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On minimizing the number of late jobs with fuzzy processing times and due dates

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1 Introduction

An efficient algorithm for sequencing a finite number of jobs through a single facility to minimize the number of late jobs was discovered by Moore (1968) more than three decades ago. It is also well known that a stochastic variant of this problem can be solved efficiently [1].

Recently Itoh and Ishii (1999) considered this problem for several special cases of fuzzy processing times and due dates. In this paper, open problems posed by Itoh and Ishii (1999) and a more general problem are solved.

2 Previous Works

Moore (1968) considered the following problem. There are n jobs $J_1, J_2, \ldots, J_n$ with (crisp) *processing times* $t_1, t_2, \ldots, t_n$ and (crisp) *due dates* $d_1, d_2, \ldots, d_n$. A schedule is a sequence of jobs. For a given schedule $S = (J_{i_1}, J_{i_2}, \ldots, J_{i_n})$, *completion time* of job J_{i_j} in S is $c_{i_j} = \sum_{k=1}^{j} t_{i_k}$. A job J_{i_j} is *late* in schedule S if $c_{i_j} > d_{i_j}$.

Problem 1 *Find a schedule S such that the number of late jobs in S is the smallest over all schedules.*

Moore (1968) presented and analyzed a polynomial time algorithm solving Problem 1.

Recently, Itoh and Ishii (1999) have considered this problem for the case with fuzzy processing times and fuzzy due dates. Each job J_i has a *fuzzy processing time* $\mathcal{T}_i$ and a *fuzzy due date* $\mathcal{D}_i$. Each $\mathcal{T}_i = (t_i, \alpha, \beta)_{LR}$ is a L-R fuzzy number whose membership function $\mu_{\mathcal{T}_i}$ is defined as follows:

$$\mu_{\mathcal{T}_i}(x) = \begin{cases} L\left(\dfrac{t_i - x}{\alpha}\right) & \text{if } x \le t_i, \\ R\left(\dfrac{x - t_i}{\beta}\right) & \text{if } x \ge t_i, \end{cases}$$

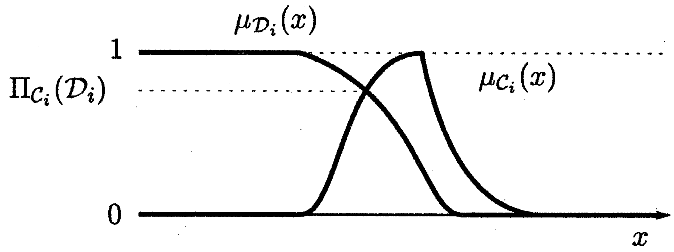

Figure 1: The possibility measure of $\mathcal{D}_i$ on $\mathcal{C}_i$.

where t_i, α, β are positive real numbers and L and R are decreasing functions from $[0,\infty]$ to $[0,1]$ satisfying $L(0) = R(0) = 1$. Each $\mathcal{D}_i = (d_i)_U$ is a fuzzy number whose membership function $\mu_{\mathcal{D}_i}$ is defined as follows:

$$\mu_{\mathcal{D}_i}(x) = \begin{cases} 1 & \text{if } x \leq d_i, \\ \max\{0, U(x - d_i)\} & \text{if } x \geq d_i, \end{cases}$$

where d_i is a non-negative real number and U is decreasing function from $[0,\infty]$ to $[0,1]$ satisfying $U(0) = 1$. Given a schedule $S = (J_{i_1}, J_{i_2}, \ldots, J_{i_n})$, the *completion time* $\mathcal{C}_{i_j}$ of job J_{i_j} in S is also a fuzzy number whose membership function $\mu_{\mathcal{C}_{i_j}}$ is defined as follows:

$$\mathcal{C}_{i_1} = \mathcal{T}_{i_1} \text{ and } \mathcal{C}_{i_j} = \mathcal{C}_{i_{j-1}} \oplus \mathcal{T}_{i_j} \text{ for } 1 < j \leq n,$$

where $\oplus$ is the extended addition defined by

$$(t, \alpha, \beta)_{LR} \oplus (t', \alpha', \beta')_{LR} = (t + t', \alpha + \alpha', \beta + \beta')_{LR}.$$

