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Optimizing multiple selection with a random number of objects - full information case

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We consider the generalization of the no-information secretary problem to the full-information case, allowing also multiple choices and a random number of objects. The goal is to maximize the probability of choosing the overall best. Previously, different authors studied no-information cases with multiple choices and fixed number of objects; or, as Porosinski(1987) did, extended Presman and Sonin's secretary problem with random number of objects to the full-information case with a single choice. He showed that if (P) holds, \( d_j(x) \geq 0 \) implies \( d_{j+k}(y) \geq 0 \) for \( k \geq 1, y \geq x \), then the problem is monotone, where \( d_j(x) = P(N=j) - \int_x^1 \Sigma_{k>j} P(N=k)y^{k-j-1}dy \). It is reasonable to expect that if single-choice problem is monotone, then two-choices, three-choices, \ldots, \( m \)-choices problems are also monotone. We investigate this monotonicity related to the condition (P) through a recursive function on \( m \) constructed from the optimality equation. As an example, the case of uniform number of objects is studied. This case satisfies (P). The optimal stopping rules is shown to be a threshold rule with multiple threshold values, which can be described as follows: The optimal stopping time, when we can make \( m \) more choices, is \( \tau_m = \min\{j \geq 1 : x \geq s_j^{(m)}\} \), where the threshold value \( s_j^{(m)} \) is a unique solution in \([0, 1]\) of the equation

\[
h_j^{(m)}(x) = h_j^{(1)}(x) + \sum_{n>j} \int_{x<s_n^{(m-1)}}^1 x^{n-j-1}h_n^{(m-1)}(y)dy
\]

with \( h_j^{(1)}(x) = \sum_{n \geq j} x^{n-j}d_n(x) \). \( s_j^{(m)} \) is nonincreasing in \( j \) and in \( m \).