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The Optimal Auto Sleep Scheduling for a Computer System with Batch Arrival of Transactions

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Abstract

This paper addresses a problem of how to determine the optimal sleep timing when the computer user should turn the hard disk or the display off in order to save the electric power after the computer has not been accessed. We consider a stochastic model to obtain the optimal sleep timing strategy which minimizes the expected power consumed per unit time in the steady-state, where access requirements arrive at the system according to a renewal process and contain some lightweight processes. Then, the approximation form for the idle period is proposed to represent the expected power consumed per unit time based on the workload process. We also derive the condition to exist the unique optimal sleep timing.

1 Introduction

Recently, the automatic sleep function of the hard disk or the display in a computer system is rapidly recognized to be important in terms of power management [1, 2]. In fact, the auto-sleep function is equipped in almost computer systems as a standard function. Then, the optimal design for the auto-sleep function is the most important problem, in particular, for notebook computers with limited capacity of battery. For example, on the hard disk of a computer, the electronic power consumed to warm up from sleep mode is larger than that consumed by the normal operation. Thus, it is not always effective to design the system such that moves its state to the sleep mode whenever there is no access requirement.

First, the optimal design problem for the auto-sleep function was considered by Sandoh, Hirakoshi and Kawai [3] and Hirakoshi and Sandoh [4]. Dohi, Kaio and Osaki [5] also proposed a nonparametric method to estimate the optimal sleep timing for the same problems. However, it is noted that the seminal works above simplified the underlying problem extremely and was incomplete for representation of stochastic behavior of the auto-sleep system. More valid formulations were made by Okamura, Dohi and Osaki [6, 7]. They proposed two kinds of models (Type I model and Type II model) with and without cancellation of access requirements arrived at the system, respectively. More specifically, Type I model with cancellation assumes that other access requirements arrived at the system when one job has been processed are canceled, and focuses on the multi-use circumstance for a desktop computer unit. On the other hand, Type II model corresponds to a buffer system in which the other access requirements arrived latter are accumulated when one job has been processed, and considers the multi-task system such as network printers. They proved that the optimal sleep timing strategies for both models are *the switching strategies*, *i.e.*, turn always the system off after the process for a job is completed or not do at all, if the access requirements arrive according to the homogeneous Poisson process.

However, notice that the actual multi-task system is required to process a huge number of transactions during a limited period. This implies that the auto-sleep scheduling problem should be considered for the buffer system with batch arrival of transactions, such that a single arrival is composed of some lightweight transactions called *threads*. Threads are parts of a single arrived process which are independently scheduled each other in the system. In this paper, we design the optimal auto-sleep schedule for a computer system with batch arrival of transactions. If a single arrival has only one thread, the model under consideration can be reduce to Type II model in Okamura, Dohi and Osaki [7]. In that sense, the model in this paper is an extension of Type II model.

The paper is planned as follows. Section 2 describes the auto-sleep model with batch arrival of transactions. In Section 3, we give an implicit form of the expected power consumed per unit time in the steady-state under the assumption that the access requirements arrive at the system following a compound renewal process. Section 4 concerns the approximation problem for the expected power consumed per unit time. Then we propose an approximation form of the idle period distribution. The condition to exist the unique optimal sleep timing is derived approximately. In Section 5, we give some numerical examples, and investigate the asymptotic property of the idle period derived in Section 4. Also, we examine the dependence on model parameters for the optimal auto-sleep timing. Finally, the paper is concluded with some remarks.

2 Auto-Sleep Model for a Computer System

We assume that the stochastic system under consideration can take the following four states:

busy state: The process is started at least for an access requirement, where $\tau\,(>0)$ time units are needed for making ready the process. The process is continued until the buffer is empty, and the system is turned immediately an idle state after completing the final process. In the busy state, we suppose that the electronic power consumed per unit time is $P_1\,(>0)$.

idle state: In this state, the system is waiting at least for an access requirement. If the access requirement arrives at the system up to the time limit $t_0\,(0 \le t_0 < \infty)$, the process is started at that time, otherwise, the system is moved to a sleep mode. In the idle state, the system also consumes the electronic power $P_1\,(>0)$ per unit time.

sleep state: Suppose that the system is dormant. If more than one access requirement come in the sleep state, the system state moves to a warm-up state immediately. To simplify the analysis, we suppose that the electronic power occurred during the sleep mode is zero.

warm-up state: After arriving an access requirement, the system is warmed up from a sleep mode, where $s\,(>0)$ time units are needed for the warm-up. After warming the system, the state is moved to a busy state immediately. Since the electronic power in the sleep mode is known to be rather small relative to that consumed during the warm-up period, we denote $P_2\,(P_2 > P_1)$ as the electronic power consumed per unit warm-up time.

