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Effective Dimension Transition of the Dynamics of Granular Materials in the Pipe.

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Granular materials exhibit some complex phenomena.[1,2] These complex phenomena strongly depend on not only the character of external forces but also that of boundary condition. When granular materials in slim pipes with fluid (air, water, and so on) flow down or are fluidized by inducing fluid from the bottom, slugging is observed[7,10] . In most of the investigations of such slugging in a slim pipe, the system's effective dimension is regarded as 1.[8-10] On the other hand, two or more dimensional phenomena, bubbling and channeling, occur when granular materials are in a wider vessel.[12] The similar tendency is also observed for granular materials in a vibrating vessel.[3-6] When the width of the vessel is small, most of particles construct solid structure in bulk and only particles near side walls flow down to the bottom.[1,4] However, in a wide vessel, most of particles are strongly fluidized and construct multi-roll structure like Bernard convection[6] From these facts, the behaviors of granular materials are extremely different between that in a slim vessel and that in a wide vessel. Then, an important problems appear; what quantity determines 'slim' (one dimensional system) or 'wide' (two or three dimensional system) for a given system? The purpose of this paper is to make a clear view on this problem.

Here, we simulate following simplified situation.[14] The system consists of two dimensional particles of mass 1 and diameter d in a two-dimensional box under uniform gravity. The

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width of the box is a, and the height of the box is infinite. We employ the following particle model which is one of the simplest model of granular materials. The equation of the motion of the ith particle is

$$\ddot{\mathbf{x}}_i = -\sum_{j=1}^{N} \theta(d - |\mathbf{x}_i - \mathbf{x}_j|)\{\nabla U(d - |\mathbf{x}_i - \mathbf{x}_j|) + \eta(\mathbf{v}_i - \mathbf{v}_j)\} + \mathbf{g} \tag{0.1}$$

$$U(d - |\mathbf{x}_i - \mathbf{x}_j|) = \frac{k}{2}(d - |\mathbf{x}_i - \mathbf{x}_j|)^2 \tag{0.2}$$

here, θ is Heviside function, N is the total number of particles, k and η are respectively the elastic constant and the viscosity coefficient,[20] and $\mathbf{x}_i(x_i, y_i)$, $\mathbf{v}_i(v_{x_i}, v_{yi})$, and $\mathbf{g} = (0, -g)$ are, respectively, the position, the velocity and the gravity of ith particles. In this model, the effect of particles' rotation is neglected. The system is driven by a simple energy source at the bottom of the box; a particle hitting the bottom with velocity (v_x, v_y) bounces back with the velocity $(v_x, V(V > 0))$. We regard the bottom of box as x axis $(y = 0)$, and $x = 0$ as the center of box. The side walls of the box are put along $x = a/2$ and $x = -a/2$, and the viscosity which works between these walls and particles is zero. At the initial condition, we put particles bed with height b on the bottom of box. We simulate this system with some combinations of parameters (η, a, b), where a and b are enough large compared to the particle's diameter d. The above equations are calculated with the Euler's scheme. The time step δt is set enough small such that δx, the displacement of the ith particle during δt, does not exceed a given value. In this paper, we set $(-g, V)$ so that the average height of the center of mass of the system $CM =< (\sum y_i)/N >_t$ keeps enough large compared to b. Figure 1 shows typical snapshots of the system for respectively, (a) $a << b$ with $\eta > \eta_*$, (b) $a >> b$ with $\eta > \eta_*$, (c) $a << b$ with $\eta < \eta_*$, and (d) $a >> b$ with $\eta < \eta_*$. Here, we fixed b, whereas $(-g, V)$ of (a) is same as that of (b), and $(-g, V)$ of (c) is same as that of (d). In cases of (a) and (c), only the particles distribution in the horizontal direction is symmetric, and the center of mass of this system moves a little only in the vertical direction. On the other hand, the particles distribution is non-uniform in vertical and horizontal direction, and one convection appears in cases of (b) and (d). In order to characterize the system, we