Thus, we have

$$\mathcal{C}_{i_j} = \bigoplus_{k=1}^{j} \mathcal{T}_{i_k} = \left(\sum_{k=1}^{j} t_{i_k}, j\alpha, j\beta \right)_{LR} \quad \text{for } 1 \leq j \leq n.$$

The *possibility measure* of $\mathcal{D}_i$ on $\mathcal{C}_i$ is denoted by $\Pi_{\mathcal{C}_i}(\mathcal{D}_i)$, and is defined as follows (see Figure 1):

$$\Pi_{\mathcal{C}_i}(\mathcal{D}_i) = \sup_x \min\{\mu_{\mathcal{C}_i}(x), \mu_{\mathcal{D}_i}(x)\}.$$

For a real number λ $(0 \leq \lambda \leq 1)$, job J_i is λ-*P late* in schedule S if $\Pi_{\mathcal{C}_i}(\mathcal{D}_i) < \lambda$.

Problem 2 *Given a real number λ $(0 \leq \lambda \leq 1)$, find a schedule S such that the number of λ-P late jobs in S is the smallest over all schedules.*

Itoh and Ishii (1999) showed that Moore's algorithm can be extended to solve Problem 2. Furthermore, they posed the following open problems.

- Problem 2 with jobwise different α, β.
- Problem 2 with jobwise different U.

In this paper, we solve these open problems and a more general problem.

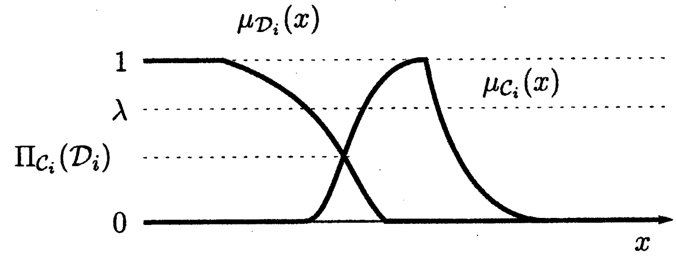

Figure 2: A λ-P late Job J_i.

3 New Results

Suppose the fuzzy processing time $\mathcal{T}_i$ and the fuzzy due date $\mathcal{D}_i$ of each job J_i are given as $\mathcal{T}_i = (t_i, \alpha_i, \beta_i)_{LR}$ and $\mathcal{D}_i = (d_i)_{U_i}$. That is, α_i, β_i and U_i are permitted to be jobwise different.

In this case, completion time C_{i_j} of J_{i_j} in schedule $S = (J_{i_1}, J_{i_2}, \ldots, J_{i_n})$, can be expressed as follows:

$$C_{i_j} = \bigoplus_{k=1}^{j} \mathcal{T}_{i_k} = \left(\sum_{k=1}^{j} t_{i_k}, \sum_{k=1}^{j} \alpha_{i_k}, \sum_{k=1}^{j} \beta_{i_k} \right)_{LR} \quad \text{for } 1 \le j \le n.$$

Since L and U are decreasing functions, we have for $1 \le j \le n$, $\Pi_{C_{i_j}}(\mathcal{D}_{i_j}) < \lambda$ if and only if

$$\sum_{k=1}^{j} t_{i_k} - \min\{x \mid L(x) = \lambda\} \sum_{k=1}^{j} \alpha_{i_k} > d_{i_j} + \max\{x \mid U(x) = \lambda\}. \tag{1}$$

Therefore, job J_{i_j} is λ-P late in schedule S if and only if (1) is satisfied (see Figure 2).