Suppose that the access requirements arrive at the system according to a compound renewal process. Denote $\{X_k : k = 1, 2, \cdots\}$ as a sequence of inter-arrival times between $(k-1)$-th and k-th arrivals. Then, X_k are non-negative i.i.d. random variables, having the probability distribution $F(t)$ with mean $1/\lambda\,(>0)$ and variance $\sigma_A\,(>0)$. There are some threads contained in an access requirement. The number of threads contained in k-th arrival is given by the random variable N_k with finite mean $\mu_B\,(>0)$ and variance $\sigma_B\,(>0)$. The other threads arrived during the processing time are accumulated in the buffer. The k-th thread contained in n-th arrival is processed for a service time $S_{n,k}$, which is the random variable having the probability distribution $G(t)$ with finite mean $1/\mu_S\,(>0)$ and variance $\sigma_S\,(>0)$. The system processes threads exhaustively until the buffer becomes empty. Since the system under consideration is very similar to the *GI/GI/1* queueing system with a server vacation (see Takagi [8]), we may analyze the stochastic properties for the auto-sleep system in the framework of queueing analysis.

3 Expected Power Formulation

To formulate the expected power consumed per unit time in the steady-state, we introduce the following two random variables (see Fig. 1);

ζ_x (time length of busy period) : time period for an exhaustive processing, provided that the server vacation period is x,

η_x (time length of idle period) : time period when there is no access requirement in the buffer, provided that the server vacation period is x.

Let us define the probability distribution function of η_x by $I(t|x) = \Pr\{\eta_x \le t\}$, where in general $\overline{\phi}(\cdot) = 1 - \phi(\cdot)$ is the survivor function. From the well-known result for queueing analysis with server vacation (see Takagi [8]), we obtain the following equilibrium equation;

$$\mathrm{E}[\zeta_x] = \rho(x + \mathrm{E}[\zeta_x] + \mathrm{E}[\eta_x]), \tag{1}$$

where $\rho = \lambda\mu_B/\mu_S$ is the traffic intensity. Since Eq. (1) holds only if $\rho < 1$, we assume in the rest part of this paper that the condition $\rho < 1$ holds.

The system is resumed at the point of time when the state moves from the sleep mode to the busy one. Then, we define the time period between the successive resuming points as one cycle. Let $T(t_0)$ denote the mean time length of one cycle, provided that the sleep timing is fixed as t_0. Noting whether the first idle period $\eta_{s+\tau}$ exceeds the sleep timing t_0, the mean time length of one cycle is written by

$$\begin{aligned} T(t_0) \;=\; & \int_0^{t_0} \{s + \tau + \mathrm{E}[\zeta_{s+\tau}] + t + T_r(t_0)\}\, dI(t|s+\tau) \\ & + \int_{t_0}^{\infty} \{s + \tau + \mathrm{E}[\zeta_{s+\tau}] + t\}\, dI(t|s+\tau). \end{aligned} \tag{2}$$

where $T_r(t_0)$ is the expected time length from the termination of the first idle period until the beginning of the next cycle. It is evident that the first idle period is we have

$$T_r(t_0) \;=\; \int_0^{t_0} \{\tau + \mathrm{E}[\zeta_\tau] + t + T_r(t_0)\}\, dI(t|\tau) + \int_{t_0}^{\infty} \{\tau + \mathrm{E}[\zeta_\tau] + t\}\, dI(t|\tau). \tag{3}$$

Hence, from Eqs. (2) and (3), the mean time length of one cycle becomes

$$T(t_0) = s + \tau + \mathrm{E}[\zeta_{s+\tau}] + \mathrm{E}[\eta_{s+\tau}] + \frac{I(t_0|s+\tau)}{\overline{I}(t_0|\tau)}\{\tau + \mathrm{E}[\zeta_\tau] + \mathrm{E}[\eta_\tau]\}. \tag{4}$$