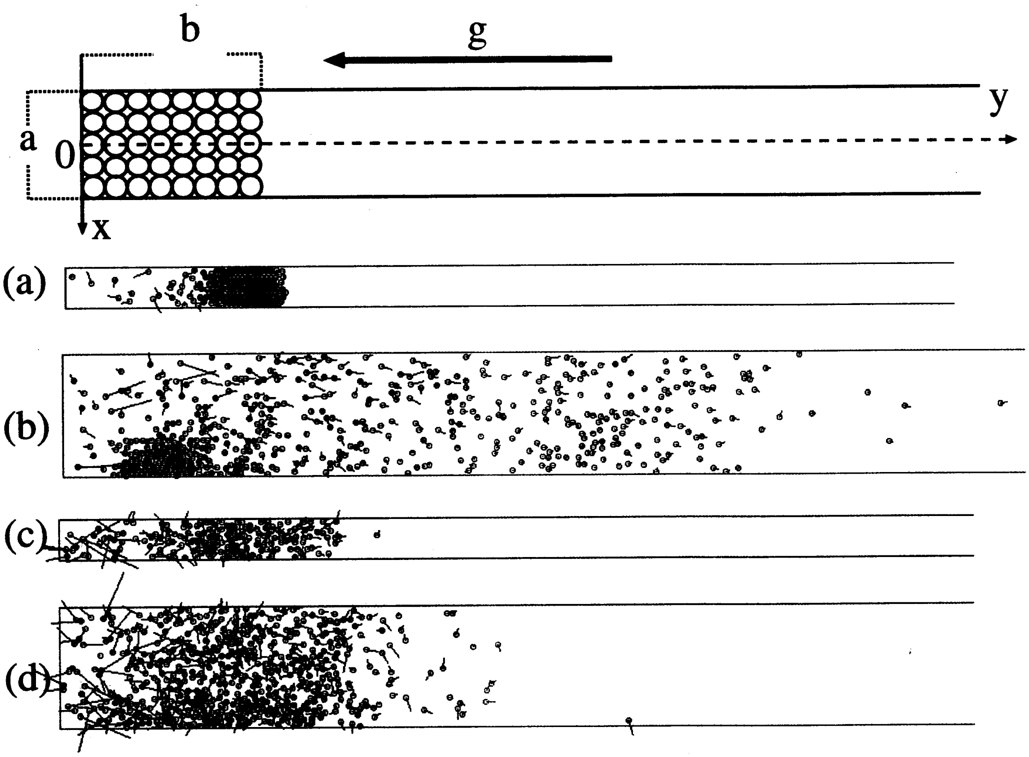

Fig. 1. Illustration of 2D pipe and typical snapshots respectively, (a) $a << b$ with $\eta > \eta_*$, (b) $a >> b$ with $\eta > \eta_*$, (c) $a << b$ with $\eta < \eta_*$, and (d) $a >> b$ with $\eta < \eta_*$.

introduce the order parameter $L(t) = |\sum_i x_i(t)v_{yi}(t)|/N$ which indicates the strength of the convection of the system. Figure 2 (a) and (b) are typical probability distributions of $L(t)$ which is given by one time series for respectively the cases of $a << b$ and those of $a >> b$. In Fig.2 (a), the peak of the probability distribution of $L(t)$ appears at $L(t) = 0$ which means there are no convection for the case $a << b$. In these cases, the effective dimension of this system can be regarded as 1. We name such states as the '1D state'. On the contrary the peak of the probability distribution of $L(t)$ appears at $L(t) > 0$ in Fig.2 (b) which means a convection with finite magnitude appears in the system. In these cases, the system is actually 2 dimensional system. We name such states with a convention as the '2D state'. Such a transition between the 1D state and the 2D state which depends on the relation between a and b is observed in a simple system.

