数理解析研究所講究録
1165 巻 2000 年 157-174

Investment Decisions and the Choice of Technique of the Firm under Imperfect Competition

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1. Introduction

THE PURPOSE of this paper is to build a model of investment decisions and the choice of technique of the firm under imperfect competition. Special features of our model are two-fold as the title indicates. First, we explicitly formulate the investment decisions of the firm under imperfect competition; second, it analyzes not only firm's investment decisions, but also its choice of technique.

Most of the investment theories developed so far assume that the firm to decide investment is under perfect competition. One of the purposes of this paper is to develop the theory of investment that explicitly take into account of the behavior of the imperfectly competitive firm. The most important behavioral difference between the competitive firm and the imperfectly competitive firm is that the former bases his decisions on price expectations, while the latter on quantity expectations. The firm under imperfect competition faces with expected demand curves over future periods when it makes investment decisions. There is scarcely any work that analyzes investment decisions of the imperfectly competitive firm.[1] In this paper we will attempt to construct a model of investment that explicitly formulates the behavior of the imperfectly competitive firm.

The second purpose of this paper is to analyze the choice of technique of the firm simultaneously with investment decisions.[2] The investment of the firm involves two kinds of decisions: how many machines to install, and what type of machines to choose. The theory of investment usually deals with the former, but not the latter explicitly. In this paper, we discuss both of these decisions. In doing so, we differentiate the long-run production from the short-run production function; the former represents a set of available techniques as the relation between labor-capital ratio and output-capital ratio, while the latter represents utilization of the existing capital stock. From the set of available techniques represented by the long-run production function, the firm chooses the best one when it installs new equipment. To take into account the putty-clay character of technology, we assume that adjustment costs are required not only for increasing productive capacity, but also for changing labor-capital ratio.

This paper is organized as follows. Section 2 discusses the relation between the long-run production function and the short-run production function. Section 3 analyzes the decisions of the rate of capital utilization of the firm. Section 4 presents a model of investment decisions and the choice of technique. Section 5 focuses on the choice of technique of the firm. Section 6 analyzes the investment decisions of the firm under imperfect competition. Section 7 summarizes the results.

2. Long-run Production Function and Short-run Production Function

In our model, we differentiate the long-run production function from the short-run production function. The former represents a spectrum of techniques available under the present state of technological knowledge, while the latter represents utilization of the existing capital stock.[3] Let $\overline{N}$ and $\overline{Y}$ be the level of employment and the level of output, respectively, when the stock of capital K is utilized at the normal level. Then the long-run production function is written as follows:

$$\overline{Y} = F(\overline{N}, K) \qquad (1).$$

In the following, we call $\overline{N}$ as the normal level of employment and $\overline{Y}$ as the normal level of output. Suppose that this production function exhibit constant returns to scale as usual, then it may be rewritten as

$$\frac{\overline{Y}}{K} = F\left(\frac{\overline{N}}{K}, 1\right) = f(n), \quad \text{where} \quad n \equiv \frac{\overline{N}}{K}. \qquad (2)$$

The notation n represents labor-capital ratio at the normal utilization of capital. The production function $f(n)$ is assumed to satisfy Inada's condition, *i.e.*,

$$f(0) = 0, \quad f(\infty) = \infty, \quad f'(n) > 0, \quad f'(0) = \infty, \quad f'(\infty) = 0, \quad f''(n) < 0 \quad (3)$$

At a given point of time, capital stock K as well as the normal labor-capital ratio n is given. Then, the normal level of employment is $\overline{N} = nK$, and the normal level of output is $\overline{Y} = f(n)K$. In practice, however, the existing capital stock may not always be utilized at the normal level. Let us denote actual employment by N, and actual output by Y. N and Y agree with $\overline{N}$ and $\overline{Y}$ respectively, only if the capital equipment is utilized at the normal level. Otherwise, actual the levels of N and Y depend not only the existing volume of capital (K) and the normal labor-capital ratio (n) but also the rate of utilization of capital. In order to know the precise relation between N and Y, we have to specify the utilization function, or the short-run production function.

As for the relation between actual output and actual employment, we follow the formulation given by Okishio (1984). Given the stock of capital and the technique embodied in it (*i.e.*, given $\overline{Y}$), Y is related to N as follows:

$$\frac{Y}{\overline{Y}} = g\left(\frac{N}{\overline{N}}\right). \tag{4}$$

This function may be called a utilization function. Define $x \equiv N/K$; then $N/\overline{N} = x/n$. Substituting this relation and (2) into (4), we have

$$\frac{Y}{K} = g\left(\frac{x}{n}\right)f(n). \tag{5}$$

This function shows the relation between actual output per unit of capital and actual employment per unit of capital for given K and n; so it may be called the short-run production function. The utilization function $g(x/n)$ is assumed to have the following properties:

(a) $g(0) = 0$

(b) $g' > 0$

(c) $g(1) = 1$

(d) $g(\infty) = \overline{u} > 1$

(e) There exists a point of inflection $(x/n)^0 < 1$ such that

if $x/n < (x/n)^0$, then $g''(x/n) > 0$;

if $x/n = (x/n)^0$, then $g''(x/n) = 0$;

if $x/n > (x/n)^0$, then $g''(x/n) < 0$.

(f) $\dfrac{(x/n)g'(x/n)}{g(x/n)} = \dfrac{nf'(n)}{f(n)}$, if and only if $x = n$.

