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Modelling of sound fields inside acoustically-structurally coupled fields by a combined method

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Abstract

In this study, the sound fields inside acoustically-structurally coupled enclosed spaces excited by simple harmonic point sound sources and structural forcing under the presence of Helmholtz resonators including the acoustic absorption is modeled by a special modal analysis technique. Two case studies are performed, in the first one, a rectangular cavity having all boundaries flexible is solved by FEM and the proposed method. In the second case study, the efficiency of Helmholtz resonators and acoustic absorptive treatment is compared.

Introduction

Study of sound fields inside acoustically-structurally coupled enclosed spaces is always an interesting but difficult subject for those who are interested in shaping and/or controlling such fields by active and/or passive means. There are several different techniques in modeling these coupled fields such as FEM, Modal Analysis, BEM, Statistical Energy Analysis etc. In this study, it is aimed to model the low frequency response of acoustically - structurally coupled enclosed spaces under the presence of simple harmonic point sound sources and/or structural forcing for shaping these fields. Acoustical absorption is also included in the model. Model also includes the Helmholtz resonators which are well-known passive systems tuned to shape sound fields [1]. Efficiency of resonators and absorptive treatment is compared in the coupled field response in low frequency range inside acoustoelastically coupled enclosed spaces. Besides, as a special case, a rectangular cavity having all boundaries flexible is solved with the proposed method and the results are compared with FEM results obtained in a previous study [2].

Theory

Since it is interested in the low frequency response of coupled fields, mathematical model is based on a special modal analysis technique, namely, Method of Acoustoelasticity [3]. The original technique is developed and improved to include all the possible excitations and resonators are also included in the mathematical model for single and multi-cavity systems. Throughout the formulations of acoustoelastically coupled enclosed spaces the following basic assumptions are done:

i. fluid inside the cavity is at rest prior to the motion of the flexible wall .

ii. sound sources are simple harmonic point sound sources.

iii. all the sound sources and Helmholtz resonators are stationary

iv. Helmholtz resonators are compact in size when compared to the size of the enclosed space.

v. harmonic variations in time are assumed for all inputs.

vi. absorption material is to be modeled as a locally reacting material for which a force at a point leads to motion of that point only

Under the presence of Helmholtz resonators there are three different coupled systems, i.e. acoustical system, structural system and Helmholtz resonators

$$\ddot{P}_n + \sum_i \frac{A_A^i}{V}\rho_o c_o^2 \frac{\alpha_{nn}^i}{M_{n,A}}\dot{P}_n + P_n = -\frac{A_F}{V}\ddot{w}_n - \sum_k \frac{S_k}{V}\ddot{\xi}_k F_n(\vec{r}_{Rk}) - \sum_l \frac{\dot{Q}_l}{V}F_n(\vec{r}_{Sl}) \tag{1}$$

second term in Equation (1) is the acoustic absorption, and the first term on the right hand side represents the structural coupling, second term represents the Helmholtz resonators which are treated as source-like, and the last term represents the simple harmonic point sound sources

$$M_m(\ddot{q}_m + \omega_m^2 q_m) = \rho_o c_o^2 A_F \sum_m \frac{P_n L_{nm}}{M_{n,A}} + Q_m^E \tag{2}$$

Equation (2) represents the structural system and the first term on the right hand side is the acoustical loading on it and the second term is the structural external loading

$$\rho_o l_k' S_k \ddot{\xi}_k + S_k R_{ik}\dot{\xi}_k + \frac{\rho_o c_o^2 S_k^2}{V_{Rk}}\xi_k = -p(\vec{r}_{Rk})S_k \tag{3}$$

the last equation, Equation (3) is the equation representing the Helmholtz resonator and as it is seen it couples with total acoustic pressure at the mouth in acoustically-structurally coupled cavities. These three system equations can be reduced into two by defining two new resonator parameters as [1]

$$H_{Rk}(\omega_E) = \frac{S_k c_o^2}{l_k' V}\frac{\omega_E^2(-\omega_E^2 + \omega_{Rk}^2)}{R_k^2\omega_E^2 + (-\omega_E^2 + \omega_{Rk}^2)^2} \tag{4}$$

$$H_{\omega k}(\omega_E) = \frac{S_k c_o^2}{l_k' V}\frac{R_k \omega_E^3}{R_k^2\omega_E^2 + (-\omega_E^2 + \omega_{Rk}^2)^2} \tag{5}$$