Furthermore, substituting Eq. (1) into Eq. (4), we have

$$T(t_0) = \frac{1}{1-\rho}\Big\{s + \tau + \mathrm{E}[\eta_{s+\tau}] + (\tau + \mathrm{E}[\eta_\tau])\frac{I(t_0|s+\tau)}{\overline{I}(t_0|\tau)}\Big\}. \tag{5}$$

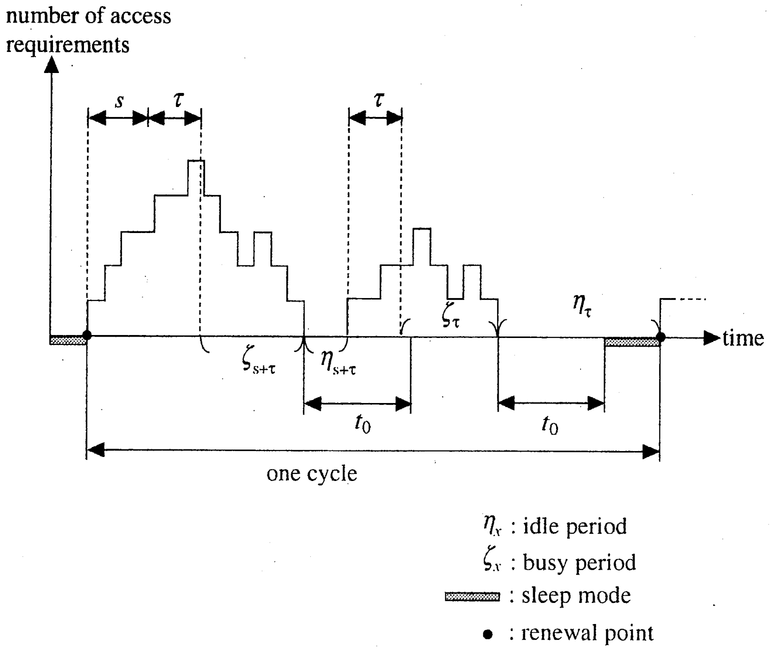

Figure 1: Possible realization of the stochastic system.

Let the random variable N denote the total number of transition from the idle to the busy during one cycle. Then, it is verified that N has the following probability mass function;

$$\Pr\{N=n\} = \begin{cases} \overline{I}(t_0|s+\tau) & \text{for } n=0, \\ I(t_0|s+\tau)\overline{I}(t_0|\tau)I(t_0|\tau)^{n-1} & \text{for } n=1,2,\cdots. \end{cases} \tag{6}$$

From the property of the geometric distribution, the expected value of N is

$$\mathrm{E}[N] = \frac{I(t_0|s+\tau)}{\overline{I}(t_0|\tau)}. \tag{7}$$

Using Eq. (7), the mean time length of one cycle has the following simple form:

$$T(t_0) = \frac{1}{1-\rho}\Big\{s+\tau+\mathrm{E}[\eta_{s+\tau}]+\mathrm{E}[N](\tau+\mathrm{E}[\eta_\tau])\Big\}, \tag{8}$$

where Eq. (8) depends only on the idle distribution $I(t|\cdot)$.

In a fashion similar to the mean time length of one cycle, we define the total expected power consumed during one cycle as

$$\begin{aligned} C(t_0) &= \int_0^{t_0}\{P_2 s + P_1(\tau+\mathrm{E}[\zeta_{s+\tau}]+t)+C_r(t_0)\}\,dI(t|s+\tau) \\ &\quad + \int_{t_0}^{\infty}\{P_2 s + P_1(\tau+\mathrm{E}[\eta_{s+\tau}]+t_0)\}\,dI(t|s+\tau), \end{aligned} \tag{9}$$

where

$$C_r(t_0) = \int_0^{t_0}\{P_1(\tau+\mathrm{E}[\zeta_\tau]+t)+C_r(t_0)\}\,dI(t|\tau)$$

$$+\int_{t_0}^{\infty}\{P_1(\tau+\mathrm{E}[\eta_{s+\tau}]+t_0)\}\,dI(t|\tau) \tag{10}$$

is the expected power consumed for the period $T_r(t_0)$. From a few algebraic manipulations, the total expected power consumed for one cycle is given by