These assumptions imply that the utilization function g is an increasing function with S-shape starting from the origin. In view of (e), the marginal productivity of labor is increasing when the rate of employment is lower than $(x/n)^0$, and is decreasing when it is above $(x/n)^0$. Assumption (c) implies that actual output is at the normal level when employment is at the normal level, and (d) implies that there exists some upper bound for output in the short-run. Finally, assumption (f) implies that the short-run production function touches the long-run production function at the normal utilization of capital.

Figure 1 illustrates the relation between the long-run and short-run production functions. As is explained above, the long-run production function shows a spectrum of available techniques as the relationship between the normal labor-capital, n, and the normal output-capital ratio, $f(n)$. Suppose that the technique embodied in the existing capital stock is represented by $(\overline{n}, f(\overline{n}))$. Then, the short-run production function touches the long-run production function at $x = \overline{n}$, since the existing capital stock is normally utilized at that point. Except that point, the short-run production function is located below the long-run production function. For the long-run production function represents efficient frontier of production.

In the following discussion, we use the inverse function of (4) for convenience. Let us define $u \equiv Y/\overline{Y}$, which is called the rate of capital utilization. Then, the inverse function of (4) may be written as

$$\frac{N}{\overline{N}} = h(u). \tag{6}$$

This function, which represents the required rate of employment for a given rate of utilization, is called the employment function in the following. The employment function $h(u)$ have the following properties:

(a) $h(0) = 0$

(b) $h' > 0$

(c) $h(1) = 1$

(d) There exists some real quantity $\overline{u} > 1$, such that $h(\overline{u}) = \infty$.

(e) There exists some real quantity $u_C < 1$, such that:

if $u < u_0$, then $h'' < 0$;

if $u = u_0$, then $h'' = 0$;

if $u > u_0$, then $h'' > 0$.

(f) $\dfrac{h(u)}{uh'(u)} = \dfrac{nf'(n)}{f(n)}$ if and only if $u = 1$

This function is illustrated by Figure 2. It increases with decreasing rate for $u < u_0$, with increasing rate for $u > u_0$, and asymptotically approaches $\overline{u} > 1$.

3. The Decisions of the Rate of Capital Utilization under Imperfect Competition

In this section, we examine how the imperfectly competitive firm determine its output and prices, given the existing stock of capital and the technique embodied in it. To determine the level of output, Y, with given stock of capital is nothing but to determine the rate of capital utilization, u. So what we examine in this section is reduced to the determination of the rate of capital utilization and the price of output by the imperfectly competitive firm.

At a given point of time, the representative firm under imperfect competition is faced with an expected demand curve with downward sloping. Let us denote the expected demand of the firm by Y^e and the price of product by p. Then the expected demand function of the firm is written as

$$Y^e = Ap^{-\eta}, \tag{7}$$

where A denotes the level of expected demand at a given level of price, and η is the price elasticity of demand. A change in A indicates a shift in the expected demand curve. We assume η to be constant in the following.

Suppose that the firm determines output Y to be equal to the expected demand Y^e. Then we can rewrite (7) in the form of inverse demand function as

$$p = \left(\frac{Y}{A}\right)^{-\varepsilon} = \left(\frac{Y}{K}\frac{K}{A}\right)^{-\varepsilon} = \{uf(n)k\}^{-\varepsilon}, \tag{8}$$

where $\varepsilon \equiv 1/\eta$ (the inverse of the price elasticity of demand) and $k \equiv K/A$ (capital per unit of expected demand). The normal labor-capital ratio n is constant in the short-run, since the technology embodied in the existing capital is given.

The short-run profit Π of the firm is given by

$$\Pi = pY - WN = [puf(n) - Wh(u)n]K, \tag{9}$$

where W is the money wage rate. The short-run decisions of the firm under imperfect competition is to set price and determine the rate of capital utilization so as to maximize the profit, given the stock of capital and technology. Maximizing Π with respect to u subject to the constraint (8) yields:

$$(1-\varepsilon)pf(n) = Wh'(u)n \tag{10}$$

This equation has a meaningful solution only if $\varepsilon < 1$ (or $\eta > 1$). So the price elasticity of demand for the representative must be greater than unity. In addition to this condition, the second order condition for profit maximization

$$\varepsilon + \frac{uh''}{h'} > 0 \tag{11}$$

must be satisfied. The second term of the left-hand side of (11) represents the elasticity of the marginal employment rate, $h''(u)$, with respect to capital utilization, u. Let us denote it by σ:

$$\sigma \equiv \frac{uh''}{h'}. \tag{12}$$

Then (11) is rewritten as

$$\varepsilon + \sigma > 0 \tag{13}$$

The value of σ may be either positive or negative. Second order condition (13) implies that even if it is negative, its absolute value cannot exceed ε. This condition restricts the degree of increasing return for the short-run production function (5).

Substituting (8) into (10), we have

$$(1-\varepsilon)\{uf(n)k\}^{-\varepsilon} f(n) = Wh'(u)n. \tag{14}$$

Since k and n are constant in the short-run, this equation determines u. So u may be expressed as a function of k and n by solving (14): $u = u(k, n)$. The elasticity of u

with respect to k is calculated to be

$$\frac{k}{u}\frac{\partial u}{\partial k} = -\frac{\varepsilon}{\varepsilon + \sigma} < 0. \tag{15}$$

Thus u is a decreasing function with respect to k. This means that the rate of capital utilization, u, increases if the level of expected demand, A, rises under a given stock of capital, K.

