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Principal blocks with extra-special defect groups of order 27

Yoko Usami 宇佐美 陽子

Ochanomizu University , Department of Mathematics

Abstract. The groups $PGU(3,q^2)$ defined over the finite field $GF(q^2)$ satisfying $q \equiv 2$ or 5 (mod 9) have a common Sylow 3-subgroup P isomorphic to the extra-special group of order 27 of exponent 3. Their principal 3-blocks are Morita equivalent to one another. The groups $PGL(3,q)$ satisfying $q \equiv 4$ or 7 (mod 9) have also a common Sylow 3-subgroup P above. Their principal 3-blocks are Morita equivalent to one another. This paper also contains an improved result on the classification of principal 3-blocks with extra-special defect groups of order 27 of exponent 3 with respect to the existence of perfect isometries and isotypies.

§1. $PGU(3,q^2)$ and $PGL(3,q)$

1.1. In modular representation theory there is an important conjecture due to M. Broué (Question 6.2 in [6]). This conjecture can be stated like the following:

Broué's conjecture. Let B and B' be p-blocks of finite groups G and G' having the same Brauer category and in particular having a common defect group P. If P is abelian, is it true that B and B' are derived equivalent?

It is known that if P is not abelian, this is not true and as counter examples there exist some principal blocks (see section 6 in [6]). Nevertheless, it seems that there are not so many derived category equivalence classes among the principal p-blocks having a fixed common Brauer category. Keeping this in mind, in this paper we offer two examples of principal 3-blocks of an infinite series of groups having the same Brauer category and the same non abelian defect group and also belonging to the same derived category equivalence class.

1.2. Note that if we consider only principal p-blocks, their defect groups are Sylow p-subgroups and their Brauer categories are equivalent to the Frovenius categories of the corresponding groups (i.e., we can assume that the above P is a Sylow p-subgroup of each group and as a Brauer category we have only to consider the fusion of p-subgroups of P in each group). Note that having the same Frobenius category is equivalent to having the same p-local structure. See the Definition in 4 in [18] : Finite groups G and H have the same p-local structure if they have a common Sylow p-subgroup P such that whenever Q_1 and Q_2 are subgroups of P and f: $Q_1 \rightarrow Q_2$ is an isomorphism, then there is an element $g \in G$ such that $f(x) = x^g$ for all $x \in Q_1$ if and only if there is an element $h \in H$ such that $f(x) = x^h$ for all $x \in Q_1$. Let ($\mathcal{K}$, $\mathcal{O}$, k) be a splitting p-modular system for all subgroups of the considering groups, that is, $\mathcal{O}$ is a complete discrete valuation ring with unique maximal ideal $\mathcal{P}$, $\mathcal{K}$ is its quotient field of characteristic zero and k is its residue field $\mathcal{O}/\mathcal{P}$ of prime characteristic p and we assume that $\mathcal{K}$ and k are big enough such that they are splitting fields for all subgroups of the considering groups (see §6 in Chapter 3 in [16]).

We denote by Z_n a cyclic group of order n and by $N \rtimes L$ a semi-direct product of a group N by a group L. Let $B_o(G)$ denote the principal p-block of a group G (i.e. the indecomposable two sided ideal of the group algebra of G over $\mathcal{O}$ to which the trivial module belongs). We set

$$\bar{B}_o(G) = k \otimes_{\mathcal{O}} B_o(G) .$$

In this paper " modules " always mean finitely generated modules. They are left modules, unless stated otherwise. For a subgroup H of a group G , let U and V be $\mathcal{O}G$- and $\mathcal{O}H$-modules. We write $U_{\downarrow H}$ for the restriction of U to H , namely

$$U_{\downarrow H} = {}_{\mathcal{O}H}\mathcal{O}G \otimes_{\mathcal{O}G} U$$

and $V^{\uparrow G}$ for the induction of V to G , namely

$$V^{\uparrow G} = {}_{\mathcal{O}G}\mathcal{O}G \otimes_{\mathcal{O}H} V \quad .$$

We use the similar notation for kG-modules and kH-modules and even for ordinary characters. Let 1_G be the trivial $\mathcal{O}G$-module and k_G be the trivial kG-module. For other notation and terminology we follow the books of Benson [4] , Landrock [13] and Nagao-Tsushima [16] . Since the Brauer homomorphism plays an important role in this paper, we state its definition here.

