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# 解析関数の非線形発展方程式への応用

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## 1 Introduction

In this note we present a survey recent progress on analyticity of solutions to nonlinear Schrödinger (NLS) equations and generalized Korteweg-de Vries (gKdV) equation. We also state some applications of analytic function spaces to these equations. Nonlinear Schrödinger equations considered in this note are written

$$\text{(NLS)} \qquad \begin{cases} i\partial_t u + \frac{1}{2}\Delta u = N(u), & (t,x) \in \mathbf{R} \times \mathbf{R}^n, \\ u(0,x) = u_0(x), & x \in \mathbf{R}^n, \end{cases}$$

where nonlinear terms $N(u)$ will be defined in each theorem in the below and $n$ denotes the spatial dimension.

The generalized Korteweg-de Vries (gKdV) equation is written

$$\text{(gKdV)} \qquad \begin{cases} \partial_t u + \frac{1}{3}\partial_{x_1}^3 u + \partial_{x_1}(|u|^{p-1}u) = 0, & t, x_1 \in \mathbf{R}, \\ u(0,x) = u_0(x_1), & x_1 \in \mathbf{R}, \end{cases}$$

where $p \in \mathbf{N}$.

Local and global in time of solutions to these equations were studied extensively by many authors in the usual Sobolev spaces (see, e.g.,[8], [20],[21], [30], [31], [33], [36] and references cited these papers). In order to state previous results we prepare some function space and notations

*Function spaces and notation.* We use the usual Lebesgue space

$$\mathbf{L}^p = \{\phi \in \mathcal{S}'; \|\phi\|_p < +\infty\}.$$

We define the weighted Sobolev space as follows

$$\mathbf{H}^{m,l,p} = \{f \in \mathbf{L}^p; \|(1+|x|^2)^{l/2}(1-\Delta)^{m/2} f\|_p < \infty\}.$$

For convenience, $\|\cdot\| = \|\cdot\|_2$ and $\|\cdot\|_{m,l} = \|\cdot\|_{m,l,2}$. $J_{x_j} = x_j + 2it\partial_{x_j}$. For each $r > 0$ we denote the strip in the complex plane $\mathbf{C}^n$ by

$$S_n(r) = \{z = (z_1, \cdots, z_n) = (x_1 + iy_1, \cdots, x_n + iy_n); -r < y_j < r, -\infty < x_j < \infty; 1 \le j \le n\}$$

We also define the sector in $\mathbf{C}^n$ by

$$\Delta_n(\alpha) = \{z = (z_1, \cdots, z_n) = (r_1 e^{i\theta}, \cdots, r_n e^{i\theta}) \quad ; 0 \le r_j < \infty, \quad -\alpha < \theta < \alpha, \\ \pi - \alpha < \theta < \pi + \alpha, \quad 0 < \alpha < \tfrac{\pi}{2}, ; 1 \le j \le n\}.$$

For $x \in \mathbf{R}^n$, if a complex-valued function $f(x)$ has an analytic continuation to $S_n(r)$ or $\Delta_n(\alpha)$, then we denote this by the same letter $f(z)$ and if $g(z)$ is an analytic function on $S(r)$ or $\Delta_n(\alpha)$, then we denote the restriction of $g(z)$ to the real axis by the same letter $g(x)$.

We let

$$\mathbf{AS_n}^{m,l}(|\beta|) = \{f(z); f(z) \quad \text{is analytic on} \quad S_n(|\beta|), \|f\|_{\mathbf{AS_n}^{m,l}(|\beta|)} < \infty\},$$

where

$$\|f\|_{\mathbf{AS_n}^{m,l}(|\beta|)} = \sup_{y \in (-|\beta|, |\beta|)^n} \|f(\cdot + iy)\|_{m,l}, \quad \|f(\cdot + iy)\|^2 = \int_{\mathbf{R}^n} |f(x+iy)|^2 dx$$