The elasticity of u with respect to n, on the other hand, is shown to be

$$\frac{n}{u}\frac{\partial u}{\partial n} = -\frac{1-(1-\varepsilon)\theta}{\varepsilon + \sigma} < 0, \tag{16}$$

where θ is defined as

$$\theta \equiv \frac{nf'(n)}{f(n)}. \tag{17}$$

It represents the elasticity of the long-run production function $f(n)$ with respect to n, and $0 < \theta < 1$ if the production function satisfies Inada's conditions (3). Thus u is a decreasing function with respect to n. It implies that, other things being equal, a higher labor-capital ratio yields a lower rate of capital utilization.

The results obtained above are summarized as follows. In the short-run, given the values of k and n, the rate of utilization, u, is determined by profit maximizing condition (14), and then, the product price is determined by the inverse demand function (8). In other words, the short-run decisions of the representative firm under imperfect competition is to determine the rate of utilization and the price of product at each point of time, given expected demand, capital stock and technology.

4. A Model of Investment Decisions and the Choice of Technique

In the last section, we dealt with the short-run decisions of the firm, given the stock of capital and technology. We now turn to the long-run decisions concerning with investment and technology.

The investment decisions of the firm are made based on the expectations of about demand and costs over the periods during which the newly installed equipment will be used. So expectations for investment decisions may be characterized as long-run expectations, differing from those for the decisions of capital utilization.

In order to make expectations about demand and costs central to investment decisions, we follow the standard theory of investment decisions that emphasizes the presence of costs to changing the capital stock. In addition, however, we assume that changes in techniques embodied in the capital stock also involve adjustment costs. So we introduce two kinds of adjustment costs in our model.

Let us first formulate the assumption about adjustment costs for investment. Following Hayashi (1983), adjustment costs per unit of investment are assumed to rise as a function of I_t / K_t, which is denoted by g_t in the following. Then, the total adjustment costs C_t is written as[4]

$$C_t = \Phi(I_t / K_t) I_t = \Phi(g_t) I_t, \tag{18}$$

where $\Phi(g_t)$ is the per-unit adjustment cost. This function is assumed to have the following properties:

$$\Phi(0) = 0, \quad \Phi' > 0, \quad \Phi'' > 0. \tag{19}$$

In other words, the per unit adjustment cost increases more than proportionally as g_t increases.

We assume the price of capital goods to be constant, putting it equal to unity for convenience. Then, the total cost of investment becomes as

$$\{q_t + \Phi(g_t)\} I_t = [\{1 + \Phi(g_t)\} g_t] K_t = \phi(g_t) K_t, \tag{20}$$

where $\phi(g_t) = \{1 + \Phi(g_t)\} g_t$. In view of (19), this function has the following properties:

$$\phi(0) = 0, \quad \phi' > 0, \quad \phi'' > 0. \tag{21}$$

If we ignore the depreciation of capital, we have

$$\dot{K}_t = g_t K_t. \tag{22}$$

Let us next consider adjustment costs accompanied by changes in the technique embodied in the capital stock. The technique embodied in the existing capital stock is expressed by its normal labor-capital ratio n_t. If capital is assumed to be completely malleable, labor-capital ratio will be adjusted instantaneously to changes in factor prices. In reality, however, factor proportions are largely embodied in existing capital: technology is putty-clay. In this case, the labor-capital ratio will not instantaneously get to the optimal level responding to changes in factor prices. To take into account of this fact in our model, rather than explicitly allowing for a putty-clay technology, we assume that the firm faces cost of adjusting factor proportions.[5] Then, what the firm can control in the short-run is not n_t but its time derivative $\dot{n}_t$. Denoting the firm's control variable by s_t, we have

$$\dot{n}_t = s_t. \tag{23}$$

We assume that the cost of adjusting the normal labor-capital ratio, n_t, depends on its rate of change, $\dot{n}_t$, and the size of capital stock, K_t; specifically, we express it as

$$C_n = \psi(\dot{n}_t) K_t = \psi(s_t) K_t. \tag{24}$$

Here, the function $\psi(\dot{n}_t)$ representing the adjustment cost per unit of capital has the following properties:

$$\psi(0) = 0, \quad \psi'(\dot{n}_t) \gtreqless 0 \text{ depending on } \dot{n}_t \gtreqless 0, \quad \psi'' > 0. \tag{25}$$

In other words, the cost of changing n_t increases with increasing rate as the degree of its change increases. The adjustment costs function satisfying (25) is shown in Figure 3.