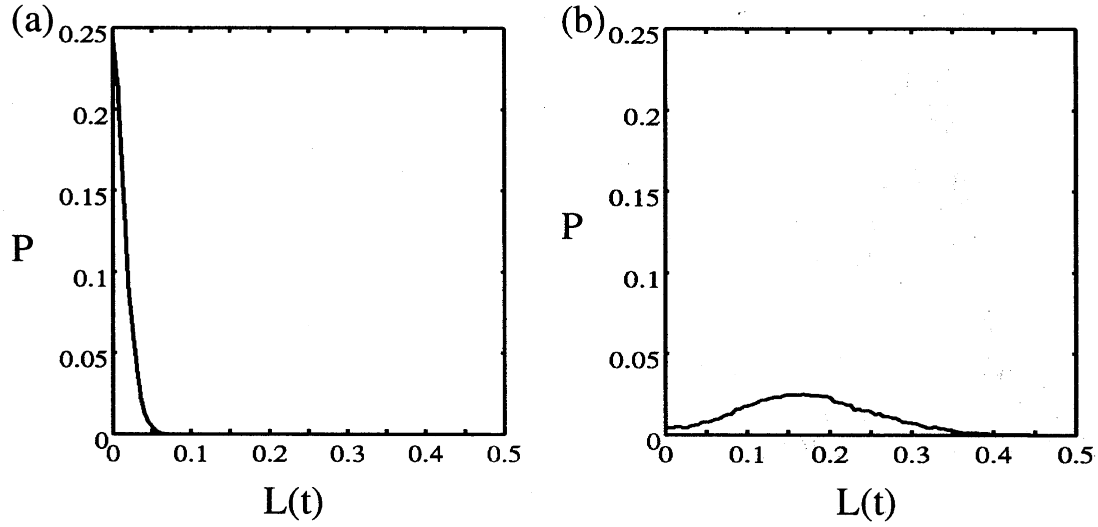

Fig. 2. Probability distributions of $L(t)$ respectively (a) $a << b$, and (b) $a >> b$.

In our simulation, particles are strongly excited by the energy source at the bottom of the box. Then, if we set a improper g for a small η or a small b, the height of particles diverge. In order to clear off such cases, we need to control g for several η and b. By the simulation, we found the fact that CM have almost same values independent of a with $a << b$ for a fixed set of g, η, b and V. Hereafter, we fix V and determine the gravity g for a given set (η, b) independent of a. In this paper, we set $V = 2$ and $g(\eta, b)$ with which $CM \sim 60d$ is realized for all cases of $a << b$. When we set $V = 3.0$ or $V = 4.0$, we can get qualitatively same results as those of following discussions with $V = 2.0$.

Before the discussion of the transition width at which the 1D-2D transition takes place or the character of the transition, we discuss the dependency of η for the 1D states. Figure 3 shows the time averaged packing fraction profile of the y direction as the function of $y - y_{CM}$ for several η with $a = 9d$ and $b = 30d$. Here, y_{CM} is the y component of center of masses of particles, and we define the packing fraction as followings. We divide the space by a lattice with the lattice constant d, and we define the number of centers of particle within each $d \times d$ square as the packing fraction in the square. The packing fraction of y direction