The Fourier transform of $\phi(x_j)$ is denoted by $\mathcal{F}_j\phi$ or $\hat{\phi}$, namely

$$\hat{\phi}(\xi_j) = \frac{1}{\sqrt{2\pi}} \int_{\mathbf{R}} e^{-i\xi_j x_j} \phi(x_j) dx_j.$$

We denote by $\mathcal{F}_j^{-1}\phi$ or $\check{\phi}$ the inverse Fourier transform of the function $\phi(\xi_j)$. the free Schrödinger evolution group $\mathcal{U}(t)$ is defined by

$$\mathcal{U}(t)\phi(x) = \frac{1}{(2\pi i t)^{n/2}} \int e^{i(x-y)^2/2t} \phi(y) dy. \tag{1}$$

It is also written as $\mathcal{F}^{-1} e^{it|\xi|^2} \mathcal{F}$. We let $M_j = M_j(t) = \exp(i|x_j|^2/2t)$ and $J_j = J_j(t) = (x_j + 2it\partial_{x_j}) = \mathcal{U}(t) x_j \mathcal{U}(-t) = M_j(t) 2it\partial_{x_j} M_j(t)^{-1}$, where $j = 1, 2$.

We organize the survay as follows. In Section 2, we present a survey results about existence of analytic solutions. Section 3 is devoted to analytic smoothing effects to some dispersive nonlinear equations. Finally we state results about asymptotic behavior and global existence in time of small solutions to nonlinear evolution equations in analytic function spaces.

## 2 Existence of solutions

Analytic function spaces are useful to prove existence theorems of various nonlinear evolution equations involving derivative of unknown functions, see [6], [9], [10], [11], [12], [32]. In [32], Kato and Masuda proved existence of analtytic solutions to (gKdV) by making use of analytic function space.

**Theorem 2.1.** *Assume that* $u_0 \in \mathbf{AS_1}^{2,0}(|a|)$. *Then there exists a time* $T > 0$ *and a unique solution* $u(t,x)$ *of (gKdV) which has an analytic continuation* $u(t,z)$ *on the strip* $S_1(|b|)$ *and* $u(t,\cdot) \in \mathbf{AS_1}^{2,0}(|b|)$, *where* $|b| < |a|$.

This result says the analytic function space of solutions is smaller than that of the data. Their method works well for local existence theorem of other nonlinear evolution equations of the form $\partial_t u = F(u)$, where $F(u)$ is a nonlinear term contains the derivatives. Their idea is to use the norm $\|\mathcal{F}^{-1}e^{a(t)|\xi|}\mathcal{F}u\|$, where $a(t)$ ia a decreasing function satisfying $a(0) = |a|$ and $a(\infty) = |b|$. If we use this norm, we get

$$\frac{d}{dt}\|\mathcal{F}^{-1}e^{a(t)|\xi|}\mathcal{F}u\| - a'(t)\|\mathcal{F}^{-1}|\xi|e^{a(t)|\xi|}\mathcal{F}u\| \leq \|\mathcal{F}^{-1}e^{a(t)|\xi|}\mathcal{F}F(u)\|.$$

The second term of the left hand side is important to treat the derivatives in the nonlinear term since we can gain regularity of one derivative from this term.

However it seems that their method does not work for global results.

In [27], [28], [18], [10], we combined a vector field method and analytic function spaces to show global existence in time of solutions to nonlinear Schrödinger equations. We only state the result of [10].

**Theorem 2.2.** *Assume that* $u_0 \in \mathbf{AS_n}^{n,0}(|a|) \cap \mathbf{AS_n}^{0,n}(|a|)$, $n \geq 2$, $\|u_0\|_{\mathbf{AS_n}^{n,0}(|a|)} + \|u_0\|_{\mathbf{AS_n}^{0,n}(|a|)}$ *is small enough and* $N$ *satisfies* $N(u, \nabla u) = e^{i\theta}N(e^{i\theta}u, e^{i\theta}\nabla u)$ *for any* $\theta \in \mathbf{R}$ *and* $N$ *is a polynomial of order p which is greater than or equal to* 3. *Then there exists a unique global solution* $u(t,x)$ *of (NLS) which has an analytic continuation* $u(t,z)$ *on the strip* $S_n(|b|)$ *and* $u(t,\cdot) \in \mathbf{AS_n}^{2,0}(|b|)$ *for any* $t \in \mathbf{R}$, *where* $|b| < |a|$.