Taking into account the cost for investment (20) and the cost for changing factor proportions (24), we can express the present value of the firm's long-run profits as

$$V_0 = \int_0^\infty [p_t u_t f(n_t) - W_t h(u_t) n_t - \phi(g_t) - \psi(s_t)] K_t e^{-rt} dt, \tag{26}$$

where we assume the real rate of interest r to be constant. In view of the inverse demand function (8), the price of products is given by

$$p_t = \{u_t f(n) k_t\}^{-\varepsilon}, \tag{27}$$

and as we discussed in the last section, the rate of capital utilization is give by

$$u_t = u(k_t, n_t), \quad u_k < 0, \quad u_n < 0 \tag{28}$$

Here, k_t is defined by

$$k_t \equiv \frac{K_t}{A_t}. \tag{29}$$

In the following discussion, we assume that the expected the firm expects the demand for their product at the given price level grows at a constant rate α. Therefore, we have

$$A_t = A_0 e^{\alpha t}. \tag{30}$$

Taking the time derivative of equation (29) and substituting from (30), we have

$$\dot{k}_t = (g_t - \alpha) k_t. \tag{31}$$

To sum up, the problem of investment decisions and the choice of technique of the imperfectly competitive firm is to maximize

$$V_0 = \int_0^\infty [p_t u_t f(n_t) - W_t h(u_t) n_t - \phi(g_t) - \psi(s_t)] k_t e^{-(r-\alpha)t} dt, \tag{32}$$

subject to the constraints

$$\dot{k}_t = (g_t - \alpha) k_t \tag{33}$$

$$\dot{n}_t = s_t, \tag{34}$$

where p_t is given by (24). The variables that the firm can control are g_t and s_t, while k_t and n_t are state variables.

To solve this problem, we set up the present-value Hamiltonian:

$$H_t = e^{-(r-\alpha)t}[\{p_t u_t f(n_t) - W_t h(u_t) n_t - \phi(g_t) - \psi(s_t)\} k_t + \lambda_t (g_t - \alpha) k_t + \mu_t s_t] \tag{35}$$

where λ_t and μ_t are shadow prices of k_t and n_t, respectively. The first order conditions for a maximum of V_0 are

$$\lambda_t = \phi'(g_t), \tag{36a}$$

$$\mu_t = \psi'(s_t)k_t, \tag{36b}$$

$$\dot{\lambda}_t = (r - g_t)\lambda_t - \{(1-\varepsilon)p_t u_t f(n_t) - W_t h(u_t)n_t - \phi(g_t) - \psi(s_t)\}, \tag{36c}$$

$$\dot{\mu}_t = (r-\alpha)\mu_t - \{(1-\varepsilon)p_t u_t f'(n_t) - W_t h(u_t)\}k_t. \tag{36d}$$

The transversality conditions are

$$\lim_{t\to\infty} k_t \lambda_t e^{-(r-\alpha)t} = 0, \qquad \lim_{t\to\infty} n_t \mu_t e^{-(r-\alpha)t} = 0. \tag{36e}$$

The system consisting of six equations (33), (34), and (35a)~(35d) include six variables: g_t, s_t, k_t, n_t, λ_t and μ_t. So it is complete. The solution of this system determines the path of those variables. But the system is too complex to solve explicitly for the general solution. In the following, therefore, we discuss investment decisions and the choice of technique separately by making some simplifying assumptions.

5. The Choice of Technique of the Firm under Imperfect Competition

We first consider the choice of technique of the firm. Equation (36b) shows that the optimum rate of change of labor-capital ratio, s_t, determines at which the shadow price of labor per unit of capital equals the marginal adjustment cost of changing labor-capital ratio. But, in view of (36c), the shadow price of labor per unit of capital, μ_t, can be expressed as follows:

$$\mu_t = \int_0^{\infty} e^{-(r-\alpha)(\tau-t)}\{(1-\varepsilon)p_\tau u_\tau f'(n_\tau) - W_\tau h(u_\tau)\}k_\tau d\tau. \tag{37}$$

This equation states that the value of labor per unit of capital at a given time equals the discounted value of its future marginal revenue products. Substituting this equation into (36b), we have

$$\psi'(s_t)k_t = \int_t^{\infty} e^{-(r-\alpha)(\tau-t)}\{(1-\varepsilon)p_\tau u_\tau f'(n_\tau) - W_\tau h(u_\tau)\}k_\tau d\tau. \tag{38}$$

At time t, k_t is given since it is a state variable. Therefore, this equation determines s_t if the firm's expectation of the future marginal revenue products is given. This result implies that the firm's expectations about future demand and costs are crucial in the determination of labor-capital ratio if the adjustment costs for its changes are taking into account.

However, the discounted value of the future marginal products of labor per unit of capital, the right-hand side expression of (38), depends not only on expected prices and wages but also on future values of n and k. But, those future values are affected by the levels of s_t and g_t to be determined at present. So, s_t cannot be determined by equation (38) alone. It is determined simultaneously with other variables in the complete system.

In order to seek a meaningful explanation for the determination of the labor-capital ratio, we focus on the steady state of the complete system. Putting $\dot{k}_t = 0$ in (33) and $\dot{n}_t = 0$ in (34), we have $g_t = \alpha$ and $s_t = 0$. Next, putting $\dot{\lambda}_t = 0$ in (36c) and $\dot{\mu}_t = 0$ in (36d), and substituting from (35a) and (35b), respectively, we have the following steady state relationships:

$$(1-\varepsilon)puf(n) - Wh(u)n = \phi(\alpha) + (r-\alpha)\phi'(\alpha), \tag{39}$$

$$(1-\varepsilon)puf'(n) - Wh(u) = 0. \tag{40}$$

In view of (27) and (28), the steady state values of p and u are determined by

$$p = \{uf(n)k\}^{-\varepsilon}, \tag{41}$$

$$u = u(k,n). \tag{42}$$

Taking these relations into consideration, the steady-state values of k and n are determined by (39) and (40). The wage rate, W, the rate of interest, r, and the expected rate of growth, α, are given exogenously.