Definition 1.3 (1 in [5], 6.C. in [7]) For an $\mathcal{O}G$-module V and a p-subgroup P of G , we set

$$Br_P(V) = V^P / \left(\sum_{Q \subsetneq P} Tr_Q^P(V^Q) + \mathfrak{p}V^P \right) \qquad (1.1)$$

where V^P denotes the set of fixed points of V under P and Q runs over all proper subgroups of P and

$$\mathrm{Tr}_Q^P(v) = \sum_{x\in P/Q} x(v) \tag{1.2}$$

for a p-subgroup Q of P and $v \in V^Q$.

1.4. In section 1 we consider only the groups which

(i) have a Sylow 3-subgroup isomorphic to M(3), an extra-special group of order 27 of exponent 3

and also

(ii) have a Frobenius category equivalent to that of $(Z_3 \times Z_3) \rtimes SL(2,3)$, the semi-direct product of the elementary abelian group of order 9 by SL(2,3) with the faithful action.

When we consider a principal p-block of a group, we may assume that the maximal normal p'-subgroup is trivial, since it is contained in the kernel of each module in a principal p-block. Hence we may add the assumption that

(iii) the maximal normal 3'-subgroup is trivial.

$PGU(3,q^2)$ defined over the finite field $GF(q^2)$ with $q \equiv 2$ or 5 (mod 9) has properties (i), (ii) and (iii) , and especially PGU(3,4) is isomorphic to the semi-direct product $(Z_3 \times Z_3) \rtimes SL(2,3)$ above in (ii) . PGL(3,q) defined over the finite field GF(q) with $q \equiv 4$ or 7 (mod 9) also has properties (i), (ii) and (iii). We remark that these projective general unitary and linear groups have a common Sylow 3-subgroup P which is satbilized as a group by any field automorphism. Using the classification of finite simple groups to find the groups having property (i), we know that

- the groups $PGU(3,q^2)$ defined over the finite field $GF(q^2)$ satisfying $q \equiv 2$ or 5 (mod 9) and extensions of them by field automorphisms which fix all the elements of P,

and

- the groups $PGL(3,q)$ defined over the finite field $GF(q)$ satisfying $q \equiv 4$ or 7 (mod 9) and extensions of them by field automorphisms which fix all the elements of P

(1.3)

are the only groups having properties (i), (ii) and (iii). Our aim in section 1 is to show the similarity of the module categories of the principal 3-blocks of these groups in (1.3). Here we state a definition of a special type of equivalence of module categories introduced by Broué (5.A in [7]) and Rickard (5.5 in [17]). After that we will state our main theorem.

Definition 1.5 (Definition 1.1 in [14]). Let A and B be $\mathcal{O}$-algebras, M (= ${}_AM_B$) an (A,B)-bimodule, N (= ${}_BN_A$) a (B,A)-bimodule. We say M and N induce a stable equivalence of Morita type between B and A , if

(i) M is projective as a left A-module and as a right B-module,

(ii) N is projective as a left B-module and as a right A-module,

(iii) $M \otimes_B N = A \oplus X$ for a projective (A,A)-bimodule X and $N \otimes_A M = B \oplus Y$ for a projective (B,B)-bimodule Y.

For k-algebras we define a stable equivalence of Morita type similarly.

Theorem 1.6 (Kunugi and Usami). Set

$\mathcal{F}_1 = \{ PGU(3,q^2)$ defined over the finite field $GF(q^2)$ $q \equiv 2$ or 5 (mod 9) $\}$

and

$\mathcal{F}_2$ = { PGL(3,q) q ≡ 4 or 7 (mod 9) } .

Then the groups G in $\mathcal{F}_1 \cup \mathcal{F}_2$ and their principal 3-blocks $B_o(G)$ have the following properties.

(i) These groups in $\mathcal{F}_1 \cup \mathcal{F}_2$ have a common Sylow 3-subgroup P isomorphic to M(3), the extra-special group of order 27 of exponent 3 , and also have common normalizer of P isomorphic to the semi-direct product of P by a cyclic group Z_2 of order 2 with the faithful action on P not fixing Z(P).

