \(c > 0\) and \(f \in \Sigma\). If

\[-\text{Re} \left\{ \frac{zf'(z)}{g(z)} \right\} > \gamma \ (0 \leq \gamma < p)\]

for some \(g \in \Sigma_p\) satisfying the condition

\[\left| \frac{zg'(z)}{g(z)} + p \right| < p;\]

then

\[-\text{Re} \left\{ \frac{zF'(z)}{G(z)} \right\} > \gamma,\]

where \(F\) and \(G\) are given by (2.9) and (2.16), respectively.

Taking \(n = 0, B \to A\) and \(g(z) = \frac{1}{z^p}\) in Theorem 2.3, we have

**Corollary 2.5.** Let \(c > 0\) and \(f \in \Sigma_p\). If
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$$|\arg (-z^{p+1}f'(z) - \gamma)| < \frac{\pi}{2} \delta, \quad (0 \leq \gamma < p; \quad 0 < \delta \leq 1)$$

then

$$|\arg (-z^{p+1}F'(z) - \gamma)| < \frac{\pi}{2} \alpha,$$

where $F$ is the integral operator given by (2.9) and $\alpha (0 < \alpha \leq 1)$ is the solution of the equation

$$\delta = \alpha + \frac{2}{\pi} \tan^{-1} \left( \frac{\alpha}{c + p - 1} \right).$$

By using the same methods as in proving Theorem 2.3, we have

**Theorem 2.4.** Let $f \in \Sigma_p$ and choose a positive number $c$ such that

$$c \geq \frac{1 + A}{1 + B} - p,$$

where $-1 < B < A \leq 1$ and $p \in \mathbb{N}$. If

$$|\arg \left( \frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} + \gamma \right) < \frac{\pi}{2} \delta \quad (\gamma > p; \quad 0 < \delta \leq 1)$$

for some $g \in \Sigma_p^{*}[n;A,B]$, then

$$|\arg \left( \frac{z(D^{n+p-1}F(z))'}{D^{n+p-1}G(z)} + \gamma \right) < \frac{\pi}{2} \alpha,$$

where $F$ and $G$ are given by (2.9) and (2.16), respectively, and $\alpha (0 < \alpha \leq 1)$ is the solution of the equation given by (2.17)

Finally, we derive

**Theorem 2.5.** Let $f \in \Sigma_p$. Choose an integer $n$ such that

$$n \geq \frac{p(1 + A)}{1 + B} - 2p,$$

where $-1 < B < A \leq 1$ and $p \in \mathbb{N}$. If

$$|\arg \left( \frac{-z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} - \gamma \right) < \frac{\pi}{2} \delta \quad (0 \leq \gamma < p; \quad 0 < \delta \leq 1)$$

for some $g \in \Sigma_p^{*}[n;A,B]$, then
where \( F \) and \( G \) are given by (2.9) and (2.16) with \( c = n + p \), respectively.

Proof. From (2.7) and (2.8) with \( c = n + p \), we have
\[
D^{n+p-1}f(z) = D^{n+p}F(z)
\]
Therefore
\[
\frac{z(D^{n+p-1}f(z))'}{D^{n+p-1}g(z)} = \frac{z(D^{n+p}F(z))'}{D^{n+p}G(z)}
\]
and the result follows.

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References


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