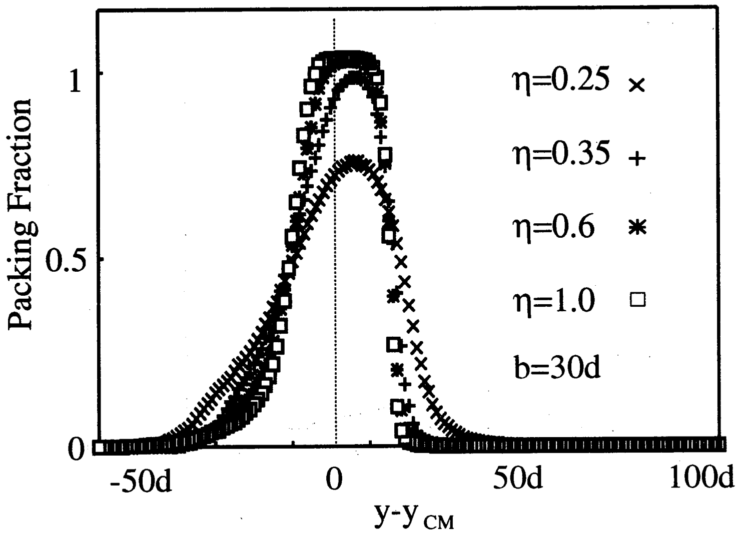

Fig. 3. Packing fraction profile for y direction respectively $\eta = 0.25, 0.35, 0.6$, and 1.0.

is given as the average of them through the x direction for each y. For large η ($\eta = 0.6, 1.0$), each profile includes a flat region with large packing fraction near the center of mass of this system. These flat regions mean the existence of the solid structure in which particles are almost completely packed. The length of this flat region decreases with decreasing η, and this length becomes 0 for $\eta = \eta^* \sim 0.38$. If $\eta < \eta^*$ ($\eta = 0.25, 0.35$), each packing fraction profile includes no flat region, and maximum packing fraction is smaller than that of $\eta > \eta^*$. It means that no solid structures are created for such small η. Thus, a transition between a state which includes a solid structure and the other state which include no solid structure occurs at the critical value $\eta = \eta^*$. Similar results are obtained in following two cases, $b = 20d$ and $b = 40d$. Now we introduce following no dimensional values: $a' = a/d$, $b' = b/d$, and $e' = 1 - e$ where $e = exp(\pi\eta/(k - \eta^2)^{\frac{1}{2}}))$, and the length of flat regions $h(e') = h'(e')d$. ($h'(e')$ has no dimensions.) Here, e indicates the coefficient of restitution for head-on collisions between two particles.[3] The relation between two rescaled values, $(e'b')$ and $\frac{h'(e')}{b'^2}$, is obtained as shown in Fig.4(a). The rescaled critical point $(e'b')^*$ is determined