More general nonlinear Schrödinger equations were treated in [18].

# 3 Smoothing property and analyticity of solutions

In the case of nonlinear heat equation

$$\text{(NLH)} \qquad \begin{cases} \partial_t u - \frac{1}{2}\partial_{x_1}^2 u = u^2, & (t, x_1) \in \mathbf{R}^+ \times \mathbf{R}, \\ u(0,x) = u_0(x_1), & x_1 \in \mathbf{R}, \end{cases}$$

it is known that the following smoothing effects of solutions to (NLH) hold.

**Theorem 3.1.** *Assume that* $u_0 \in \mathbf{L}^2$, *Then there exists a time* $T > 0$ *and a unique global solution* $u(t,x_1)$ *of (NLH) such that* $u$ *has an analytic continuation* $u(t,z_1)$ *on the strip* $S_1(\sqrt{t})$ *and an analytic continuation* $u(t+i\tau, x_1)$ *on the sector* $\{t + i\tau; -\alpha < \frac{\tau}{t} < \alpha, 0 < \alpha < \frac{\pi}{2}\}$ *for any* $t < T$.

*Proof.* See, e.g., [2].

Linear heat equation on the half line was used to research of isometrical identities for the Bergman space on a sector [3]. In [4] we extented the result of [3].

Let $\Delta_1(\alpha) = \{z_1; |\arg z_1| < \alpha\}$. We considered in [4] the Bergman space

$$B_{\Delta_1(\alpha)} = \{F; F \quad \text{is analytic on} \quad \Delta_1(\alpha), \|F\|_{B_{\Delta_1(\alpha)}} < \infty\},$$

where

$$\|F\|_{B_{\Delta_1(\alpha)}} = \left\{ \int\int_{\Delta_1(\alpha)} |F(x_1 + iy_1)|^2 dx_1 dy_1 \right\}^{1/2}.$$

In the case of $\alpha = \pi/4$ we showed that $\|F\|_{B_{\Delta_1(\alpha)}}$ is represented as a series of weighted square integrals of the derivatives of the trace of $F$ on the positive real axis in [3]. The proof worked only in the case of $\alpha = \pi/4$. In [4] we presented a general result for $0 < \alpha < \pi/2$ by a completely different proof. More precisely we showed

**Theorem 3.2.** *We have the isometrical identity*

$$\int\int_{\Delta_1(\alpha)} |F(x_1 + iy_1)|^2 dx_1 dy_1 = \sin(2\alpha) \sum_{j=0}^{\infty} \frac{(2\sin\alpha)^{2j}}{(2j+1)!} \int_0^\infty x_1^{2j+1} |\partial_{x_1}^j f(x_1)|^2 dx_1,$$

*where $f$ stands for the trace of $F$ on the positive real axis.*

This result shows function spaces of the data considered in [16], [18] are not empty. For related results of [3] and [4], see , [1], [25].

We next state a smoothing property of solutions to (NLS) obtained in [17], [26] which is considered as a similar smooting property property of solutions as in (NLH). We also give time analyticity of solutions to (NLS) and (gKdV) obitained in [5], [16].

The following result says an analytic smoothing property in space variables

**Theorem 3.3.** *Assume that $n = 1$, $e^{|a||x_1|}u_0 \in \mathbf{L}^2$ and $N = \lambda|u|^2u$, where $a \neq 0$ and $\lambda \in \mathbf{C}$. Then there exists a time $T > 0$ and a unique solution $u(t,x)$ of (NLS) such that $u$ has an analytic continuation $u(t, z_1)$ on the strip $S_1(|a|t)$ for any $|t| < T$.*

*Proof.* See [26].