(ii) P contains a subgroup Q isomorphic to the elementary abelian group $Z_3 \times Z_3$ of order 9 such that these groups in $\mathcal{F}_1 \cup \mathcal{F}_2$ have a common normalizer H of Q. This H is isomorphic to the semi-direct product of $Z_3 \times Z_3$ by SL(2,3) with the faithful action, and we have $\mathcal{O}H = B_o(H)$. Furthermore, H belongs to $\mathcal{F}_1$ as PGU(3,4)

(iii) In each group G in $\mathcal{F}_1 \cup \mathcal{F}_2$, H (= $N_G(Q)$) contains $N_G(P)$ and H controls the fusion of 3-subgroups of P in G. Then the groups in $\mathcal{F}_1 \cup \mathcal{F}_2$ have a common Frobenius category.

(iv) Fix any group G in $\mathcal{F}_1 \cup \mathcal{F}_2$. Let M be the Green correspondent of an $\mathcal{O}(G\times G)$-module $B_o(G)$ with respect to ($G\times G$, $\Delta(P)$, $G\times H$). Then

$$M \text{ is an indecomposable direct summand of } 1_{\Delta(P)}{}^{\uparrow G\times H} \quad (1.4)$$

where $\Delta(P)$ means the diagonal group of P in $G\times G$. Let N be the Green correspondent of an $\mathcal{O}(G\times G)$-module $B_o(G)$ with respect to ($G\times G$, $\Delta(P)$, $H\times G$). Then

$$N \text{ is an indecomposable direct summand of } 1_{\Delta(P)}{}^{\uparrow H\times G} \quad (1.5)$$

Furthermore, M and N induce a stable equivalence of Morita type between $B_o(H)$ and $B_o(G)$ as a $(B_o(G),B_o(H))$-bimodule and a $(B_o(H),B_o(G))$-bimodule respectively.

(v) If G is in $\mathcal{F}_1$, then M and N in (iv) induce a Morita equivalence between $B_o(H)$ and $B_o(G)$.

(vi) Let G be any fixed group in $\mathcal{F}_2$ and set

$$G_1 = PGL(3,4) \ . \quad G_2 = G \ , \quad M_2 = M \ \text{ and } \ N_2 = N$$

as (iv) and let M_1 and N_1 be bimodules defined for G_1 like M and N for G. Then the unique non-projective indecomposable direct summand M_0 of

$$M_2 \underset{B_o(H)}{\otimes} N_1$$

induces a Morita equivalence between $B_o(G_1)$ and $B_o(G_2)$. Furthermore

$$M_0 \ \text{ is a direct summand of } \ 1_{\Delta(P)}{}^{\uparrow G_2 \times G_1} . \qquad (1.6)$$

Remark 1.7. Let F be a functor which induces a Morita equivalence between blocks B and B' and M be a (B',B)-bimodule such that F is realized by

$$M \underset{B}{\otimes} -$$

Then by Theorem 22.1 in [3] the inverse functor of F is given by a (B,B')-bimodule

$$\mathrm{Hom}_{B'}(M,B')$$

and by 3.A.3 in [7] this bimodule is isomorphic to

$$M^* = \mathrm{Hom}_{\mathcal{O}}(M,\mathcal{O}) \qquad (\text{ i.e. } \mathcal{O}\text{-dual of M }).$$

Then a Morita equivalence in Theorem 1.6 (v) is induced by bimodules M and M^* and furthermore, M is satisfying (1.4). Also a Morita equivalence in Theorem 1.6 (vi) is induced by bimodules M_0 and M_0^* and furthermore, M_0 is satisfying (1.6). Then by Puig and Scott's Theorem 1.6 in Marcus's paper [15] , each of these Morita equivalences is a so-called Puig equivalence (i.e. it implies the coincidence of their source algebras). On the other hand, for any extension G in (1.3) of a group G_o in $\mathcal{F}_1 \cup \mathcal{F}_2$ $\bar{B}_o(G)$ and $\bar{B}_o(G_o)$ are Morita equivalent to each other by a bimodule

$$ {}_{\bar{B}_o(G_o)}{\bar{e}_o \bar{B}_o(G)}_{\bar{B}_o(G)} \quad \text{(i.e. the restriction)} $$

by Alperin and Dade's isomorphic blocks (see Theorem 2 in [1]), where $\bar{e}_o$ is the central idempotent corresponding to $\bar{B}_o(G_o)$. As we will explain a similar case in 1.13 below , this bimodule is a $\Delta(P)$-projective trivial source module and then liftable. Hence the uniquely lifted bimodule

