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Optimal Aid in Environmental Policy: A Real Options Approach*

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In this paper, we analyze an environmental policy that is designed to reduce the emission of pollutants when there is uncertainty over the social costs of environmental damage. We first establish a model that incorporates direct aid in which there are two noncooperative agents: a developing country and a developed country. Next, we derive closed-form solutions for the optimal levels of direct aid that the developed country provides the developing country. Finally, we compare the social welfare level implied by this model with those implied by two additional models. In one of the models, two agents cooperate, and in the other, the agents do not cooperate and there is no direct aid. As a result, the means of providing direct aid plays an important role in reducing the decline in social welfare caused by external effects.

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1. Introduction

The Kyoto protocol on global warming came into effect on 16 February 2005. According to this agreement, each country's emissions target is to be achieved

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during the period 2008–2012. For example, Japan will be obliged to reduce its 1990 level of greenhouse gas emissions by 6% (as will Canada, Hungary, and Poland); meanwhile Switzerland, most central and eastern European states, and the European Union will have to reduce their emissions by 8%. However, there are concerns that many developing countries have disregarded this protocol. Hence, it is important that the developed countries investigate feasible solutions as soon as possible.

There has been considerable research on effective political solutions to environmental problems, based on different approaches to environmental economics. For an extensive survey of the literature on environmental policy instruments, see Cropper and Oates (1992). However, almost all the existing research applies cost-benefit analysis. Standard cost-benefit analysis cannot simultaneously explain three important characteristics of most of the environmental problems described in Pindyck (2000). First, there remains an uncertainty regarding the future costs and benefits of adopting a particular policy. Second, environmental policies designed to reduce ecological damage are typically irreversible; i. e., are associated with *sunk costs*. Third, policy adoption is rarely a "now-or-never" proposition. In most cases, it is feasible to delay action and wait for new information.

The real options approach has become a useful tool for evaluating irreversible investments under uncertainty (see Dixit and Pindyck (1994)). Pindyck (2000, 2002) explains how irreversibilities and uncertainty interact to affect the timing and design of policy by using a real options approach. Adopting a different perspective, Arrow and Fisher (1974), Henry (1974), and Kolstad (1996) examine the implications of irreversibility and uncertainty with regard to environmental policy from a different point of view. More recent studies have investigated the problem of strategic countries implementing environmental policies (see Barrieu and Chesney (2001) and Ohyama and Tsujimura (2008)). In regard to the strategic framework, the occurrence of an external effect constitutes a potential problem. If one agent implements an environmental policy, this affects the other agent's environment. That is, the implementation of an environmental policy by one agent improves not only the agent's environment, but also that of the other agent. Barrieu and Chesney (2001) analyzed an environmental problem in the presence of strategic interactions between two agents within a real options framework. Ohyama and Tsujimura (2008) demonstrate how an external effect delays policy implementation. This has crucial implications for international policies and regulations. That is, if each country considers the timing of implementing environmental policy in a noncooperative framework, environmental problems are not resolved optimally.

To resolve this problem, several political measures are available. These include environmental subsidies (see, e. g., Dewees and Sims (1976)), environmental taxes (see Baumol and Oates (1988)), emission trading systems (see, e. g., Maeda (2004), and Montgomery (1972)). Hanley, Shogren and White (1997) and Kolstad (1999) summarize the effectiveness of these political measures

regarding environmental problems. In the context of incorporating an emissions trading system within a real options framework, Insley (2003) examines the optimal decisions of a firm that has the option of retrofitting its plant to reduce pollution and thereby, avoiding the purchase of emission allowances. Ohyama and Tsujimura (2006) extend Pindyck (2002) by discussing the optimal values of such political instruments and examine their influence on the behavior of strategic agents. Most studies suggest that the policies mentioned above effectively resolve the problem of external effects. However, it is difficult to maintain an environmental taxation system or an emission trading system in practice, because this requires an agreement between several countries and firms. Hence, high adjustment costs are expected to accompany the implementation of these systems. Moreover, developing countries cannot be included in such a system. This is because developed and developing countries disagree with regard to the levels of pollution that the former has occasional.