$$\begin{aligned}
C(t_0) &= P_2 s + P_1\Big\{\tau+\mathrm{E}[\zeta_{s+\tau}]+\mathrm{E}[\eta_{s+\tau}\wedge t_0]\Big\}+P_1\mathrm{E}[N]\Big\{\tau+\mathrm{E}[\zeta_\tau]+\mathrm{E}[\eta_\tau\wedge t_0]\Big\}\\
&= \Big\{\frac{\rho}{1-\rho}P_1+P_2\Big\}s+\frac{P_1\tau}{1-\rho}+P_1\Big\{\frac{\rho}{1-\rho}\mathrm{E}[\eta_{s+\tau}]+\mathrm{E}[\eta_{s+\tau}\wedge t_0]\Big\}\\
&\quad+\Big\{\frac{P_1\tau}{1-\rho}+P_1\Big(\frac{\rho}{1-\rho}\mathrm{E}[\eta_\tau]+\mathrm{E}[\eta_\tau\wedge t_0]\Big)\Big\}\frac{I(t_0|s+\tau)}{\overline{I}(t_0|\tau)},
\end{aligned} \tag{11}$$

where

$$\mathrm{E}[\eta_x\wedge t_0]=\mathrm{E}[\min(\eta_x,t_0)]=\int_0^{t_0}u\,dI(u\mid x)+t_0\overline{I}(t_0\mid x). \tag{12}$$

Finally, the expected power consumed per unit time in the steady-state is, from the well-known renewal reward argument,

$$V(t_0)=\lim_{t\to\infty}\frac{\mathrm{E}[\text{total consumed power in }(0,t]]}{t}=\frac{C(t_0)}{T(t_0)}. \tag{13}$$

Then, the problem is to seek the optimal sleep timing t_0^* which minimizes $V(t_0)$.

4 The Approximation of the Idle Period

To represent the expected power consumed per unit time $V(t_0)$ explicitly, we have to derive the probability distribution $I(t|x)$ for the idle period. It is, however, difficult to obtain the explicit form in the ordinary renewal arrival case. Thus, we propose an approximation method for the idle period based on the workload process. Let us denote the arrival time of the n-th access requirement by $\{T_n;\ n=0,1,2,\cdots\}$, where

$$T_n=\sum_{k=0}^{n}X_k. \tag{14}$$

Then, for an arbitrary time $t\in(T_n<t<T_{n+1})$, we define the workload process $\{W(t):\ t\geq 0\}$, where

$$W(t)=W(T_n+)-(t-T_n). \tag{15}$$

$$W(T_n+)=W(T_n-)+\sum_{k=1}^{N_n}S_{n,k}, \tag{16}$$

where $W(t+)=\lim_{\epsilon\to 0}W(t+\epsilon)$ and $W(t-)=\lim_{\epsilon\to 0}W(t-\epsilon)$ (see Gross and Harris [9]). Notice that if $W(t)>0$ then the system is busy at time t, otherwise, the system is idle. Suppose that the first arrival of access requirements occurs at time $t=0$ and that the server vacation period is $s+\tau$. If the number of access requirements which experience the first empty buffer in the system is n^*, then the idle period can be represented as

$$\eta_{s+\tau}=-W(T_{n^*}-). \tag{17}$$

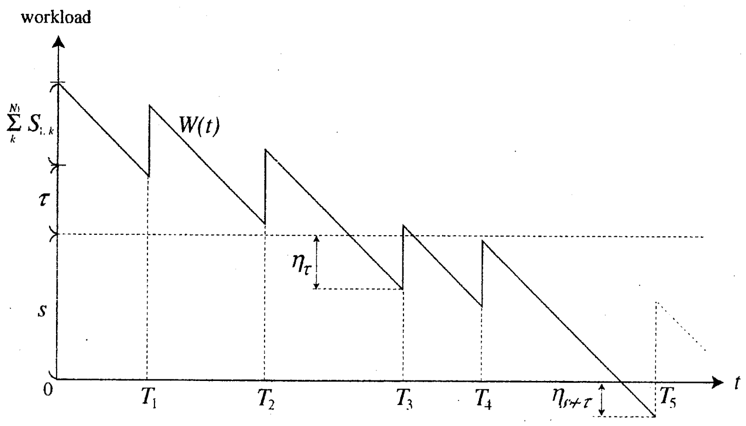

Figure 2: Behavior of the workload process.