$$ {}_{B_o(G_o)}{e_o B_o(G)}_{B_o(G)} $$

and its $\mathcal{O}$-dual induce a Morita equivalence between $B_o(G)$ and $B_o(G_o)$ (Furthermore, this is also a Puig equivalence by the same reason as above .) Hence if we use the classification of finite simple groups as in 1.4 , we can conclude that there are at most two derived equivalence classes of principal 3-blocks having the same Brauer (Frobenius) category as that of $(Z_3 \times Z_3) \rtimes SL(2,3)$ with the faithful action. Here we remark that all of these blocks have the same number ot irreducible ordinary characters and the same number of simple modules. Furthermore, there is a perfect isometry between the principal 3-blocks of PGL(3,4) and PGU(3,4).

1.8. This work is subsequent to Koshitani and Kunugi's papers [10], [12] and the main task is to determine the Green correspondents of simple modules and for this task we use the same tools effectively such as Scott's result that a trivial source module is liftable (Theorem 12.4 in II in [13] and Theorem 14.8 in I in [13] or Lemma 1.2 in [10]) and Robinson's lemma (Theorem 3 in [19] or Lemma 1.4 in [10]) and Knorr's relatively projective cover ([11] or section 2 in [12]). As for Theorem 1.6, assuming (i) through (iii) which we can prove by direct calculations, we will explain the frame of the proof of (iv) through (vi). For (iv) we need Broué's Theorem 6.3 in [7] in special situation stated in Theorem 1.9 below. But in order to state it we need the Brauer homomorphisms defined in two ways. Note that for any non-trivial p-subgroup R of P and for any p-block with defect group P we have the following. Here we state them only for the principal block $B_o(G)$. First, with an action of R by a conjugation in (1.2) we have

$$Br_R(\mathcal{O}G) = kC_G(R) \quad \text{and} \quad Br_R(B_o(G)) = kC_G(R)\bar{b}_o(R) = \bar{B}_o(C_G(R)) \qquad (1.7)$$

where $\bar{b}_o(R)$ is the block idempotent of $kC_G(R)$ corresponding to the principal p-block by Theorem 3.13 (2) in [2]. This implies that we have

$$Br_{\Delta(R)}(\mathcal{O}G) = kC_G(R) \quad \text{and} \quad Br_{\Delta(R)}(B_o(G)) = \bar{B}_o(C_G(R)), \qquad (1.8)$$

(Since the proof of Theorem 6.3 was omitted in [7], we show an outline of the proof of Theorem 1.9 here. By assumption (ii) and the argument in page 344 in [18], $Br_{\Delta(R)}(M^* \otimes_A M) \simeq Hom_{kC_G(R)}(\bar{M}_R, \bar{M}_R) \simeq kC_H(R)\bar{b}'_R$. Since $Br_{\Delta(N_P(R))}(\bar{b}'_R) \neq 0$ by Corollary 4.5 in [2] , non-projective indecomposable direct summand of $M^* \otimes_A M$ is unique and it has vertex $\Delta(P)$ by Theorem 3.2 in [5] . By assumption (i) B is a summand of $M^* \otimes_A M$ as it is the unique indecomposable direct summand of $A_{\downarrow H\times H}$ with vertex $\Delta(P)$. From Theorem 2.1 in [18] the conclusion follows.) Theorem 1.6 (v) and (vi) are based on a Linckelmann's theorem.

Theorem 1.9 (cf. Broué, Theorem 6.3 in [7]). Let G be a finite group with a Sylow p-subgroup P and H be a subgroup of G containing $N_G(P)$. Assume that G and H have the same fusion on p-subgroups contained in P. Let b and b' be central idempotents of $\mathcal{O}G$ and $\mathcal{O}H$ respectively such that there is a Brauer correspondence between

$$A = \mathcal{O}Gb \quad \text{and} \quad B = \mathcal{O}Hb'$$

having common defect group P. For a subgroup R of P , set

$$\bar{b}_R = Br_R(b) \quad , \quad \bar{b}'_R = Br_R(b') \ . \tag{1.9}$$

Let M be an (A,B)-bimodule and N be a (B,A)-bimodule. For eacn subgroup R of P set

$$\bar{M}_R = Br_{\Delta(R)}(M) \quad \text{and} \quad \bar{N}_R = Br_{\Delta(R)}(N) \tag{1.10}$$

Assume that

(i) M is a direct summand of the restriction of A from $G\times G$ to $G\times H$.