In this paper, as a second-best solution to the problem of the external effect, we examine the effectiveness of direct aid from a developed country in inducing a developing country to implement its environmental policy more optimally with respect to the overall system. We extend the single-agent model of Pindyck (2000) to the case of two asymmetric agents: the developing country and the developed country. We assume that the developed country is more sensitive to pollution than is the developing country. We show that the external effect and the difference in the evaluation of pollution with regard to the two countries imply a level of direct aid that the developed country is willing to provide the developing country. However, it should be noted that both the developing and the developed country can behave strategically. Direct aid from a developed country does not necessarily accelerate the implementation of environmental policy by a developing country. Taking strategic behavior into account, we derive closed-form optimal levels of direct aid and the optimal timing with which developed and developing countries individually implement their environmental policies.

It is important to realize that the direct aid framework proposed in this paper is selfmotivated, rather than forced. This has the practical implication that the developed country may voluntarily give direct aid to the developing country to induce the developing country to implement its environmental policy sooner. Our numerical examples indicate that direct aid benefits boht developed and developing countries. Furthermore, we show that providing direct aid is effective in reducing the loss in social welfare that is caused by the external effect. We suggest that the results obtained in this paper can provide additional insights into the role of direct aid in addressing environmental economic problems.

The paper is organized as follows. In Section 2, we describe the model, which is based on that of Pindyck (2000). In Section 3, we extend the model of Pindyck (2000) to one with two asymmetric agents, which we refer to as the developing country and the developed country. In Section 4, we consider the effects of incorporating direct aid into the two-agent model and derive the optimal level of direct aid. In Section 5, we compare the levels of social welfare from the

cooperative model, the noncooperative model, and the noncooperative model that incorporates direct aid. Section 6 concludes the paper.

2. A Model for Environmental Policy

In this section, we first describe the model, which is based on those of Dixit and Pindyck (1994) (Chapter 12, Section 3), Pindyck (2002), and Ohyama and Tsujimura (2008). All these authors consider an environmental economics problem that concerns the optimal timing corresponding to the adoption of a cost-bearing environmental policy for reducing the flows of a pollutant. The objective of the underlying agent is to choose the optimal timing that minimizes the expected total environmental damage (in monetary terms) caused by the pollutant.

Suppose that the underlying agent i emits the pollutant by π_t^i, which is the flow of the pollutant over time, and that its stock Y_t^i is governed by

$$dY_t^i=(\pi_t^i-\delta Y_t^i)dt, \quad Y_0^i=y^i, \tag{1}$$

where $\delta>0$ is the rate of natural decay of the stock of the pollutant and y^i constitutes the initial stock of the agents. The initial stock of the pollutant such as greenhouse gases does not depend on the attributes of the agents. Let $B^i(X_t, Y_t^i)$ denote agent i's damage (or negative benefit) function associated with the stock of the pollutant Y_t^i. It is assumed that

$$B^i(X_t, Y_t^i)=a^i X_t Y_t^i,$$

where $a^i>0$ is a constant parameter that denotes each agent's degree of subjective sensitivity to pollution. X_t is a state variable that reflects changes in tastes and technologies. In other words, we can regard X_t as the assessment or the social cost arising from a unit amount of the pollutant stock. For example, consider the problem of CO_2. Since the market for tradable emission permits has recently expanded and evolved, it is now possible to evaluate the social cost arising from a unit amount of the pollutant (CO_2) stock, using the price of the tradable emission permit among the countries that join in the market. In fact, the variable X_t can be replaced by the price of the tradable emission permit in the actual implementation of this model (See Insley (2003)). X_t is assumed to be governed by

$$dX_t=\mu X_t dt+\sigma X_t dW_t, \quad X_0=x,$$

where $\mu\in\mathbb{R}$ and $\sigma\in\mathbb{R}^{++}$ are constants. x is an initial value of X_t. $(W_t)_{t\geq 0}$ is a standard Brownian motion process. $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$: *i. e.*, $\mathcal{F}_t=\sigma(W_s, s\leq t)$. Note that X_t is independent of the characteristics of agents.

For simplicity, we assume that π_t^i remains constant at its initial level π_0^i until

agent i implements the policy. When agent i implements the policy, π_0^i reduces to π_1^i with $0 \leq \pi_1^i < \pi_0^i$. Thus, equation (1) becomes

$$dY_t^i = \begin{cases} (\pi_0^i - \delta Y_t^i)dt, & 0 \leq t < \tau^i, \\ (\pi_1^i - \delta Y_t^i)dt, & \tau^i \leq t < \infty, \end{cases} \tag{2}$$