Harmonic inputs are assumed for all the acoustical and structural parameters and some matrices are defined for the equations above [1]. The coupled system equations then obtained in the simple matrix form as

$$\left[[WNA] + \sum_k H_{Rk}(\omega_E)[FRMNA]_k\right]\{\overline{P}_{nr}\} + \left[\sum_k H_{\omega k}(\omega_E)[FRMNA]_k - \sum_i [\alpha nn^{(i)}]\right]\{\overline{P}_{ni}\} = \omega_E^2 \frac{A_F}{V}[LNM]\{qmr\} \tag{6.a}$$

$$\left[[WNA] + \sum_k H_{Rk}(\omega_E)[FRMNA]_k\right]\{\overline{P}_{ni}\} - \left[\sum_k H_{\omega k}(\omega_E)[FRMNA]_k + \sum_i [\alpha nn^{(i)}]\right]\{\overline{P}_{nr}\} = \omega_E^2 \frac{A_F}{V}[LNM]\{qmi\} - \sum_l \frac{\omega_E}{V}Q_l\{FNRS\}_l \tag{6.b}$$

$$[WMS]\{qmr\} = \rho_o c_o^2 A_F [LMNA]\{Pnr\} + \{Q_m^E\} \tag{7.a}$$

$$[WMS]\{qmi\} = \rho_o c_o^2 A_F [LMNA]\{Pni\} \tag{7.b}$$

Equations (6.a) to (7.b) are the general equations representing the acoustical structural system and Helmholtz resonators. As it is seen that, sound fields should contain phase information as well. Solution of these equations requires only simple matrix operations. Calculation of the eigenvalues are performed by bisection method with a dynamic step length, thus the error in obtaining the eigenvalues is proportional with the step size used in the calculations. Besides, the computational time is very short compared with other methods.

Moreover, since the theoretical solutions are employed in the solution of system equations, it is easy to determine the cost function for the optimization procedures for shaping purposes.

Case Study

Within the scope of the present study, two case study is performed. In the first case study, a rectangular cavity having all boundaries flexible of dimensions of 1m x 1m x 1m excited by unit harmonic structural forcing is solved by the proposed model and the results are compared with the results obtained from solving the same system by FEM by Ma and Hagiwara [2]. In their study, only lowest 12 modes were held, but the present model considers all the modes, for which the mode numbers changing from 1 to 8 for each structural flexible plate within the frequency range of interest. The results obtained from the present analysis are tabulated in Table 1. In Table 2, modes calculated by FEM, and the present model are compared and percentage error is given in the last column. Study of the Table 1, yields that, the coupled system at low frequency range is highly dominated by structural modes with a very high modal density and parallel plates can act like a piston ~~in the~~ in the system response. The frequencies which are very close to zero on the other hand can be considered as rigid body motion of the cavity. Table 2 illustrates that for the modes retained in the previous study, the results obtained from FEM and the results obtained from the proposed method are quite close to each other. The reason of the percentage difference between the two methods on the other hand, is due to truncation of some lower and higher modes in the FEM analysis which affects the frequency response.

In the second case study, the efficiency of Helmholtz resonators and absorptive treatment are compared in coupled field response. Although in the mathematical model, Helmholtz resonators are treated as source-like, they also introduce an absorptive mechanism to the acoustical system which is very similar to acoustic absorption provided by absorptive materials and are coupled with total acoustic pressure within the cavity. Therefore, even Helmholtz resonators are tuned to a single frequency, they affect the frequency range of interest. This can be seen in Figure 2. In Figure 3, the response of the cavity having a sound source and resonator is given. In Figure 4, the coupled acoustic pressure distribution is given.

Comparison of the figures yield that, in the absence of sound sources resonators are very promising in shaping the cavity response. When there is sound source, although the efficiency of resonators seems to be decreased compared with the absorptive treatment, this can be improved by optimizing the resonator equivalent resistance and dimensions. Besides, for the efficiency, it is necessary to optimize the resonator-source-receiver combinations. When the application of these two passive methods are compared, depending on the purpose, Helmholtz resonators can be still more attractive with proper combinations, since they are much less bulky and cheap compared with absorptive treatment. Therefore, in shaping the sound fields inside vehicle passenger cabins such as automobiles or airplane fuselages for which weight has importance, the possibility of employing the resonators as a promising passive method should also be considered having an appropriate mathematical modeling of the system.