(ii) For each non-trivial subgroup R of P , $\bar{M}_R$ and $\bar{N}_R$ induce a Morita equivalence between $kC_H(R)\bar{b}'_R$ and $kC_G(R)\bar{b}_R$.

Then M and its $\mathcal{O}$-dual induce a stable equivalence of Morita type between B and A.

Theorem 1.10 (Linckelmann , Theorem 2.1 in [14]). Let G and H be two finite groups and b and b' be central idempotents of $\mathcal{O}G$ and $\mathcal{O}H$ respectively. Set

$$A = \mathcal{O}Gb \ , \ B = \mathcal{O}Hb' \ , \quad \bar{A} = k \otimes_{\mathcal{O}} A \quad \text{and} \quad \bar{B} = k \otimes_{\mathcal{O}} B \ .$$

Let M be an (A,B)-bimodule which is projective as left and right module, such that the functor

$$M \otimes_B -$$

induces an $\mathcal{O}$-stable equivalence between B and A.

(i) Up to isomorphism, M has the unique indecomposable non-projective direct summand M' as an (A,B)-bimodule and then $k\otimes_{\mathcal{O}}M'$ is, up to isomorphism, the unique indecomposable non-projective direct summand of $k\otimes_{\mathcal{O}} M$ as a $(\bar{A},\bar{B})$-bimodule.

(ii) If M is indecomposable, for any simple B-module S, the A-module $M\otimes_B S$ is indecomposable and non-projective as an $\bar{A}$-module,

(iii) If for any simple B-module S, the A-module $M\otimes_B S$ is simple, then the functor $M\otimes_B -$ is a Morita equivalence.

1.11. Fix any group G in $\mathcal{F}_1 \cup \mathcal{F}_2$ and set

$$H = N_G(Q)$$

as in Theorem 1.6(ii) and then we have

$$\mathcal{O}H = B_o(H) \quad (1.11)$$

Note that a $G\times G$-module $B_o(G)$ has a vertex $\Delta(P)$ and the source $1_{\Delta(P)}$, and $G\times H$ contains $N_{G\times G}(\Delta(P))$. Then there exists the Green correspondent M of $B_o(G)$ with respect to ($G\times G$, $\Delta(P)$, $G\times H$) and M satisfies (1.4) (and then it satisfies Theorem 6.3 (i) in [7] with $X = 1_{\Delta(P)}$) and

M is a direct summand of the restriction of $B_o(G)$ to $G\times H$.(1.12)

Similarly we can define N. We will apply Theorem 1.9 for $B_o(G)$, $B_o(H)$ and M (and N). Let R be a non-trivial subgroup of P. By (1.8) and (1.10)

$\bar{M}_R$ is a direct summand of a $(\bar{B}_o(C_G(R)),\bar{B}_o(C_H(R)))$-bimodule $\bar{B}_o(C_G(R))$ (1.13)

and

$\bar{N}_R$ is a direct summand of a $(\bar{B}_0(C_H(R)),\ \bar{B}_0(C_G(R))')$-bimodule

$\bar{B}_0(C_G(R))$ (1.14)

and by (1.7) and (1.9) the second condition in Theorem 1.9 is equivalent to the following:

For each non-trivial subgroup R of P, $\bar{M}_R$ and $\bar{N}_R$ induce a Morita equivalence between $\bar{B}_0(C_H(R))$ and $\bar{B}_0(C_G(R))$, (1.15)

which will be proved locally. Then we will conclude that M and N induce a stable equivalence of Morita type between $B_0(H)$ and $B_0(G)$ by Theorem 1.9 and then Theorem 1.6 (iv) will follow. Set

$$\bar{M} = k \underset{\mathcal{O}}{\otimes} M \quad \text{and} \quad \bar{N} = k \underset{\mathcal{O}}{\otimes} N \ .$$

Note that each of $\bar{M}$ and $\bar{N}$ is projective as left and right module and by definition a stable equivalence of Morita type between $B_0(H)$ and $B_0(G)$ by bimodules M and N guarantees a stable equivalence of Morita type between $\bar{B}_0(H)$ and $\bar{B}_0(G)$ by bimodules $\bar{M}$ and $\bar{N}$. Also note that $\bar{M}$ and $\bar{N}$ are indecomposable by Theorem 1.10 (i).