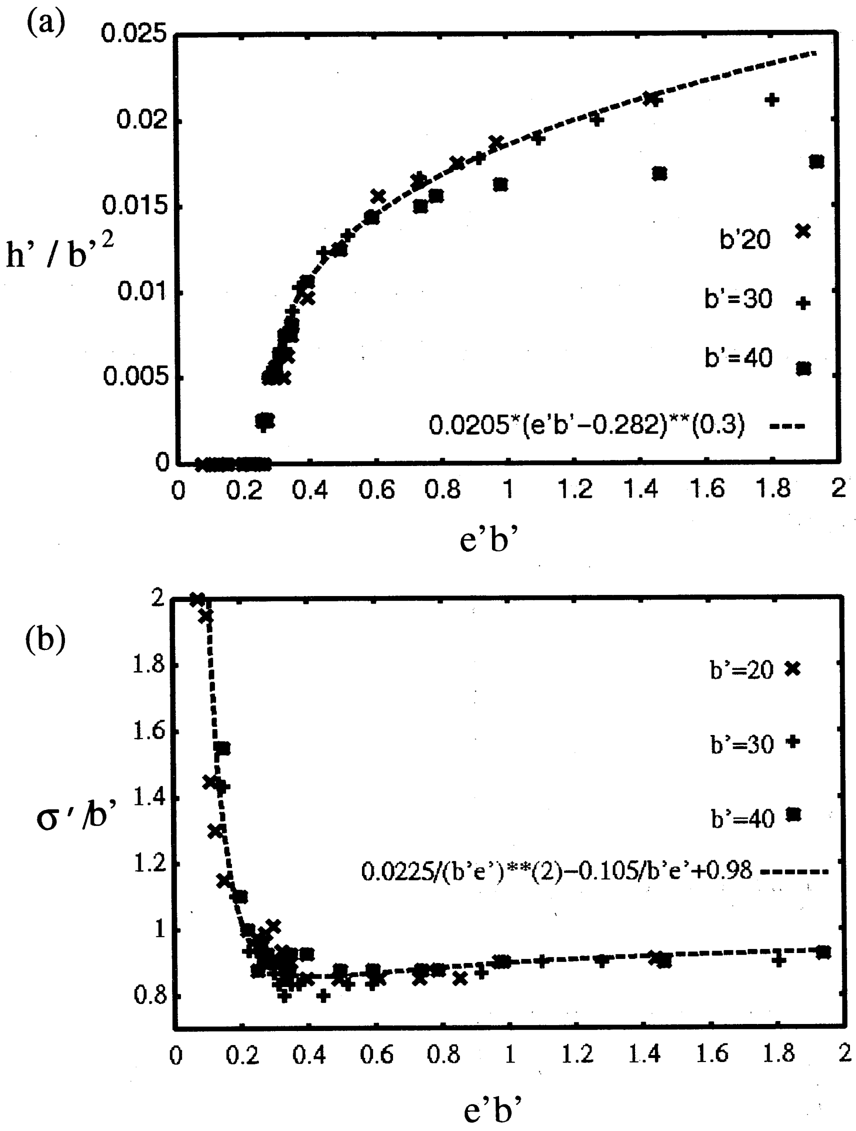

Fig. 4. Relation between two rescaled values, (a) $e'b'$ and $\frac{h'(e')}{b'^2}$, and (b) $e'b'$ and $\sigma'(e')/b'$

independent of the system size, and the profile of this relation fits with

$$\frac{h'(e')}{b'^2} = h'_o|e'b' - (e'b')^*|^{0.3} \tag{0.3}$$

near $(e'b')^* \sim 0.282.$(h'_o is constant.) Moreover, we introduce the cluster length $\sigma(e') = \sigma'(e') * d$ ($\sigma'(e')$ has no dimensions.) which is defined as the distance between two nearest inflection points from the maximum point of the packing fraction profile. Thus, the relation between two rescaled values, $e'b'$ and $\sigma'(e')/b'$, are obtained as shown in Fig.4 (b). The profile of this relation fits with

$$\frac{\sigma'(e')}{b'} = \frac{0.0255}{(e'b')^2} - \frac{0.105}{e'b'} + 0.98. \tag{0.4}$$

We expect $\sigma'(e')/b' \to \infty$ for $e'b' \to 0$, and $\sigma'(e')/b' \sim 1$ for $e'b' \to \infty$.

Now, we discuss the character of transition between the 1D state and the 2D state. Here, we define this transition width a^* as the maximum width that the 2D state cannot be observed