If we use equation (10) to substitute out p and W in equation (40), we have

$$\frac{h(u)}{uh'(u)} = \frac{nf'(n)}{f(n)}. \tag{43}$$

Notice the assumptions that were made on the utilization function $h(u)$ in section 2. Specifically, assumption (f) states that (43) holds if and only if $u = 1$. In other words, the rate of utilization is at the normal level in the steady state. On this condition, we can rewrite (39) and (40) as follows:

$$(1-\varepsilon)pf(n) - Wn = \phi(\alpha) + (r-\alpha)\phi'(\alpha) \tag{44}$$

$$(1-\varepsilon)pf'(n) - W = 0 \tag{45}$$

Eliminating p from these two equations, we have

$$\frac{f'(n)}{f(n) - nf'(n)} = \frac{W}{\phi(\alpha) + (r-\alpha)\phi'(\alpha)}. \tag{46}$$

This equation determines the normal labor-capital ratio, n, at the steady state, given W, r and α.

Calculating the effect of a change in W or r on n from equation (46), we have the following results:

$$\frac{W}{n}\frac{\partial n}{\partial W}=\frac{f'(n)[f(n)-nf'(n)]}{nf(n)f''(n)}<0 \tag{47}$$

$$\frac{1}{n}\frac{\partial n}{\partial r}=-\frac{f'(n)[f(n)-nf'(n)]}{nf(n)f''(n)}\frac{\phi'(\alpha)}{\phi(\alpha)+(r-\alpha)\phi'(\alpha)}>0 \tag{48}$$

Thus, the labor-capital ratio decreases with an increase in the wage rate, increases with an increase in the rate of interest. It should be noted that these results have been obtained from the comparison of steady states. Since the normal labor-capital ratio is fixed in the short-run in our model, it does not respond instantaneously to changes in the wage rate or interest rate. Corresponding to given factor prices, the optimum labor-capital ratio is attained only at the steady state. But, it takes quite a long time for the transition from one steady state to another. So changes in factor prices can lead to changes in factor proportions only in the long-run.

6. Investment Decisions of the Firm under Imperfect Competition

Let us next consider investment decisions of the firm by assuming that the normal labor-capital ratio, n, is given. Since changes in the normal labor-capital ratio take quite a long time as is mentioned above, this simplifying assumption may be justified. It should be noted, however, that the actual labor-capital ratio changes with the rate of capital utilization as is already explained in section 2.

With this simplifying assumption, investment decisions of the firm is formulated as the problem of maximizing

$$V_0=\int_0^\infty [p_t u_t f(n)-W_t h(n_t)n-\phi(g_t)]k_t A_0 e^{-(r-\alpha)t}dt\,, \tag{49}$$

subject to

$$\dot{k}_t=(g_t-\alpha)k_t\,. \tag{50}$$

where the product price p_t is given by (25), and the rate of capital utilization u_t by (26). Since the normal labor-capital ratio, n, is here assumed to be constant, those equations are written as

$$p_t=\{u_t f(n)k_t\}^{-\varepsilon} \tag{51}$$

$$u_t=u(k_t) \tag{52}$$

For analytical convenience, we rewrite the objective function of the firm, (49), in terms of the rate of return on capital defined by $\pi_t=(p_tY_t-W_tN_t)/q_tK_t$. Since we put $q_t=1$ without the loss of generality, the rate of return on capital can be expressed as

$$\pi_t = \frac{p_t Y_t - W_t N_t}{K_t} = p_t u_t f(n) - W_t h(u_t) n. \tag{53}$$

Substituting (51) and (10) into this equation yields

$$\pi_t = \frac{\varepsilon + \xi - 1}{\xi} \{u_t f(n) k_t\}^{-\varepsilon} u_t f(n), \tag{54}$$

where ξ is defined by

$$\xi \equiv \frac{uh'}{h} \tag{55}$$

Thus, the rate of return, π_t, is expressed as a function of u_t, k_t and n. But, n is assumed to be constant and u_t is expressed as a function of k_t in view of (52). Hence, π_t is reduced to a function of k_t:

$$\pi_t = \pi(k_t). \tag{56}$$

Calculating the elasticity of the rate of return, π_t, with respect to k_t from (53) and (14) yields

$$\omega(k_t) \equiv -\frac{k_t}{\pi_t}\frac{d\pi_t}{dk_t} = \frac{\varepsilon\xi}{\varepsilon + \xi - 1}. \tag{57}$$

In view of (54), $\varepsilon + \xi - 1 > 0$ must be satisfied if $\pi_t > 0$. With this condition, therefore, $\omega(k_t) > 0$. This implies that π_t is a decreasing function of k_t.

Using (56), we can rewrite the objective function of the firm, (49), as follows:

$$V_0 = \int_0^\infty [\pi(k_t) - \phi(g_t)] k_t A_0 e^{-(r-\alpha)t} dt \tag{58}$$

Thus, investment decisions of the firm become the problem of determining g_t so as to maximize (58) subject to the constraint (50).

To solve this problem, we set up the present value Hamiltonian:

$$H_t = e^{-(r-\alpha)t} [\{\pi_t(k_t) - \phi(g_t)\} + \lambda_t (g_t - \alpha)] k_t, \tag{59}$$

where λ_t is the shadow price of k_t. We put $A_0 = 1$ without the loss of generality.