Theorem 1 *Moore's algorithm can be extended to solve Problem 2 with jobwise different α, β, and U.*

Proof. Consider an instance of Problem 2 given by $\mathcal{T}_i = (t_i, \alpha_i, \beta_i)_{LR}$, $\mathcal{D}_i = (d_i)_{U_i}$ for $1 \le i \le n$, and λ. Such an instance can be solved by applying Moore's algorithm to the (crisp) instance of Problem 1 with processing times $\hat{t}_i$ and due-dates $\hat{d}_i$ which are given as follows: for $1 \le i \le n$,

$$\hat{t}_i = t_i - \alpha_i \min\{x \mid L(x) = \lambda\}$$

and

$$\hat{d}_i = d_i - \max\{x \mid U_i(x) = \lambda\}.$$

This follows directly from the fact that job J_{i_j} is late in schedule $S = (J_{i_1}, J_{i_2}, \ldots, J_{i_n})$ for the constructed instance of Problem 1 if and only if (1) is satisfied, that is, if and only if job J_{i_j} is λ-P late in the same schedule S for the corresponding instance of Problem 2. ∎

4 Further Extension

Now let us consider a more general situation such that jobwise different functions L and R are also permitted. Then, the fuzzy processing time $\mathcal{T}_i$ of job J_i is denoted by $\mathcal{T}_i = (t_i)_{L_iR_i}$ and its membership function $\mu_{\mathcal{T}_i}$ is defined as follows:

$$\mu_{\mathcal{T}_i}(x) = \begin{cases} L_i(t_i - x) & \text{if } x \leq t_i, \\ R_i(x - t_i) & \text{if } x \geq t_i, \end{cases}$$

where t_i is a positive real number, and L_i and R_i are decreasing functions from $[0, \infty]$ to $[0, 1]$ satisfying $L_i(0) = R_i(0) = 1$. Notice that in this situation α_i and β_i are not necessary for definition of $\mathcal{T}_i$.

In this case, in order to define the completion times of jobs in a schedule, a more general sum $\tilde{\oplus}$ is used instead of the extended sum $\oplus$. In a schedule $S = (J_{i_1}, J_{i_2}, \ldots, J_{i_n})$, the completion time $\mathcal{C}_{i_j}$ of job J_{i_j} is defined as follows:

$$\mathcal{C}_{i_1} = \mathcal{T}_{i_1} \text{ and } \mathcal{C}_{i_j} = \mathcal{C}_{i_{j-1}} \tilde{\oplus} \mathcal{T}_{i_j} \text{ for } 1 < j \leq n,$$

where $\tilde{\oplus}$ is defined as follows:

$$(t)_{LR} \tilde{\oplus} (t')_{L'R'} = (t + t')_{\hat{L}\hat{R}},$$

such that

$$\hat{L}(z) = \sup_{z=x+y} \min\{L(x), L'(y)\}, \text{ and } \hat{R}(z) = \sup_{z=x+y} \min\{R(x), R'(y)\}.$$

Notice that $\tilde{\oplus}$ is an extension of $\oplus$, i.e., $\tilde{\oplus}$ can be used instead of $\oplus$ for the problem considered in the previous section.

Since for any real number λ $(0 \leq \lambda \leq 1)$,

$$\min\{z \mid \hat{L}(x) = \lambda\} = \min\{x \mid L(x) = \lambda\} + \min\{y \mid L'(y) = \lambda\},$$

job J_{i_j} is λ-P late in schedule S if and only if

$$\sum_{k=1}^{j} t_{i_k} - \sum_{k=1}^{j} \min\{x \mid L_{i_k}(x) = \lambda\} > d_{i_j} + \max\{x \mid U_{i_j}(x) = \lambda\}. \tag{2}$$

Theorem 2 *Moore's algorithm can be extended to solve Problem 2 with jobwise different L, R, and U.*

Proof. Consider an instance with $\mathcal{T}_i = (t_i)_{L_iR_i}$, $\mathcal{D}_i = (d_i)_{U_i}$ for $1 \leq i \leq n$, and λ. Such an instance can be solved by applying Moore's algorithm to the crisp instance of Problem 1 with processing times $\hat{t}_i$ and the due-dates $\hat{d}_i$ which are given as follows: for $1 \leq i \leq n$,

$$\hat{t}_i = t_i - \min\{x \mid L_i(x) = \lambda\}$$

and

$$\hat{d}_i = d_i - \max\{x \mid U_i(x) = \lambda\}.$$

It is obvious that job J_i is late in schedule S if and only if (2) is satisfied. ∎

References

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[3] J.M. Moore (1968), An n job, one machine sequencing algorithm for minimizing the number of late jobs, Management Science 15 (1), 102-109.