Summary

In this study, sound fields inside coupled enclosed spaces excited by acoustical and/or structural excitations such as sound sources, structural loadings were modeled. Acoustical absorption was also included in the model. Helmholtz resonators were also introduced to the coupled cavity as an additional system, but defining two new parameters, system was reduced into coupling problem between acoustical system and structural system only.

Two case studies were performed. In the first case study, the FEM model and the model obtained from the proposed study were compared. Acoustical pressure distribution inside the cavity and all the coupled modes were calculated. Comparison was realized for the modes retained in FEM. It was seen that there was a percentage difference of maximum 4.8% because of the truncation of lower and higher modes in FEM analysis as well as some numerical modeling errors introduced by FEM. In the second case study, the efficiency of Helmholtz resonators was compared with absorptive treatment, and it was illustrated that, Helmholtz resonators were also promising in shaping the sound fields in coupled enclosed spaces.

References

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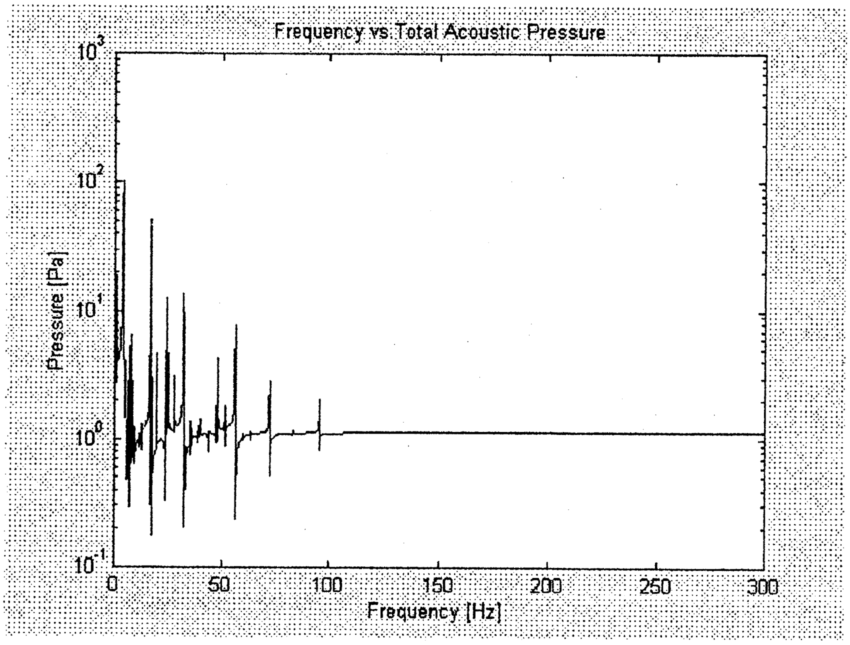

Figure 1. Acoustic Pressure Distribution Inside the Rectangular Cavity

Table 1. Coupled Natural Frequencies calculated by the Proposed method

Calculated Frequencies [Hz]
0.2
0.3
0.5
0.7
3.9
4.9
6.7
7.6
8
8.1
8.3
8.4
8.7
9.8
11.1
11.2
11.3
12.3
12.6
12.7
15.8
16
16.2
16.5
16.6
17.1
17.4
19.3
23.7
23.9
24.4
24.5
25.1
25.3
27.7
28
30.9
31.2
32.3
32.8
35.4
35.5
35.8
36.1
37.5
38.7
39.8
39.9
43.6
43.8
47.3
48
48.1
48.2
48.4
48.8
50.4
50.6
51.1
51.5
55.9
56.2
56.5
56.6
70.1
70.2
71.9
72.1
95.4
95.5

Table 2. Comparison of Coupled Frequencies Calculated by the Proposed Method and FEM

Calculated Frequencies [Hz]	Frequencies Calculated by FEM [Hz]	Percentage Difference (%)
25.1	26.596	5.62
25.3	26.597	4.88
28	29.412	4.80
30.9	29.423	5.02
39.8	39.547	0.64
71.9	73.754	2.51
72.1	73.76	2.25