Figure 2 illustrates the configuration of workload process. For the idle period with the server vacation, we obtain the following renewal-type equation for an arbitrary measurable function f;

$$\mathrm{E}[f(\eta_{s+\tau})] = \int_0^s \mathrm{E}[f(\eta_{s-x})]dI(x|\tau) + \int_s^\infty f(x-s)dI(x|\tau). \tag{18}$$

Lemma 1 *For an arbitrary measurable function f, the asymptotic property of the idle period is given by*

$$\lim_{s\to\infty} E[f(\eta_{\tau+s})] = \frac{E[\int_0^{\eta_\tau} f(z)dz]}{E[\eta_\tau]}. \tag{19}$$

Proof. Taking the Laplace transform to both sides of Eq. (18), we have

$$\int_0^\infty \exp(-\alpha s)\mathrm{E}[f(\eta_{s+\tau})]ds = \frac{\mathrm{E}[\int_0^{\eta_\tau} \exp\{-\alpha(\eta_\tau - z)\}f(z)dz]}{\mathrm{E}[1-\exp(-\alpha\eta_\tau)]}. \tag{20}$$

where α is any complex number. Letting $\alpha \to 0$ in Eq. (20), the result is due to the well-known Abel's lemma [10]. The proof is completed. Q.E.D.

If $f(z) = \exp(-\alpha z)$ in Lemma 1, then it is seen that

$$\lim_{s\to\infty} \mathrm{E}[\exp(-\alpha\eta_{\tau+s})] = \frac{1-\mathrm{E}[\exp(-\alpha\eta_\tau)]}{\alpha\mathrm{E}[\eta_\tau]}. \tag{21}$$

Since the right-hand side of Eq. (21) is the same Laplace transform as the equilibrium distribution of η_τ, we can approximate the idle period for sufficiently larger s than τ, that is,

$$I(t|s+\tau) \approx \frac{1}{\mathrm{E}[\eta_\tau]}\int_0^t \bar{I}(s|\tau)ds. \tag{22}$$

Substituting Eq. 22 into Eqs. (4) and (11), we derive approximately the expected power consumed per unit time in the steady-state as

$$V(t_0) \approx V_a(t_0) = \frac{C_a(t_0)}{T_a(t_0)}, \tag{23}$$

where

$$T_a(t_0) = \frac{1}{1-\rho}\left\{s+\tau+\frac{\mathrm{E}[\eta_\tau^2]}{2\mathrm{E}[\eta_\tau]}+(\tau+\mathrm{E}[\eta_\tau])\frac{\int_0^{t_0}\overline{I}(t|\tau)dt}{\overline{I}(t_0|\tau)\mathrm{E}[\eta_\tau]}\right\} \tag{24}$$

and

$$\begin{aligned} C_a(t_0) \quad &= \quad \left\{\frac{\rho}{1-\rho}P_1+P_2\right\}s+\frac{P_1\tau}{1-\rho}+P_1\left\{\frac{\rho}{1-\rho}\mathrm{E}[\eta_{s+\tau}]+\int_0^{t_0}\int_t^{\infty}\overline{I}(u|\tau)dudt\right\} \\ &+\left\{\frac{P_1\tau}{1-\rho}+P_1\left(\frac{\rho}{1-\rho}\mathrm{E}[\eta_\tau]+\int_0^{t_0}\overline{I}(t|\tau)dt\right)\right\}\frac{\int_0^{t_0}\overline{I}(t|\tau)dt}{\overline{I}(t_0|\tau)\mathrm{E}[\eta_\tau]}. \end{aligned} \tag{25}$$

To obtain the optimal sleep timing, we define the mean residual life $R(t_0|\tau)$ and hazard rate $r(t_0|\tau)$;

$$R(t_0|\tau) = \frac{\int_{t_0}^{\infty}\overline{I}(t|\tau)dt}{\overline{I}(t_0|\tau)} \tag{26}$$

and

$$r(t_0|\tau) = \frac{1}{\overline{I}(t_0|\tau)}\left(\frac{dI(t_0|\tau)}{dt_0}\right). \tag{27}$$

Further, define

$$\hat{t}_0 = \inf\left\{t_0>0;\ \frac{d\log R(t_0|\tau)}{d\log r(t_0|\tau)}+\frac{\mathrm{E}[\eta_\tau]}{\overline{I}(t_0|\tau)R(t_0|\tau)}>1\right\}. \tag{28}$$

The following result gives the optimal auto sleep timing which minimizes approximately the expected power consumed per unit time in the steady-state.