where $\tau^i \in \mathbb{T}$ is the (in general, unknown) time that corresponds to the adoption of the policy by agent i and $\mathbb{T}$ is the class of all values of implementation time relative to $(\mathcal{F}_t)_{t \geq 0}$. Furthermore, in this paper, we assume $\mu < r$, where $r^i \in \mathbb{R}_{++}$ is the rate of time preference. This is the normal assumption with regard to the real options approach. Dixit and Pindyck (1994) discuss this issue in detail. Let K^i be the constant cost of implementing the environmental policy for agent i. Therefore, the agent's problem is with regard to choosing $\tau^i \in \mathbb{T}$ to minimize the net present value function (see Øksendal (1998)),

$$V^i(x, y^i) = \inf_{\tau^i \in \mathbb{T}} \mathbb{E}\left[\int_0^\infty e^{-r^i t} B^i(X_t, Y_t^i)dt + e^{-r^i \tau^i} K^i\right],$$

subject to equation (2).

3. The Optimal Timing of Environmental Policy

In this section, we discuss the optimal timing of an environmental policy in the developing and the developed country by extending the model of Ohyama and Tsujimura (2006). We study the case wherein there are two competing agents: $i = L, F$. Suppose that both agents have experienced damage from pollutants and wish to determine the time period within which they should implement an environmental policy to reduce pollutant emissions. Agent L represents a developed country and agent F represents a developing country. We assume that if one of the agents (say L) implements the environmental policy, it affects the environment of the other agent (say F). That is, agent F experiences environmental improvement because agent L implements an environmental policy. Barrieu and Chesney (2001) term this effect the *induced effect*. Let $\pi_{N_i N_j}^i$ be the emission flow of agent i, where, for $\xi \in \{i, j\}$, we specify

$$N_\xi = \begin{cases} 0, \text{ if agent } \xi \text{ has not implemented the policy,} \\ 1, \text{ if agent } \xi \text{ has implemented the policy.} \end{cases}$$

We assume that the total emission reduction does not depend on the magnitude of the external effect. For simplicity, we also assume that the emission structure of each agent is as follows:

$$(\pi_{00}^{L}-\pi_{11}^{L})=(\pi_{00}^{L}-\pi_{10}^{L})+\underbrace{(\pi_{00}^{F}-\pi_{01}^{F})}_{\text{external effect}}=(\pi_{01}^{L}-\pi_{11}^{L})+\underbrace{(\pi_{10}^{F}-\pi_{11}^{F})}_{\text{external effect}}\ ;$$
$$(\pi_{00}^{F}-\pi_{11}^{F})=(\pi_{00}^{F}-\pi_{10}^{F})+\underbrace{(\pi_{00}^{L}-\pi_{01}^{L})}_{\text{external effect}}=(\pi_{01}^{F}-\pi_{11}^{F})+\underbrace{(\pi_{10}^{L}-\pi_{11}^{L})}_{\text{external effect}}. \qquad \text{(AS.1)}$$

In practice, the magnitude of the external effect depends on the type of pollutant under consideration. In our model, the type of underlying pollutant is reflected by the strength of the external effect (see Ohyama and Tsujimura (2008)). For example, suppose that the agents emit greenhouse gases such as CO_2 or methane and that an increase in greenhouse gases is assumed to raise the earth's average temperature. Under these conditions, it is likely that the following equation holds: $\frac{1}{2}(\pi_{00}^{L}-\pi_{11}^{L}) = (\pi_{00}^{L}-\pi_{10}^{L}) = (\pi_{01}^{L}-\pi_{11}^{L})$, $\frac{1}{2}(\pi_{00}^{F}-\pi_{11}^{F}) = (\pi_{00}^{F}-\pi_{10}^{F})=(\pi_{01}^{F}-\pi_{11}^{F})$ (namely, complete external effect). Suppose that the agents emit either SO2 or NOx as primary causes of acid rain. It seems reasonable that the following relationships hold: $\frac{1}{2}(\pi_{00}^{L}-\pi_{11}^{L}) < (\pi_{00}^{L}-\pi_{10}^{L}) = (\pi_{01}^{L}-\pi_{11}^{L})$, $\frac{1}{2}(\pi_{00}^{F}-\pi_{11}^{F}) < (\pi_{00}^{F}-\pi_{10}^{F}) = (\pi_{01}^{F}-\pi_{11}^{F})$ (namely, a partial external effect). Furthermore, we assume that implementation costs in the developing and developed countries are K^F and K^L, respectively.

Because of the external effect, the implementation of environmental policies takes longer when there are two competing agents than when agents cooperate (see Proposition 3.2 of Ohyama and Tsujimura (2006)). This has crucial implications for international policies and regulations. It means that if the implementation of environmental policies is noncooperative, environmental problems are not resolved optimally.