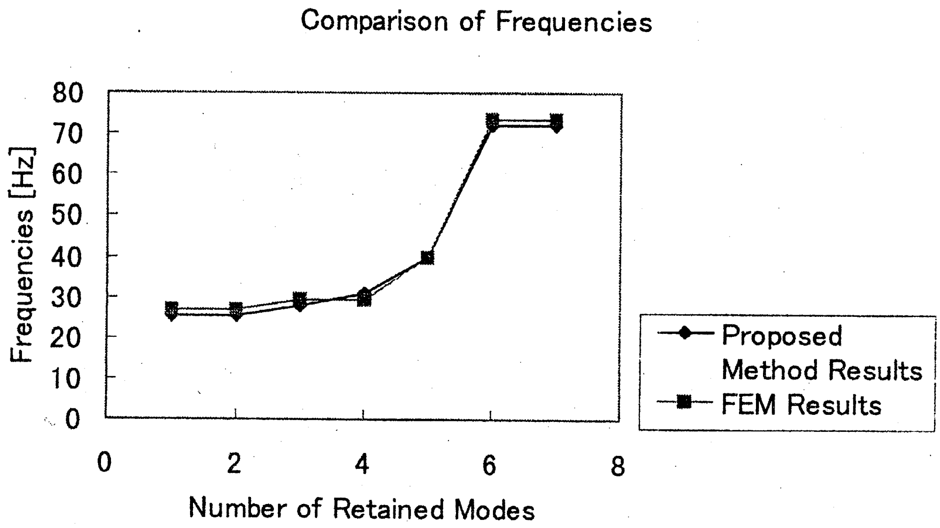

Figure 2. Percentage Difference Between Proposed Method and FEM

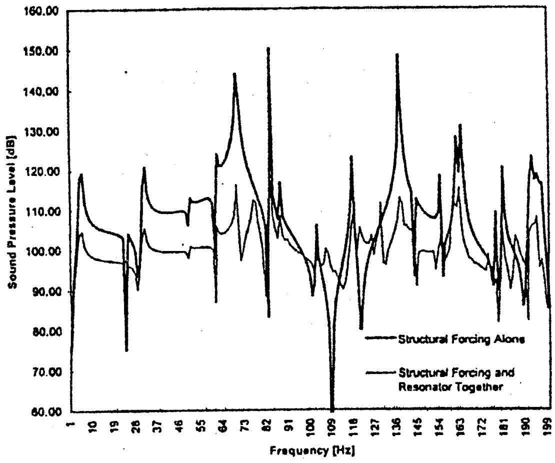

Figure 2. Effect of helmholtz Resonator without Sound Source

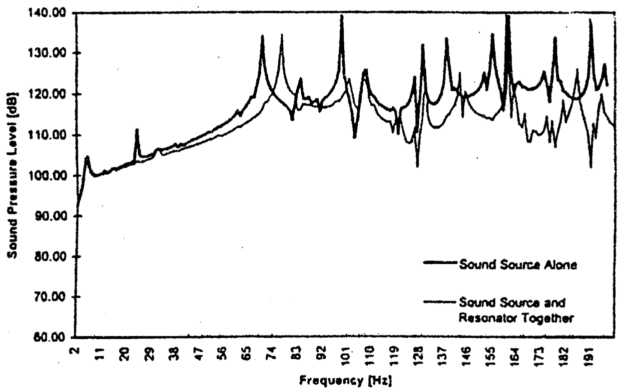

Figure 3. Effect of Helmholtz Resonator with Sound Source

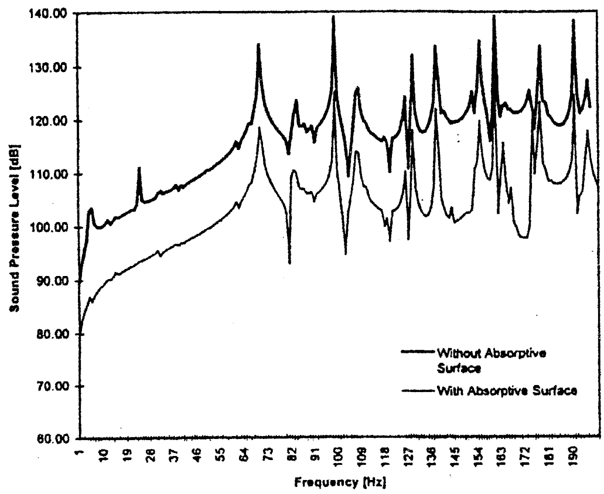

Figure 4. Effect of Absorbtive Treatment