The first order conditions for a maximum of V_0 are

$$\lambda_t = \phi'(g_t) \tag{60a}$$

$$\dot{\lambda}_t = \lambda_t (r - g_t) - [\pi(k_t)\{1 - \omega(k_t)\} - \phi(g_t)], \tag{60b}$$

where θ is defined by (57) above. The transversality condition is

$$\lim_{t\to\infty} k_t \phi'(g_t) e^{-(r-\alpha)t} = 0 \tag{60c}$$

Eliminating λ_t from (60a) and (60b), we obtain

$$\dot{g}_t = \frac{\phi'(g_t)(r - g_t) - [\pi(k_t)\{1 - \omega(k_t)\} - \phi(g_t)]}{\phi''(g_t)}. \tag{61}$$

Equations (61), (50) and (60c) characterize the firm's investment behavior.

Let us analyze the system consisting of these equations by using phase diagram. The locus of points where $\dot{g}_t = 0$ satisfies

$$\phi'(g) = \frac{\pi(k)\{1 - \omega(k)\} - \phi(g)}{r - g} \tag{62}$$

The slope of this locus on Okg plane is calculated from this equation as follows:

$$\left.\frac{dg}{dk}\right|_{\dot{g}=0} = \frac{\pi'(k)\{1 - \omega(k)\} - \pi(k)\omega'(k)}{(r - g)\phi''(g)}. \tag{63}$$

The second order condition for a maximum of (49) implies that the right-hand side expression of (63) is negative. Hence, the locus of $\dot{g} = 0$ is downward sloping. The locus of points where $\dot{k}_t = 0$ satisfies

$$g = \alpha \tag{64}$$

This locus is a horizontal line on Okg plane. The intersection of these loci denoted by $E(k^*, g^*)$ represents the steady state.

In view of (61), we see that $\dot{g}_t > 0$ above the $\dot{g} = 0$ line, and $\dot{g}_t < 0$ below it. Similarly, it is obvious from (50) that $\dot{k}_t > 0$ above the $\dot{k} = 0$ line, and $\dot{k} < 0$ below it. Thus the direction of the movement of the system in each phase becomes as Figure 4. The steady state $E(k^*, g^*)$ becomes a saddle point, and there is a unique path that converges to it. Though we omit the proof, it can be shown that on all other path, either the optimality condition (61) eventually fails or the transversality condition (60c) is not satisfied.

The solution to the optimal investment decision of the firm under imperfect competition is summarized by the saddle path PP. This implies that there is a unique initial level of investment per unit of capital, g, for each initial value of k (capital per unit of expected demand). For instance, if the initial capital per unit of expected demand, k_0, is lower than its steady state value, k^*, the optimal initial level of investment per unit of capital, g_0, is higher than its steady state value, g^*. On the contrary, if the initial capital per unit of expected demand, k_1, is higher than the steady state value, k^*, the optimal investment per unit capital, g_1, is lower than the steady state value, g^*. As the figure shows, the saddle path PP is downward sloping, and starting at any point on the path g and k converges monotonically to g^* and k^*. This implies that g decreases with k monotonically. But, as we have shown before, π is a decreasing function of k. Therefore, g increases with π monotonically. Therefore, the investment per unit of capital, g, is an increasing function of the rate of profit, π.

Let us next examine how the rate of interest, r, affects the level of investment per unit of capital, g. When r rises, the $\dot{g}=0$ line will shift downwards as is shown in figure 5. Then, the saddle path PP shifts down to $P'P'$. Therefore, for any given initial value of k, g will decrease responding to a rise in r. For instance, if the initial value of k is k_0, then g will decrease from g_0 to g_0'. Thus, investment per unit of capital changes inversely with the rate of interest.

Finally, we shall examine the effect of the expected growth rate of demand, α, on investment. When α increases, the $\dot{k}=0$ line will shift upward as is shown in figure 6. Then, the saddle path PP shift up to $P'P'$. Therefore, for any given initial value of k, g will increase responding to an increase in α. For instance, if the initial value of k is k_0, then g will increase from g_0 to g_0'. Thus, the expected growth rate of demand has a positive influence on investment per unit of capital.

To summarize the above results, investment per unit of capital, g, is related positively to π and α, and negatively to r. Thus, the investment function of the firm under imperfect competition may be represented as

$$g_t = G(\pi_t, r, \alpha), \tag{65}$$

where

$$\frac{\partial G}{\partial \pi_t} > 0, \quad \frac{\partial G}{\partial r} < 0, \quad \frac{\partial G}{\partial \alpha} > 0. \tag{66}$$

A special feature of this investment function is that in addition to the rate of profit and the rate of interest, the expected growth rate of demand plays an important role as a determinant of investment. The firms under imperfect competition make investment decisions based on expected future demands for their products. It is not price expectations but quantity expectations. While the firm under perfect competition holds price expectations, the firm under imperfect competition holds quantity expectations in determining investment. The expected growth rate of demand, α, is a parameter that represents the rate of shifts in expected demand curves over time. This parameter may be interpreted to correspond to what Keynes called 'animal sprits' of entrepreneurs.

In the end, we should mention to the relation between the choice of technique and investment decisions. As we have seen in the last section, changes in the wage rate, W, or the rate of interest, r, will lead to changes in the normal labor-capital ratio, n, in the long-run. But, changes in n will lead to changes in the rate of profit, π_t, as is obvious from (54), and then to changes in investment. To be more precise, an increase in W decreases π, and so tends to decrease g, while an increase in r increases π, and so tends to increase g. It should be noted, however, that these indirect effects on investment though the choice of technique will work only in the long-run. Moreover, as for effects of r on investment, its negative effect discussed above will certainly exceed the positive effect through the choice of technique.