Theorem 1 *Suppose that, for all $t_0 \in [\hat{t}_0, \infty)$, the idle period η_τ has decreasing hazard rate. If an auto-sleep timing t_0 satisfies $P_1 > V_a(t_0)$ for all $t_0 \in [\hat{t}_0, \infty)$, then there exists an unique optimal sleep timing $t_0^* \in [\hat{t}_0, \infty)$ which minimizes $V_a(t_0)$.*

Proof. In order to prove the convexity of $V_a(t_0)$, define

$$\tilde{C}_a(t_0) \equiv (1-\rho)\overline{I}(t_0|\tau)C_a(t_0) \tag{29}$$

and

$$\tilde{T}_a(t_0) \equiv (1-\rho)\overline{I}(t_0|\tau)T_a(t_0), \tag{30}$$

where $V_a(t_0) = \tilde{C}_a(t_0)/\tilde{T}_a(t_0)$. Further, define the numerator of the derivative of $V_a(t_0)$ with respect to t_0, divided by $\overline{I}(t_0|\tau)r(t_0|\tau)$ as $q(t_0)$, *i.e.*,

$$q(t_0) \quad = \quad P_1(1-\rho)\tilde{T}_a(t_0)\left\{\frac{1}{r(t_0|\tau)}\int_0^{t_0}\overline{I}(t|\tau)dt-\int_0^{t_0}\int_t^{\infty}\overline{I}(u|\tau)dudt\right\}$$

$$+\left\{\frac{\tilde{C}_a(\infty)}{\mathrm{E}[\eta_\tau]r(t_0|\tau)}-\tilde{C}_a(0)\right\}\tilde{T}_a(t_0)-\left\{\frac{\tilde{T}_a(\infty)}{\mathrm{E}[\eta_\tau]r(t_0|\tau)}-\tilde{T}_a(0)\right\}\tilde{C}_a(t_0). \quad (31)$$

Differentiating $q(t_0)$ with respect to t_0 yields

$$\begin{aligned}\frac{d}{dt_0}q(t_0) &= \frac{P_1(1-\rho)\tilde{T}_a(t_0)\bar{I}(t_0|\tau)}{r(t_0|\tau)}\Bigg\{1-r(t_0|\tau)R(t_0|\tau)\\ &\quad -\frac{dr(t_0|\tau)/dt_0}{r(t_0|\tau)}\left(\frac{\mathrm{E}[\eta_\tau]}{\bar{I}(t_0|\tau)}-R(t_0|\tau)\right)\Bigg\}\\ &\quad -\frac{dr(t_0|\tau)/dt_0}{\mathrm{E}[\eta_\tau]r(t_0|\tau)^2\tilde{T}_a(\infty)\tilde{T}_a(t_0)}\left\{P_1-V_a(t_0)\right\}. \end{aligned} \quad (32)$$

If $dr(t_0|\tau)/dt_0 < 0$ holds for all $t_0 \in [\hat{t}_0,\infty)$, the first term of Eq. (32) is strictly positive for all $t_0 \in [\hat{t}_0,\infty)$. Therefore, for the range satisfying $P_1 > V(t_0)$, it can be seen that $q(t_0)/dt_0 > 0$. Since this implies $d^2V_a(t_0)/dt_0^2 > 0$, the convexity of $V_a(t_0)$ is proved for $t_0 \in [\hat{t}_0,\infty)$.
Q.E.D.

Remark 1 Theorem 1 shows that there is a local optimal sleep timing for strong conditions $P_1 > V(t_0)$ and $t_0 \in [\hat{t}_0,\infty)$. Hence the local optimal sleep timing can be calculated using any numerical optimization method such as the deepest decent method, Newton-Raphson method, *etc.* On the other hand, to find the global optimal sleep timing, we have to check the global behavior of the function $V_a(t_0)$ for all t_0. Since $V_a(\infty) = P_1$, the solution space can be limited to the range at which $P_1 > V(t_0)$ holds. This result leads to the fact that the numerical optimization method should be applied for all $t_0 \in [\hat{t}_0,\infty)$.