In the context of the amount of direct aid, we need to consider the following two cases. In the first case, the developed country implements the environmental policy first; whereas in the second case, the developing country is the first to implement the policy. The order in which countries implement their policies is influenced by the parameters r^i, a^i, K^i, k, b, and $\pi_1^i-\pi_0^i$, and by the scale of the external effect, where k is the amount of direct aid and b reflects the technology level in the developed country. When the developed country provides direct aid of k to the developing country, it also simultaneously provides technological skills. Therefore, the direct aid of k reduces the implementation cost for the developing country by the amount bk. It should be noted that although the developed country adjusts the parameter k in order to achieve the maximum value function for the developed country, we fix the parameter k in this section.

The parameter a^i denotes each country's degree of sensitivity to pollution, which plays an important role in this paper. We assume that $a^L \geq a^F$, because developed countries are generally more sensitive to pollution than developing countries.

3.1 The developing country's problem: the case wherein the developed country is the first to implement its policy

We examine the case wherein the developed country implements its policy first. The developing country experiences the external effect of the developed country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^F=\begin{cases} d\check{Y}_t^F=(\pi_{00}^F-\delta\check{Y}_t^F)dt, & 0\leq t<\tau_1^L, \\ d\hat{Y}_t^F=(\pi_{01}^F-\delta\hat{Y}_t^F)dt, & \tau_1^L\leq t<\tau_1^F, \\ d\bar{Y}_t^F=(\pi_{11}^F-\delta\bar{Y}_t^F)dt, & \tau_1^F\leq t<\infty, \end{cases}$$

where τ_1^L denotes the (generally unknown) time period wherein the developed country L
adopts the policy. This is defined by

$$\tau_1^L=\inf\{t>0\ ;\ X_t\geq x_1^L\}$$

for given x_1^L. τ_1^F represents the (generally unknown) time period by which the developing country F adopts the policy. Note that we assume that $\tau_1^L<\tau_1^F$. The developing country's problem is given by

$$V_1^F(x,\ y^F)=\inf_{\tau_1^F\in\mathbb{T}}\mathbb{E}\Big[\int_0^{\tau_1^L}e^{-r^Ft}B^F(X_t,\ \check{Y}_t^F)dt+\int_{\tau_1^L}^{\tau_1^F}e^{-r^Ft}B^F(X_t,\ \hat{Y}_t^F)dt$$
$$+e^{-r^F\tau_1^F}(K^F-bk)+\int_{\tau_1^F}^{\infty}e^{-r^Ft}B^F(X_t,\ \bar{Y}_t^F)dt\Big].$$

Note that the cost of implementing the environmental policy is reduced by the amount of direct aid, which is accompanied by the transfer of technological skills.

By using the strong Markov property and the recursive property of conditional expectations, we obtain the developing country's value function as follows:

$$V_1^F(x,\ y^F)=\begin{cases} \dfrac{a^Fx\pi_{00}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^Fxy^F}{r^F-\mu+\delta}+\left(\dfrac{x}{x_1^L}\right)^{\beta^L}\left[-\dfrac{a^Fx_1^L(\pi_{00}^F-\pi_{01}^F)}{(r^F-\mu)(r^F-\mu+\delta)}\right] \\ -\left(\dfrac{x}{x_1^F}\right)^{\beta^F}\left[\dfrac{a^Fx_1^F(\pi_{01}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}-(K^F-bk)\right], & x<x_1^L, \\ \dfrac{a^Fx\pi_{01}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^Fxy^F}{r^F-\mu+\delta}-\left(\dfrac{x}{x_1^F}\right)^{\beta^F}\left[\dfrac{a^Fx_1^F(\pi_{01}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}-(K^F-bk)\right], & x_1^L\leq x\leq x_1^F, \\ \dfrac{a^Fx\pi_{11}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^Fxy^F}{r^F-\mu+\delta}+(K^F-bk), & x\geq x_1^F. \end{cases} \tag{3}$$

where β^i is the positive root of the following characteristic equation:

$\frac{1}{2}\sigma^2\beta^i(\beta^i-1)+\mu\beta^i-r^i=0$. For $x_1^L<x<x_1^F$, the sum of the first and second terms on the right-hand side represents the present value of the environmental damage. In this region, because the leader has implemented its environmental policy and because there is an external effect, the follower's emission flow of the pollutant decreases from π_{00}^F to π_{01}^F. The remaining terms represent the value of having the option of implementing the environmental policy in the future. When $x\geq x_1^F$, the sum of the first and second terms continues to represent the cost of environmental damage. Note that the emission flow of the pollutant decreases from π_{01}^F to π_{11}^F because of the implementation of the policy by the follower. The last term represents the policy implementation cost.