Note that in [26] we do not give the statement of the above result. However the same proof as in [26] does work well for the problem. This result is considered as an analytic version of the results obtained in [20] [21]. We also showed a global existence in time of solutions to (NLS). More precisely, we showed the next result.

**Theorem 3.4.** *Assume that $n \geq 2$, $e^{|a||x|}u_0 \in \mathbf{H}^{m,l}$, $m + l > [\frac{n}{2}] + 1$ and $N = \lambda|u|^2u$, where $a \neq 0$ and $\lambda \in \mathbf{C}$. Then there exists a unique global solution $u(t,x)$ of (NLS) such that $u$ has an analytic continuation $u(t,z)$ on the strip $S_n(|a|t)$ for any $t \in \mathbf{R}$.*

The following result says an analytic smoothing property in time variable.

**Theorem 3.5.** *Assume that* $n = 1$, $e^{|a|x_1^2}u_0 \in \mathbf{L}^2$ *and* $N = \lambda|u|^2u$, *where* $a \neq 0$ *and* $\lambda \in \mathbf{C}$ *Then there exists a time* $T > 0$ , *a constant* $C_0 > 0$ *and a unique solution* $u(t,x)$ *of (NLS) such that* $u$ *has an analytic continuation* $u(t+i\tau, x)$ *on the complex domain* $\{t+i\tau; -C_0t^2 < \tau < C_0t^2$ *for any* $|t| < T$.

*Proof.* See [17].

These two theorems denpend on the special operator $J_{x_1} = x_1 + 2it\partial_{x_1}$ and so the method is not applicable to nonlinearities which do not satisfy the gauge condition. For more general nonlinearities, we showed analyticity in time of solutions of (NLS) in [16] and the Gevrey smoothing property in [5].

In [16] we considered the regularity of solutions to nonlinear Schrödinger equations

$$\begin{cases} i\partial_t u + \frac{1}{2}\Delta u = F(u,\bar{u}), & (t,x) \in \mathbf{R} \times \mathbf{R}^n, \\ u(0,x) = u_0(x), \quad x \in \mathbf{R}^n \end{cases}$$

where $F$ is a polynomial of degree $p$ with complex coefficients. Roughly speaking, our result is stated as follows.

**Theorem 3.6.** *If the initial function* $u_0$ *is in some Gevrey class (for the defintiton of Gevrey class, see [16] ), then there exists a positive constant* $T$ *such that the solution* $u$ *of (NLS) is in the Gevrey class of the same order as in the initial data in time variable* $t \in [-T,T]\backslash 0$. *In particular we showed that if the initial function* $u_0$ *has an analytic continuation on the complex domain*

$$\begin{aligned} \Gamma_{A_1,A_2} \quad &= \{z \in \mathbf{C}^n; z_j = x_j + iy_j, -\infty < x_j < +\infty, \\ &-A_2 - (\tan\alpha)|x_j| < y_j < A_2 + (\tan\alpha)|x_j|, j = 1,2,...,n,\ A_2 > 0\}, \end{aligned}$$

*where* $0 < \alpha = \sin^{-1} A_1 < \pi/2$ *and* $0 < A_1 < 1$, *then there exist positive constants* $T$ *and* $\beta$ *such that the solution* $u$ *of (NLS) is analytic in time variable* $t \in [-T,T]\backslash 0$ *and has an analytic continuation on* $\{z_0 = t+i\tau; |\arg z_0| < \beta < \frac{\pi}{2}, |t| < T\}$ , *where* $\sin\beta < \min\left\{\frac{\sqrt{2}A_1}{1+\sqrt{2}A_1}, \frac{2A_2}{3A_2+\sqrt{2e}(1+R)}\right\}$ *when* $|x| < R$.