5 Numerical Examples

In this section, we investigate the asymptotic properties of the approximated idle period, and examine the dependence on model parameters for the optimal sleep timing. Suppose that the time interval between the successive access requirements obeys the phase-type distribution. The phase-type distribution represents the time until the absorption in the continuous time Markov chain, with the following distribution function;

$$F(t) = 1 - \boldsymbol{\alpha}\exp(Tt)\boldsymbol{e}, \quad (33)$$

where T is a $m \times m$ matrix having negative diagonal components. $\boldsymbol{\alpha}$ is a probability vector and $\boldsymbol{e}$ is a column vector of 1s. Also, we suppose that the workload of an access requirement obeys the following exponential distribution with a parameter μ:

$$G(t) = 1 - \exp(-\mu t). \quad (34)$$

Under the condition that the server vacation is x, the idle period has the following distribution (see Neuts [11]);

$$I(t|x) = 1 - \frac{\boldsymbol{\alpha}\exp\left\{(T+\boldsymbol{T^0}\boldsymbol{\alpha}G)x\right\}G\exp(Tt)\boldsymbol{e}}{\boldsymbol{\alpha}\exp\left\{(T+\boldsymbol{T^0}\boldsymbol{\alpha}G)x\right\}G\boldsymbol{e}}, \quad (35)$$

where $\boldsymbol{T^0} = -T\boldsymbol{e}$, and G is a transition probability matrix with components $[G]_{ij}$ which is the probability that the phase makes a transition from i to j.

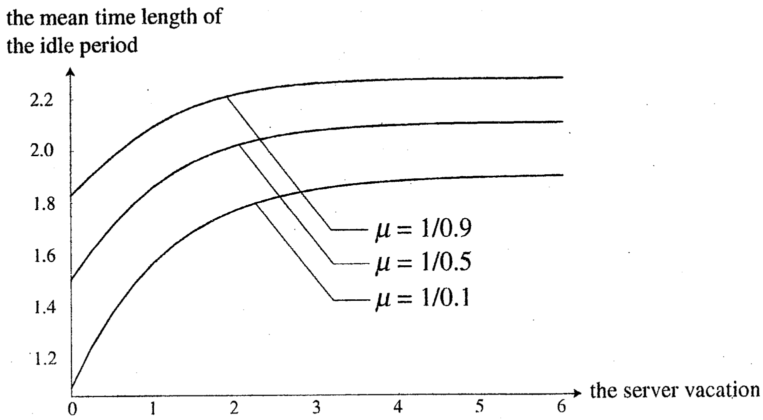

Figure 3: Asymptotic behavior of the mean time length of the idle period with varying service rate.

In this section, we assume the following model parameters;

$$T = \lambda \begin{bmatrix} -2.22 & 0.44 \\ 0.22 & -0.44 \end{bmatrix}, \qquad \alpha = (1.0, 0.0),$$

$$\lambda = 1.0,\ 0.3,\ 0.5 \text{ or } 0.7, \qquad \mu = 1.0,\ 1/0.1,\ 1/0.5 \text{ or } 1/0.9.$$

Figures 3 and 4 depict the behavior of the mean time length of the idle period for the

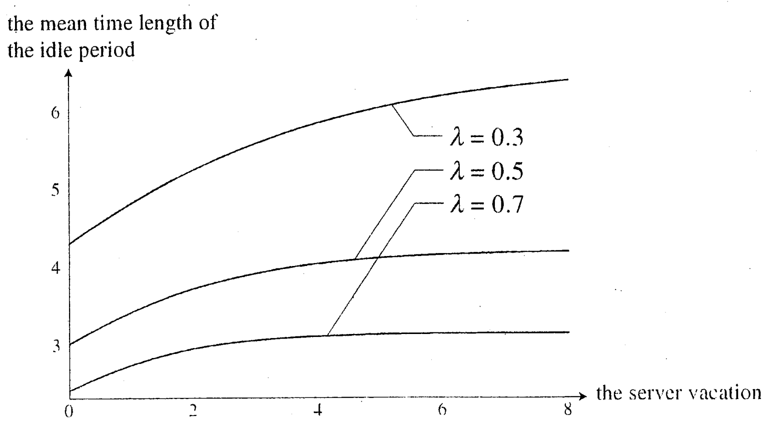

Figure 4: Asymptotic behavior of the mean time length of the idle period with varying arrival rate.

server vacation with varying μ and λ, respectively. These results explain that the mean time length of the idle period depends on the arrival of the access requirements rather than the workload. Also, taking account of the mean inter-arrival of access requirements, we observe that the mean time length of the idle period converges to a value asymptotically when the server vacation is about 4~5 times as long as the mean inter-arrival of access requirements.

Next, we investigate the dependence on model parameters for the optimal sleep timing,

Table 1: Dependence on the arrival rate for the optimal sleep timing.