The optimal stopping time is

$$\tau_1^F=\inf\{t>0\ ;\ X_t\geq x_1^F\},$$

where the threshold x_1^F is determined by the value-matching and smooth-pasting conditions as follows:

$$x_1^F=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk)}{a^F(\pi_{01}^F-\pi_{11}^F)}\right). \tag{4}$$

By observing the above equation, it is important to check whether the threshold depends on the amount of direct aid k. However, recall that we treat k as a constant or fixed in this section.

3.2 The developing country's problem: the case wherein the developing country is the first to implement its policy

We examine the case wherein the developing country implements its policy first. The developing country experiences the external effect of the developed country's policy implementation. Consequently, the dynamics of the pollutant stock given by equation (2), become

$$dY_t^F=\begin{cases} d\check{Y}_t^F=(\pi_{00}^F-\delta\check{Y}_t^F)dt, & 0\leq t<\tau_2^F,\\ d\hat{Y}_t^F=(\pi_{10}^F-\delta\hat{Y}_t^F)dt, & \tau_2^F\leq t<\tau_2^L,\\ d\bar{Y}_t^F=(\pi_{11}^F-\delta\bar{Y}_t^F)dt, & \tau_2^L\leq t<\infty,\end{cases}$$

where τ_2^L is the (generally unknown) time period by which the developed country L adopts the policy. This is defined by

$$\tau_2^L=\inf\{t>0\ ;\ X_t\geq x_2^L\}$$

for given x_2^L. τ_2^F represents the time period by which the developing country implements its policy. Note that we assume that $\tau_2^F<\tau_2^L$. The developing

country's problem is solved in the same manner as that for the developed country in Subsection 3.1. We obtain the developing country's value function as follows:

$$V_2^F(x, y^F)=\begin{cases} \dfrac{a^F x\pi_{00}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^F xy^F}{r^F-\mu+\delta}+\left(\dfrac{x}{x_2^L}\right)^{\beta^L}\left[-\dfrac{a^F x_2^L(\pi_{10}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}\right] & \\ \quad -\left(\dfrac{x}{x_2^F}\right)^{\beta^F}\left[\dfrac{a^F x_2^F(\pi_{00}^F-\pi_{10}^F)}{(r^F-\mu)(r^F-\mu+\delta)}-(K^F-bk)\right], & x<x_2^F, \\ \dfrac{a^F x\pi_{10}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^F xy^F}{r^F-\mu+\delta}+K^F-bk-\left(\dfrac{x}{x_2^L}\right)^{\beta^L}\left[\dfrac{a^F x_2^L(\pi_{01}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}\right], & x_2^F\leq x\leq x_2^L, \\ \dfrac{a^F x\pi_{11}^F}{(r^F-\mu)(r^F-\mu+\delta)}+\dfrac{a^F xy^F}{r^F-\mu+\delta}+K^F-bk, & x\geq x_2^L. \end{cases} \tag{5}$$

The optimal stopping time is

$$\tau_2^F=\inf\{t>0\ ;\ X_t\geq x_2^F\},$$

where the threshold x_2^F is determined by the value-matching and smooth-pasting conditions:

$$x_2^F=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk)}{a^F(\pi_{00}^F-\pi_{10}^F)}\right). \tag{6}$$

***Lemma* 3.1.** *Suppose that assumption* (*AS*.1) *holds. Moreover, we assume that*

$$\pi_{00}^F-\pi_{10}^F=\pi_{01}^F-\pi_{11}^F. \tag{AS.2}$$

Then, we obtain

$$x_1^F=x_2^F.$$

Proof. We obtain this result from equations (4) and (6). □

This lemma implies that the timing of the developing country's policy implementation does not depend on the country that implements the environmental policy first. In the remainder of this paper, we assume that (AS.2) holds and we let x^F and τ^F denote $x_1^F=x_2^F$ and $\tau_1^F=\tau_2^F$, respectively. Notice that x^F depends on k, so that the developed country can maximize the value function by adjusting the amount of aid k. In the next section, we determine the optimal level of aid.