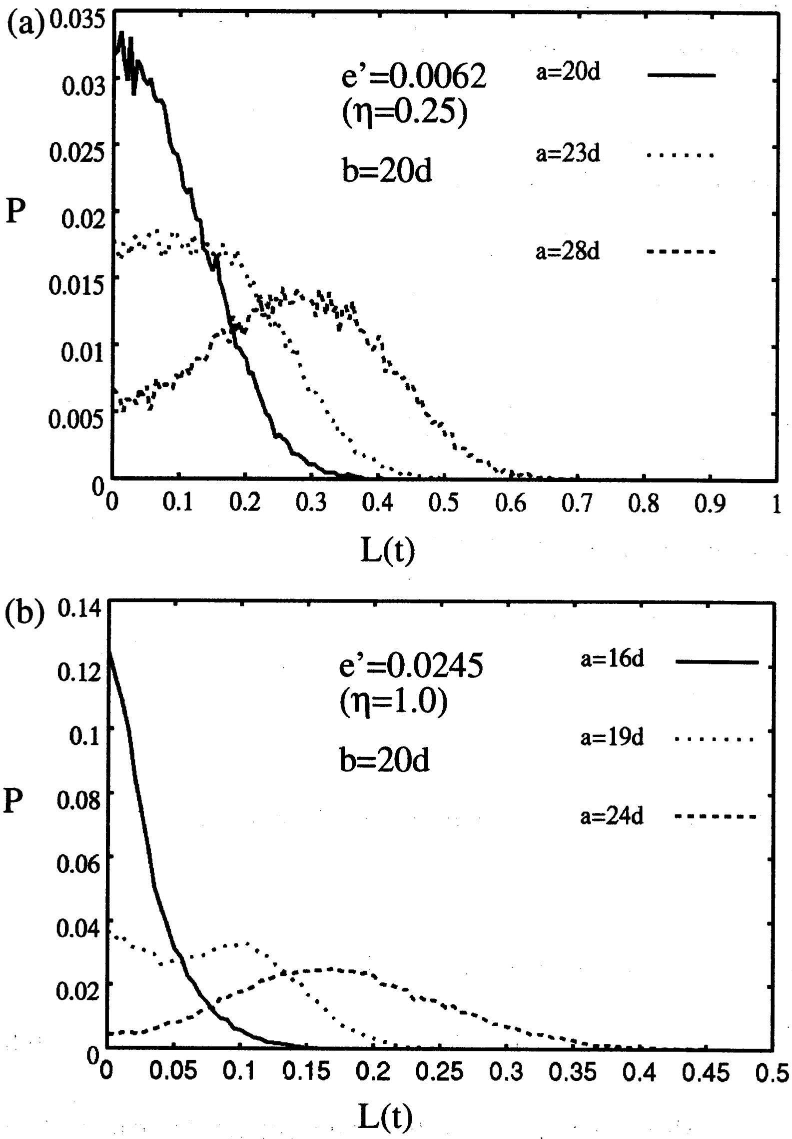

Fig. 5. Typical probability distributions of $L(t)$ of (a) $e' < e'^*$ $(\eta < \eta^*)$ and (b) $e' > e'^*$ $(\eta > \eta^*)$ for several width around a critical value $a = \sigma(e')$.

in the system. Figure 5 shows the typical probability distributions of $L(t)$ for several width a around $a = \sigma(e')$. Here, (a) indicates for the case $e' < e'^*$ ($\eta < \eta^*$) and (b) indicates for the case $e' > e'^*$ ($\eta > \eta^*$)both for $b = 20d$. In Fig.5 (a), each profile of probability distribution of $L(t)$ includes only one peak. The position of peak is at $L(t) = 0$ for $a < \sigma(e')$. For $a > \sigma(e')$, however, the peak appears at $L(t) > 0$ and this peak moves with the width of the system. Thus the continuous transition between the 1D state and the 2D state appears for the case $e' < e'^*$ ($\eta < \eta^*$), and the transition width is given as $a^* \sim \sigma(e')$. In Fig.5 (b), on the contrary, probability distributions of $L(t)$ for a, which is a little larger than $\sigma(e')$, include two peaks at $L(t) = 0$ and $L(t) > 0$. This means that two locally stable states, one is the 1D state and the other is the 2D state, coexist and they appear periodically for the case of $e' > e'^*$ ($\eta > \eta^*$). In such cases, the system includes a solid structure when 1D state is realized. In this solid structure, the friction between particles are strong because particles are densely packed. Hence, the solid structure is break-proof, and this originates the stability of the 1D state for $a > \sigma(e')$. Figure 6 shows the semi-log scale profiles of probability distributions of $L(t)$ for respectively $a < \sigma(e')$, $a \sim \sigma(e')$ and $a > \sigma(e')$. When $a' < \sigma'(e')$, the profile