λ	t_0^*	$V(t_0^*)$
0.1	0.00	0.541
0.2	0.00	0.770
0.3	0.00	0.889
0.4	0.00	0.957
0.5	1.99	0.994
0.6	∞	1.000

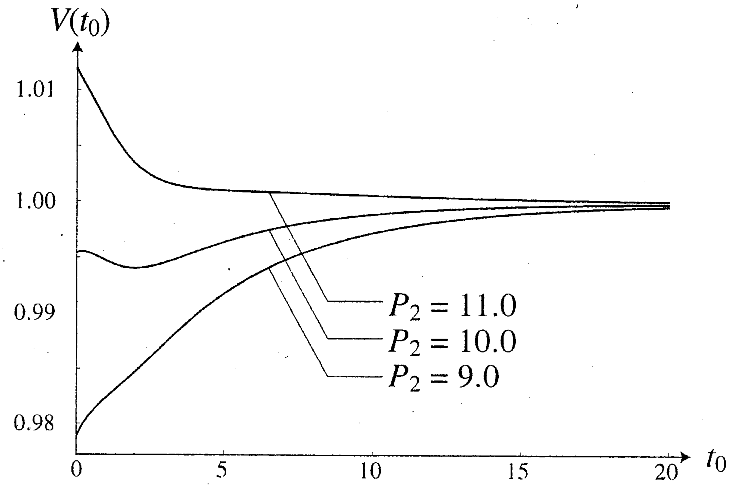

Figure 5: Dependence on the electronic power P_2 for the expected power consumed per unit time in steady-state.

where the following parameters are assumed;

$$P_1 = 1, \quad P_2 = 10, \quad s = 10.0, \quad \tau = 0.5. \tag{36}$$

and where the arrival and service parameters are the same as the previous example. In Table 1 we give the numerical value of the optimal sleep timing and the associated minimum expected power for several values of the arrival parameter λ. From Table 1, it is found that the minimum expected power increases monotonically for the increasing λ, and that the variation of the optimal sleep timing is greater than that of the mean inter arrival. These results indicate that the optimal sleep timing depends strongly on the arrival parameter, and that the auto-sleep function is a more effective power-saving function when access requirements have a light traffic. Also, in Figs. 5 and 6, we give the behavior of the expected power $V(t_0)$ for the sleep timing t_0 with $P_2 = 9.0, 10.0, 11.0$ and $s = 10.0, 12.0, 14.0$. In Fig. 5, it is seen that the shape of expected power changes sensitively as the power consumed in warm-up P_2 increases. On the other hand, it is clear from Fig. 6 that the optimal sleep timing is insensitive to warm-up time.

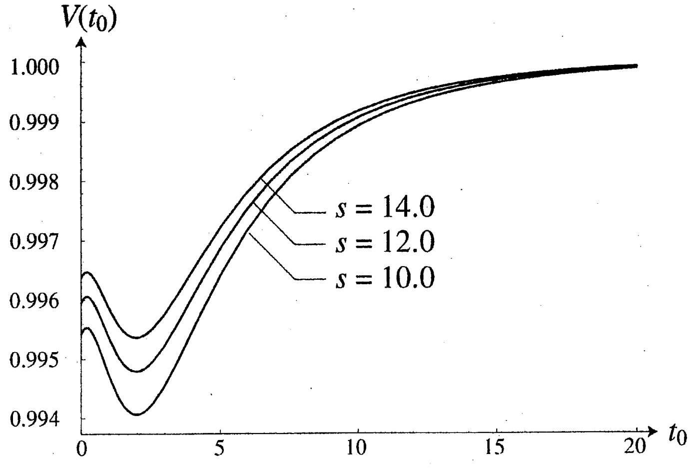

Figure 6: Dependence on the warm-up time for the expected power consumed per unit time in steady-state.

6 Concluding Remarks

In this paper, we have proposed the stochastic model to generate the auto sleep schedule for a computer system with batch arrival of transactions. In the case of the renewal access requirements, the expected power consumed per unit time in steady-state has been formulated. Also, we have derived the approximation form of the idle period from its asymptotic property. With the proposed approximation method, we have derived a sufficient condition to exist the unique optimal sleep timing which minimizes the expected power consumed per unit time in steady-state. In numerical examples, we have investigated asymptotic properties of the idle period, and have examined the sensitivity of model parameters for the optimal sleep timing.

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