# The Effect of Crystal Dispersion on the X-ray Emission Spectrum Observed using a Double Crystal Spectrometer

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The effect of the rocking curve on the x-ray emission spectrum observed using a double-crystal spectrometer was discussed. The results for Si (220) crystal at 1.54056Å (Cu  $K\alpha_1$ ) show that the x-ray of wavelength  $\lambda_0$ - $\Delta\lambda$  ( $\Delta\lambda$ =0.00007Å ) is much more reflected by the double crystal put in the (++) position of  $\theta_B$  than the x-ray of wavelength  $\lambda_0$  which exactly satisfies the Bragg condition. In the last we mentioned what we observe in our measurement with a double-crystal spectrometer.

Keywords: double-crystal spectrometer/ rocking curve/ x-ray emission spectrum

X-ray emission spectroscopy is known as a valuable tool for estimating the level width and the probabilities of various multi-electron transitions. In spite of its usefulness the instrumental function which is specific for the spectrometer used makes a precise analysis difficult. The instrumental function for a double-crystal spectrometer is relatively small compared with that for a single crystal spectrometer because the first crystal plays the same role as a narrow slit. But even for a double crystal spectrometer the instrumental function cannot be neglected. The aim of our work is to evaluate the effect of the crystal dispersion on the x-ray emission spectrum observed by means of a double-crystal spectrometer and to establish the way to analyze it. The crystal dispersion is considered to be a main component of the instrumental function

in a double-crystal spectrometer and we may assume that the contributions from other components are almost negligible.

First we consider the monochromatic x-ray which has the wavelength  $\lambda_0$  satisfying the Bragg condition with the Bragg angle  $\theta_B$  determined by the positions of two crystals. As can be seen from the Figure 1, the beam reflected on the first crystal by the angle of  $\theta = \theta_B + d\theta$  makes an incidence angle of  $\theta' = \theta_B - d\theta$  with the surface of the second crystal. This gives us the expression for the rocking curve for double crystal  $(R_D(\theta_B; \lambda_0; \theta') = R_D(\theta_B; \lambda_0; 2\theta_B - \theta))$  as follows.

$$R_D(\theta_B; \lambda_0; \theta') = R_S(\theta_B; \lambda_0; \theta') \times R_S(\theta_B; \lambda_0; 2\theta_B - \theta'). \tag{1}$$

Here,  $R_s(\theta)$  expresses the rocking curve for single crys-

## STATES AND STRUCTURES — Atomic and Molecular Physics —

### Scope of Research

In order to obtain fundamental information on the property and structure of materials, the electronic states of atoms and molecules are investigated in detail using X-ray, SR, ion beam from accelerator and nuclear radiation from radioisotopes. Theoretical analysis of the electronic states and development of new radiation detectors are also performed.



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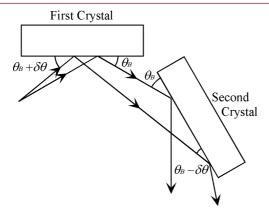
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**Figure 1.** Schematic diagram of the reflections by double-crystal with (++) position

tal which we define as the average of the rocking curve for the normal polarization and that for the parallel polarization. In the normal polarization the electric vector  $\boldsymbol{E}$ , at the moment when the x-ray is reflected, is normal to the plane containing the wave vector for the incident beam  $\boldsymbol{k}_{\theta}$  and that for the diffracted beam  $\boldsymbol{k}_{H}$ , while in the parallel polarization the electric vector  $\boldsymbol{E}$  lies in that plane. The curves of  $R_D(\theta') = R_D(2\theta_B - \theta)$  for Si(220) at Cu K $\alpha_1$  energy are shown in the Figure 2- (a).