In [5] we considered regularizing effects of solutions to the (generalized) Korteweg-de Vries equation

$$\begin{cases} \partial_t u + \partial_x^3 u = \lambda u^{p-1}\partial_x u, & (t,x) \in \mathbf{R} \times \mathbf{R}, \\ u(0) = \phi, \quad x \in \mathbf{R}, \end{cases}$$

and nonlinear Schrödinger equations in one space dimension

$$\begin{cases} i\partial_t u + \frac{1}{2}\partial_x^2 u = G(u,\bar{u}), & (t,x) \in \mathbf{R} \times \mathbf{R}, \\ u(0) = \psi, \quad x \in \mathbf{R}, \end{cases}$$

where $p$ is an integer satisfying $p \geq 2$, $\lambda \in \mathbf{C}$ and $G$ is a polynomial of $(u,\bar{u})$. We proved

**Theorem 3.7.** *If the initial function $\phi$ is in a Gevrey class of order 3 defined in Section 1 of [5] , then there exists a positive time $T$ such that the solution of the (generalized) Korteweg-de Vries equation is analytic in space variable for $t \in [-T,T]\backslash\{0\}$, and if the initial function $\psi$ in a Gevrey class of order 2, then there exists a positive time $T$ such that the solution of nonlinear Schrödinger equations is analytic in space variable for $t \in [-T,T]\backslash\{0\}$. For more precise statements of the results, see the original paper [5].*

Kato and Taniguchi [29] extended the result of nonlinear Schrödinger equation to the general spatial dimension.

# 4 Asymptotic behavior and global existence in time of solutions

As in [10], [13], [18], [14], [15], [27], [28] analytic function spaces are useful to the study of global existence and asymptotic behavior in time of solutions to nonlinear Schrödinger equations.

In [14], we studied the scattering problem and asymptotics for large time of solutions to the Cauchy problem for the nonlinear Schrödinger and Hartree type equations with subcritical nonlinearities

$$\begin{cases} i\partial_t u + \frac{1}{2}\partial_{x_1}^2 u = f(|u|^2)u, & (t,x_1) \in \mathbf{R}^2 \\ u(0,x_1) = u_0(x_1), & x \in \mathbf{R}, \end{cases}$$

where the nonlinear interaction term is $f(|u|^2) = V * |u|^2, V(x_1) = \lambda|x_1|^{-\delta}, \lambda \in \mathbf{R}, 0 < \delta < 1$ in the Hartree type case, or $f(|u|^2) = \lambda|t|^{1-\delta}|u|^2$ in the case of the cubic nonlinear Schrödinger equation.

We showed

**Theorem 4.1.** *We suppose that the initial data $e^{\beta|x_1|}u_0 \in L^2$ , $\beta > 0$ with sufficiently small norm $\epsilon = \|e^{\beta|x_1|}u_0\|$. Then we proved the sharp decay estimate $\|u(t)\|_p \leq C\epsilon t^{\frac{1}{p}-\frac{1}{2}}$, for all $t \geq 1$ and for every $2 \leq p \leq \infty$. Furthermore we showed that for $\frac{1}{2} < \delta < 1$ there exists a unique final state $\hat{u}_+ \in L^2$ such that for all $t \geq 1$*

$$\|u(t) - \exp(-\frac{it^{1-\delta}}{1-\delta}f(|\hat{u}_+|^2)(\frac{x}{t}))U(t)u_+\| = O(t^{1-2\delta})$$

*and uniformly with respect to $x_1$*

$$u(t,x_1) = \frac{1}{(it)^{\frac{1}{2}}}\hat{u}_+(\frac{x_1}{t})\exp(\frac{ix_1^2}{2t} - \frac{it^{1-\delta}}{1-\delta}f(|\hat{u}_+|^2)(\frac{x_1}{t})) + O(t^{1/2-2\delta}).$$

*The function $e^{i|x_1|^2/2t}u$ has an analytic continuation on the strip $S_1(a|t|)$, where $a < \beta$.*