1.12. This $\bar{M}$ for G in $\mathcal{F}_1$ (respectively, $\bar{N}$ for G in $\mathcal{F}_2$) sends a simple module to an indecomposable module by Theorem 1.10 (ii) and furthermore we claim that this module is its Green correspondent with respect to (G,P,H) over k . In order to prove this claim, first we will assure that

$$H = N_G(Q) \supset N_G(P)$$

from the explicit structure of G , and that any simple $\bar{B}_o(H)$-module has vertex P or Q and that any simple $\bar{B}_o(G)$-module has vertex P or Q for G in $\mathcal{F}_2$. By the definition of a stable equivalence of Morita type our claim will follow. Using this claim effectively we will prove Theorem 1.6 (v) and (vi) separatedly. We remark that by definition (see (1.4) and (1.5)), M , N , $\bar{M}$ and $\bar{N}$ are trivial source modules.

1.13. Fix any group in $\mathcal{F}_1$. In order to show Theorem 1.6 (v) , it suffices to show that

$\bar{M}$ and $\bar{N}$ induce a Morita equivalence between $\bar{B}_o(H)$ and $\bar{B}_o(G)$. (1.16)

Indeed , by the remark in 1.12 they are trivial source modules and they are liftable to M and N. Then M and N induce a Morita equivalence between $B_o(H)$ and $B_o(G)$ (see Rickard § 5 in [18]). In order to prove (1.16) we will show that the Green correspondent of a simple module is also simple from character calculation. Then from our claim in 1.12 we can conclude that $\bar{M}$ induces a Morita equivalence between $\bar{B}_o(H)$ and $\bar{B}_o(G)$ by Theorem 1.10 (iii) (for $\bar{M}$). More precisely, $\bar{M}$ and its k-dual $\bar{N}$ induce it as a pair of bimodules and (1.16) will follow.

1.14. For G in $\mathcal{F}_2$ the method is little bit different. We will show that the set of Green correspondents of the simple $\bar{B}_o(G)$-modules in kH do not depend on q (i.e. the choice of G in $\mathcal{F}_2$). Using this fact we will find a suitable bimodule from the direct summands of

$$\bar{B}_o(G) \otimes_{kH} \bar{B}_o(PGL(3,4)) .$$

§ 2. Classification

2.1. Do the principal p-blocks of two groups having the same p-local structure have the same number of irreducible ordinary characters and the same number of simple modules ? As a test case for this question , the author and M.Kiyota classified the principal 3-blocks with extra-special defect groups of order 27 of exponent 3 , according to their 3-local structures and obtained an affirmative answer for this case, using the classification of finite simple groups (see Theorems 1 and 2 in [22]). We add some results to this investigation.

Theorem 2.2 (Using the classification of finite simple groups for the latter case with $E \simeq Z_8$). Let G be a finite group with an extra-special Sylow 3-subgroup P of order 27 of exponent 3. If $N_G(P) \subseteq C_G(Z(P))$ or $Z(P)$ is normal in G , then $B_0(G)$ and $B_0(P \rtimes E)$ are isotypic with $E = N_G(P)/PC_G(P)$.

Proposition 2.3. The groups in (i) (respectively (ii), (iii) and (iv)) have the same 3-local structure with Sylow 3-subgroup P in Theorem 2.2 and there is a perfect isometry between the principal 3-blocks of any two of them.

(i) M_{24} , He , Aut(He) (with the author's student M. Nakabayashi)

(ii) Aut(M_{12}) , Aut(PSL(3,3))

(iii) Ru , J_4 (by M. Nakabayashi)

(iv) $G_2(q)$ $q = 3k \pm 1$ $(3, k) = 1$ (with M. Nakabayashi),

On the other hand, the groups in (i)' (respectively (ii)',(iii)' and (iv)') have the same 3-local structure with Sylow 3-subgroup P in Theorem 2.2, but there is no perfect isometry between their principal 3-blocks.

(i)' M_{24}, Aut(M_{12})

(ii)' Ru , ${}^2F_4(2)$

(iii)' $G_2(2)$, $Aut(J_2)$

(iv)' $Aut(J_2)$, $P \rtimes SD_{16}$ with the faithful action (SD_{16} denotes the semidihedral group of order 16 .).

(Here we check only perfect isometries which send the trivial character to the trivial character.)

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