Now we consider the beam having the wavelength  $\lambda'$  which is very close to  $\lambda_0$ . The Bragg angle for this beam  $\theta_B$ ' is given by

$$\theta_{B}' \cong \theta_{B} + \left(\frac{d\theta}{d\lambda}\right)_{\lambda = \lambda_{0}} (\lambda' - \lambda_{0}) = \theta_{B} + \frac{\lambda' - \lambda_{0}}{\lambda_{0}} \tan \theta_{B}$$
 (2)

This leads to the expression for the rocking curve in a double-crystal (The attention should be paid to the fact that the positions of the two crystals are not for  $\theta_B$ ' but for  $\theta_B$ .) at the wavelength of  $\lambda$ ' as follows.

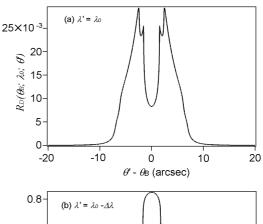
$$R_{D}(\theta_{B}; \lambda'; \theta') = R_{S}(\theta_{B}; \lambda'; \theta') \times R_{S}(\theta_{B}; \lambda'; 2\theta_{B} - \theta')$$

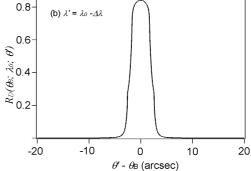
$$\cong R_{S}\left(\theta_{B}; \lambda_{0}; \theta' - \frac{\lambda' - \lambda_{0}}{\lambda_{0}} \tan \theta_{B}\right)$$

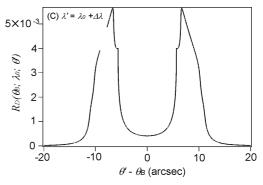
$$\times R_{S}\left(\theta_{B}; \lambda_{0}; 2\theta_{B} - \theta' + \frac{\lambda' - \lambda_{0}}{\lambda_{0}} \tan \theta_{B}\right). \tag{3}$$

We can easily check that  $R_D(\theta_B;\lambda';\theta)$  corresponds to equation (1) when  $\lambda'$  is equal to  $\lambda_0$ . For  $\lambda' = \lambda_0 - \Delta \lambda$ ,  $\lambda_0 + \Delta \lambda$  ( $\Delta \lambda = 0.00007$  Å), the curve of  $R_D(x;\theta)$  is shown in the Figure 2-(b),(c) respectively. Figure 2-(a),(b),(c) show that the x-ray of wavelength  $\lambda_0 - \Delta \lambda$  is reflected most strongly compared with the others. In fact S(a):S(b):S(c), the ratio of the area under the curve, is approximately 4:67:1. When the intensity distribution of the incident beam on the wavelength  $\lambda$  is  $I_{in}(\lambda)$ , the intensity distribution of the diffracted beam becomes

$$I_{out}(\theta_B; \lambda) = \int I_{in}(\lambda) R_D(\theta_B; \lambda; \theta') d\theta'. \tag{4}$$







**Figure 2.**  $R_D(\theta_B; \lambda'; \theta')$  for Si (220) at (a)  $\lambda' = \lambda_0 (\lambda_0 = 1.54056\text{\AA})$ , (b)  $\lambda' = \lambda_0 - \Delta \lambda (\Delta \lambda = 0.00007 \text{\AA})$ , (c)  $\lambda' = \lambda_0 + \Delta \lambda (\Delta \lambda = 0.00007 \text{\AA})$ 

Inversely, the the intensity distribution of the diffracted beam in the direction  $\theta$ ' is expressed as follows .

$$I_{out}(\theta_B; \theta') = \int I_{in}(\lambda) R_D(\theta_B; \lambda; \theta') d\lambda.$$
 (5)

The range of angle or wavelength having a significant intensity is so small that the detector can count all the photons in this range. Then the expression for what we observe is written as

$$I_{out}(\theta_B) = \iint I_{in}(\lambda) R_D(\theta_B; \lambda; \theta') d\lambda d\theta' \qquad (6)$$

The observed spectrum is considered to be the trace of  $I_{out}(\theta_B)$  at each point of  $\theta_B$  which we change during the scan.