We only state our main idea in [14]. Applying the operator $\mathcal{F}_1 M_1 \mathcal{U}(-t)$ to the both sides of equation and putting $v = \mathcal{F}_1 M_1 \mathcal{U}(-t)u$, we obtain

$$\begin{cases} i\partial_t v + \frac{1}{2t^2}\partial_{\chi_1}^2 v = t^{-1} f(|v|^2)v, \\ v(1,\chi_1) = \mathcal{F}_1 e^{ix_1^2/2}\mathcal{U}(-1)u_0(x_1), \quad \chi_1 \in \mathbf{R}, \end{cases}$$

To eliminate the term $t^{-1}f(|v|^2)$ we make use of a tansformation $w = e^{ig}v$, where $g$ satisfies

$$\begin{cases} g_t = t^{-\delta} f(|v|^2) + \frac{1}{2t^2}(g_\chi)^2, \quad t > 1, \\ g(1) = 0. \end{cases}$$

We easily see that $w$ satisfies the Cauchy problem

$$\begin{cases} w_t = \frac{1}{t^2} w_\chi g_\chi + \frac{i}{2t^2} w_{\chi\chi} + \frac{1}{2t^2} w g_{\chi\chi}, \quad t > 1, \\ w(1) = v(1) = \mathcal{F}_1 e^{\frac{ix_1^2}{2}} \mathcal{U}(-1)u(1). \end{cases}$$

Thus we removed the nonlinear term which does not have sufficient time decay but instead we now encounter the derivative loss. This is the reason why we need an analytic function space. We consider the system of equations

$$\begin{cases} w_t = \frac{1}{t^2} w_\chi g_\chi + \frac{i}{2t^2} w_{\chi\chi} + \frac{1}{2t^2} w g_{\chi\chi}, \quad t > 1, \\ g_t = t^{-\delta} f(|w|^2) + \frac{1}{2t^2}(g_\chi)^2, \quad t > 1, \\ g(1) = 0, \quad w(1) = v(1) = \mathcal{F}_1 e^{\frac{ix_1^2}{2}} \mathcal{U}(-1)u(1). \end{cases}$$

in an analytic function space.

In [19], we extented the above result to some Gevrey class. We supposed that $e^{\beta|x|^\sigma}u_0 \in L^2$ , $\beta > 0$, $1 - \frac{\delta}{2-\delta} < \sigma < 1$ and the norm $\epsilon = \|e^{\beta|x|^\sigma}u_0\|$ is sufficiently small. Then we proved the same results as in [14] except an analytic smoothing properties of solutions.

In [18] we considered the nonlinear Schrödinger equations

$$\begin{cases} i\partial_t u + \frac{1}{2}\Delta u = F(u, \nabla u, \bar{u}, \nabla \bar{u}), \quad (t,x) \in \mathbf{R} \times \mathbf{R}^n, \\ u(0,x) = \epsilon_0 \phi, \quad x \in \mathbf{R}^n, \end{cases}$$

where $F : \mathbf{C}^{2n+2} \to \mathbf{C}$ is quadratic and $\epsilon_0$ is sufficiently small constant. We proved that small analytic solutions exist globally in time when $n \geq 3$ and $F$ satisfies

$$|\partial_u F| + |\partial_{\bar{u}} F| \leq C|\nabla u|.$$

We also showed an almost global existence of small analytic solutions when $n = 2$ and $F$ is written as

$$F = F(\nabla u, \nabla \bar{u}).$$

Furthermore we proved a global existence result of small analytic solutions when $n = 2$ and

$$F = \lambda(\partial_1 u \partial_2 \bar{u} - \partial_1 \bar{u} \partial_2 u),$$

where $\lambda \in \mathbf{C}$.

Our results show that we can handle a wider class nonlinear terms compared with the previous results [34], [35], [37] in lower dimensional cases if we assume a certain analytical condition on the data.

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