7. Conclusions

In this paper, we investigated investment decisions and the choice of technique of the firm under imperfect competition. Our model has two special features. First, we analyzed firm's investment decisions simultaneously with the choice of technique; and second, we analyzed the behavior of imperfectly competitive firms. We distinguish explicitly between the normal labor-capital ratio and the actual labor-capital ratio. The former is determined by the choice of technique of the firm, and the latter by the rate of utilization of existing capital. We assumed that the normal labor-capital ratio is fixed in the short-run, and adjustment costs are needed for changing the ratio towards an optimal level. So, in our model, adjustment costs are involved not only with investment as usual, but also with changes in factor proportions. '

As for the choice of technique, the comparison of steady states has revealed that a rise in the wage rate decreases the labor-capital ratio, while a rise in the interest rate increases its ratio. These results do not seem surprising. It should be noted, however, that in our model the labor-capital ratio attains its optimal level corresponding to factor prices only in the long-run. For, it takes quite a long time for the transition from one steady state to another.

As for investment decisions, we have shown that the expected growth rate of demand is an important determinant of investment in case the firm is under imperfect competition. This is due to the fact that the imperfectly competitive firm bases his investment decisions on quantity expectations unlike the perfectly competitive firm who bases his decisions on price expectations. The expected growth rate of demand may be interpreted to correspond to Keynes' animal spirits that reflect the state of long-run expectations of the firm.

Lastly, we have shown that the wage rate and the interest rate affect investment indirectly through their effects on the normal labor-capital ratio. These indirect effects on investment through the choice of technique will work only in the long-run.

NOTES

1. Uzawa (1972) gives an outline of the investment model for the case of the imperfect competition. But he does not analyze the model in detail.
2. Okishio (1984) constructed a model of the simultaneous decisions of capital utilization, investment and technique, and discussed some Keynes's assertions given in 'The General Theory'. This paper owes much to his model. But his model deals with only the case of two or three periods. Besides, our model focus on different problems from his.
3. Most of the investment models presented so far do not differentiate between the long-run production function and the short-run production function. This distinction is made clear in the following literature: Okishio (1984), Malinvaud (1989), Malinvaud (1998).
4. This formulation of adjustment costs follows Hayashi (1983).
5. A similar assumption is made by Blanchard (1997).