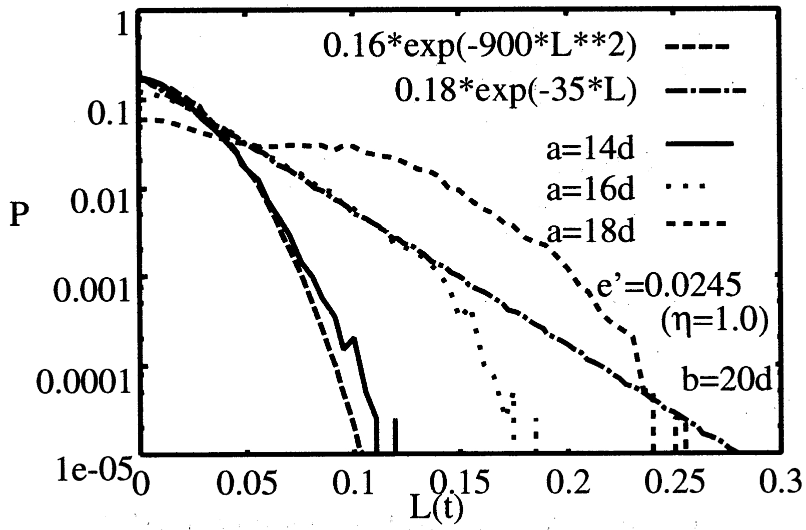

Fig. 6. Semi-log scale profiles of probability distributions of $L(t)$ for respectively $a < \sigma(e')$, a is a little smaller than $\sigma(e')$ and $a \geq \sigma(e')$ with $e' \sim 0.247$ ($\eta = 1.0$)

is proportion to $exp(-\gamma_0 L^2)$, which means that fluctuations of $L(t)$ are so small that they can be neglected. However, a profile which proportion to $exp(-\gamma_1 L)$ is obtained when a is close to $\sigma(e')$, which means that the fluctuation from $L(t) = 0$ become large. Moreover, the profile includes two peaks at $L(t) = 0$ and $L(t) > 0$ when $a \geq \sigma(e')$. Then, the transition width a^* is regarded as $a^* \sim \sigma(e')$ for also $e' > e'^*$ $(\eta > \eta^*)$. These results mean following two facts. I) The critical width which the 2D state can appear, is equal to the cluster length. II) The magnitude of dissipation separates the type of the transition between the 1D state or the 2D state. We obtain the similar results for the case $b' = 30$. Then, by using rescaled parameter $e'b'$ and a'/b', the phase diagram is obtained as shown in Fig.7.

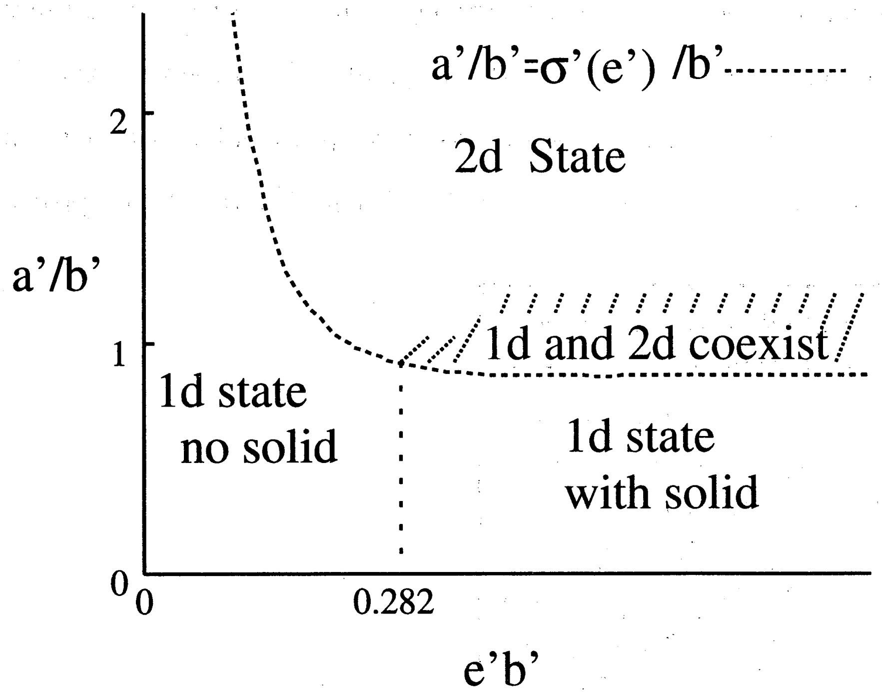

Fig. 7. Phase diagram of the effective dimension of the system which depends on $e'b'$ and a'/b'