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FIGURE

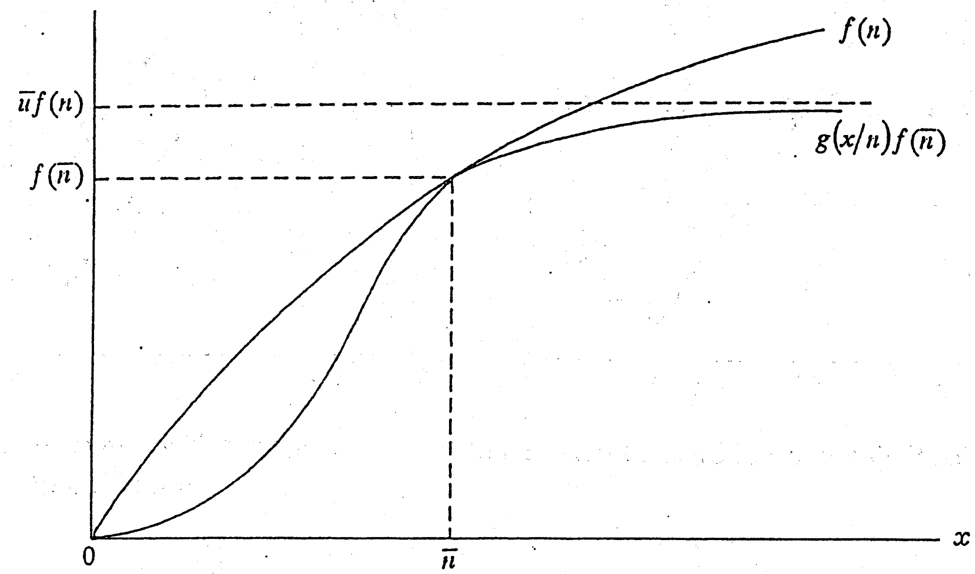

Figure 1. The Long-run Production Function and the Short-run Production Function

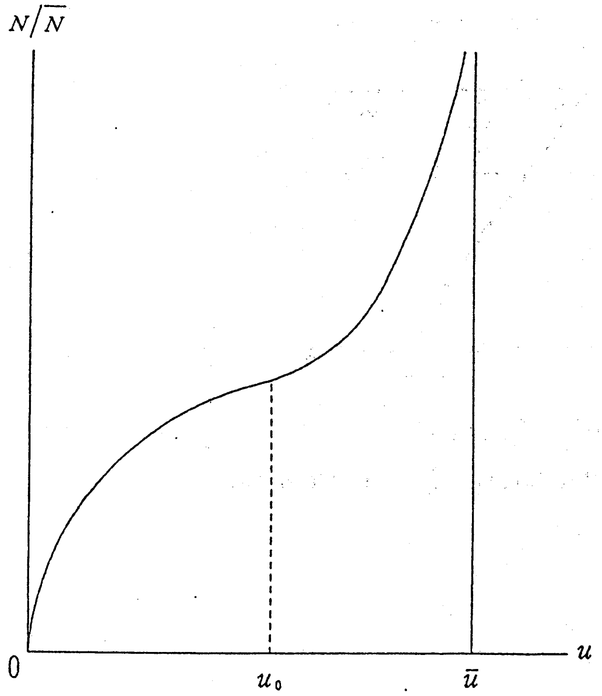

Figure 2. The Employment Function

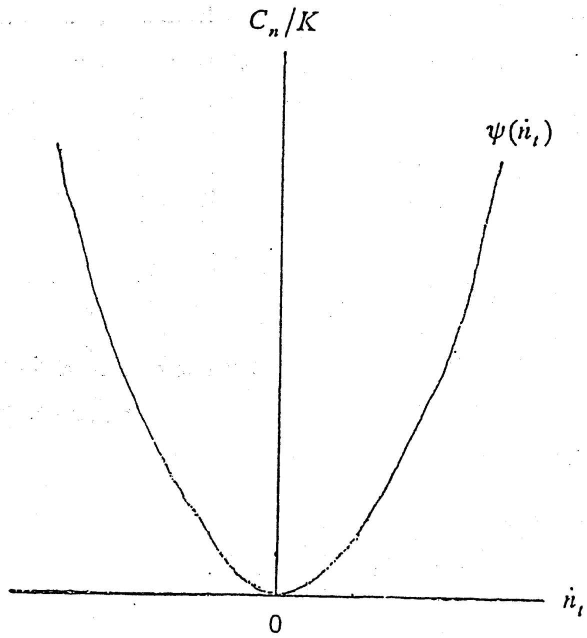

Figure 3. The Adjustment Costs for Changing Labor-Capital Ratio

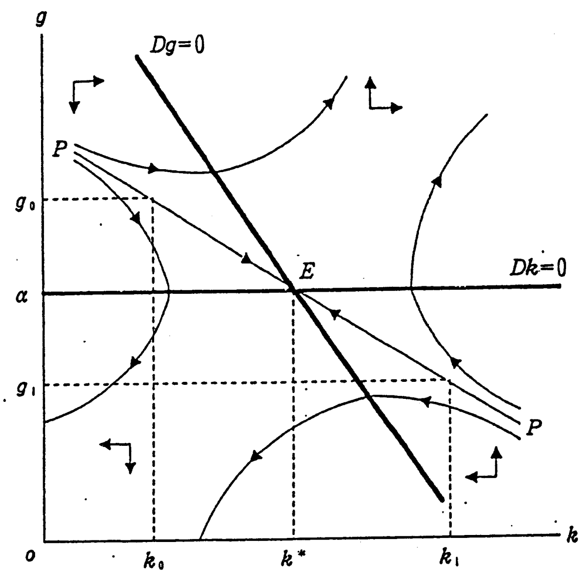

Figure 4. The Saddle Path of Optimal Investment

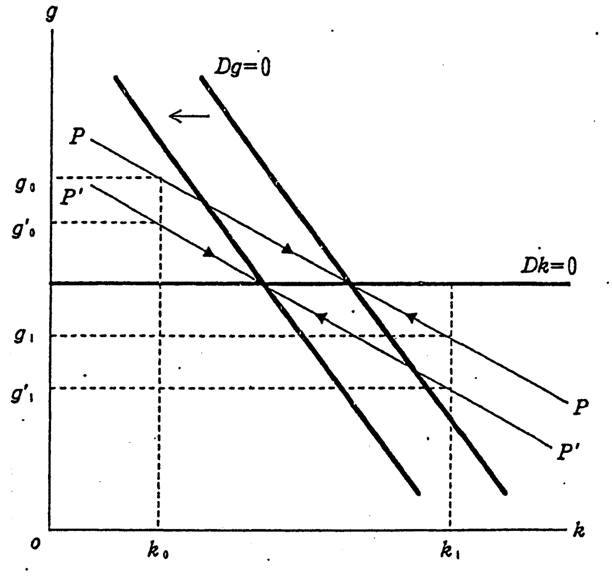

Figure 5. The Effects of a Rise in the Interest Rate

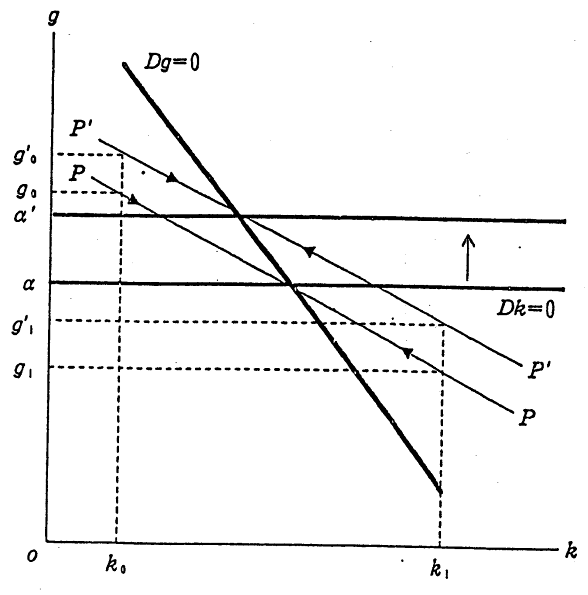

Figure 6. The Effects of an Increase in the Expected Growth Rate of Demand