In this paper, we simulated strongly excited granular materials in a 2 dimensional pipe. When the width of the system is small, a characteristic state, in which the effective dimension of particles' dynamics is regarded as 1, appears. We named this state as the 1D state. However, when the width of the system is larger than a critical width, the actual 2 dimensional state in which convection appears. We named this state as the 2D state. Moreover, we found that the critical width is equal to the length of the cluster which appears in the 1D state. The character of the 1D state depends on the magnitude of dissipation as followings. When the magnitude of dissipation is larger than the critical value, the system include solid structure. On the contrary, such solid structure disappears when the magnitude of dissipation is smaller than the critical value. According to the differences of the magnitude of dissipation, following two types of behaviors appears in the system near the critical width. When the magnitude of dissipation is smaller than the critical value, the continuous transition between the 1D state and the 2D state appears. On the contrary, when the magnitude of dissipation is larger than the critical value, two meta-stable states, the 1D state and the 2D state, appear periodically for a little over the critical width. By another simulation, the existence of such a critical magnitude of dissipation is reported.[18,19] The analytical derivation of this critical value is one of the most important issue for the research of granular materials. Moreover, simulations of more highly excited systems, larger systems, and analytical study of the critical width for several magnitude of dissipation are important future issues.

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1) H.Hayakawa, H.Nishimori, S.Sasa, Y-h. Taguti, Jpn. J. Appl. Phys. **34** (1995), 397
2) H.M.Jaeger and S.R.Nagel, Science **255** (1990) 1523
3) Y.-h.Taguchi, Phys. Rev. Lett. **69**, 1367 (1992) ;J.A.C.Gallas, H.J.Herrmann and S.Sokolowski Phys. Rev. Lett. **69**, 1371 (1992)
4) E.L.Grossman, Phys.Rev.E **56** (1997) 3290.
5) K.M.Aoki, T.Akiyama, Y,Maki and T.Watanabe Phys.Rev.E **54** (1996) 874.
6) K.M.Aoki and T.Akiyama Phys.Rev.E **58**, (1998) 4629.
7) S.Horikawa, T.Isoda, T.Nakayama, A.Nakahara and M.Matsushita, Physica **A233**(1996),699 ;A.Nakahara and T.Isoda: Phys.Rev.E **55**(1997) 4264.
8) T.S.Komatu and H.Hayakawa Phys.Lett. A **183** (1993) 56.
9) G.Peng and H.J.Herrmann: Phys.Rev.E **51**(1995) 1745.
10) O.Moriyama, N,Kuroiwa, M,Matsushita, and H,Hayakawa: Phys.Rev.lett,**80**(1998)2833.
11) A.Awazu, J.Phys.Soc.Jpn. **67** (1998) 1071.
12) K.Ichiki and H.Hayakawa, Phys.Rev.E **52**, (1995) 658. ;Phys.Rev.E **57**, (1998) 1990.

13) T.Ooshida and T.Kawahara, Phys.Rev.E **56**, (1997) 511.

14) Similar simplified situations have been investigated to discuss the statistical and hydro-dynamical properties of granular materials.[15–17]

15) Y.DU, H.Li and K.P.Kadanoff Phys. Rev. Lett. **74**, (1995) 1268

16) E.L.Grossman, T.Zhou and E.Ben-Naim Phys.Rev.E **55**, (1997) 4200.

17) M.Isobe and H.Nakanishi, J.Phys.Soc.Jpn. **68**, (1999) 2882.

18) T.S.Komatu, J.Phys.Soc.Jpn. **69**, (2000) 5.

19) R.Ramirez, D.Risso and P.Cordero cond-matt/0002433

20) The elastic constant k and the viscosity coefficient η are related with the coefficient of restitution e and the collision time t_{col}, time period during collision 3).