3.3 The developed country's problem: the case in which the developed country implements its policy first

We examine the case wherein the developed country implements its policy first. The developed country experiences the external effect of the developing country's policy implementation. The dynamics of the pollutant stock, given by equation

(2), become

$$dY_t^L=\begin{cases} d\check{Y}_t^L=(\pi_{00}^L-\delta\check{Y}_t^L)dt, & 0\le t<\tau_1^L,\\ d\hat{Y}_t^L=(\pi_{10}^L-\delta\hat{Y}_t^L)dt, & \tau_1^L\le t<\tau^F,\\ d\tilde{Y}_t^L=(\pi_{11}^L-\delta\tilde{Y}_t^L)dt, & \tau^F\le t<\infty,\end{cases}$$

where τ^F is the (generally unknown) time period by which the developing country F adopts the policy, as we derived in Subsection 3.1. The developed country's problem is given by

$$V_1^L(x,y^L)=\inf_{\tau_1^L\in\mathbb{T}}\mathbb{E}\left[\int_0^{\tau_1^L}e^{-rt}B^L(X_t,\check{Y}_t^L)dt+\int_{\tau_1^L}^{\tau^F}e^{-rt}B^L(X_t,\hat{Y}_t^L)dt+e^{-r\tau_1^L}K^L+\int_{\tau^F}^{\infty}e^{-rt}B^L(X_t,\tilde{Y}_t^L)dt+e^{-r\tau^F}k\right].$$

Note that the cost of implementing the environmental policy is reduced by the amount of direct aid k and not by bk. This is because the parameter b denotes the difference in the technological skill levels between the developing and the developed country. The developed country does not concern itself with the cost of providing technological skills to the developing country.

By using the strong Markov property and the recursive property of conditional expectations, we obtain the developed country's value function as follows:

$$V_1^L(x,y^L)=\begin{cases} \dfrac{a^Lx\pi_{00}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^Lxy^L}{r^L-\mu+\delta}+\left(\dfrac{x}{x_1^L}\right)^{\beta^L}\left[-\dfrac{a^Lx_1^L(\pi_{00}^L-\pi_{10}^L)}{(r^L-\mu)(r^L-\mu+\delta)}+K^L\right] & \\ \quad\underbrace{-\left(\dfrac{x}{x_1^F}\right)^{\beta^F}\left[\dfrac{a^Lx_1^F(\pi_{10}^L-\pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)}-k\right]}_{G(k)}, & x<x_1^L,\\ \dfrac{a^Lx\pi_{10}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^Lxy^L}{r^L-\mu+\delta}+K^L\underbrace{-\left(\dfrac{x}{x_1^F}\right)^{\beta^F}\left[\dfrac{a^Lx_1^F(\pi_{10}^L-\pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)}-k\right]}_{G(k)}, & x_1^L\le x\le x_1^F\\ \dfrac{a^Lx\pi_{11}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^Lxy^L}{r^L-\mu+\delta}+(K^L+k), & x\ge x_1^F.\end{cases} \tag{7}$$

The optimal stopping time is

$$\tau_1^L=\inf\{t>0\ ;\ X_t\ge x_1^L\},$$

where the threshold x_1^L is determined by the value-matching and smooth-pasting conditions:

$$x_1^L=\frac{\beta^L}{\beta^L-1}\left(\frac{(r^L-\mu)(r^L-\mu+\delta)K^L}{a^L(\pi_{00}^L-\pi_{10}^L)}\right). \tag{8}$$

Note that the trigger x_1^L does not depend on the amount of direct aid k. It seems

that offering direct aid is not relevant to the policy implementation of the developed country. However, with regard to strategic behavior, we can not conclude that this observation is alway true. We examine the strategic problems in section 4. To explain the terms in the value function that can be controlled by the developed country, we define the following equation:

$$G(k) := \left(\frac{x}{x_1^F}\right)^{\beta^F}\left[\frac{a^L x_1^F(\pi_{10}^L-\pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)}-k\right]. \tag{9}$$

Note that because the developed country controls the timing of the developing country's implementation by providing direct aid, the developed country maximizes its value function accordingly.

3.4 The developed country's problem: the case in which the developing country implements its policy first

We examine the case wherein the developing country implements its policy first. The developed country experiences the external effect of the developing country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^L=\begin{cases} d\check{Y}_t^L=(\pi_{00}^L-\delta\check{Y}_t^L)dt, & 0\le t<\tau^F,\\ d\hat{Y}_t^L=(\pi_{01}^L-\delta\hat{Y}_t^L)dt, & \tau^F\le t<\tau_2^L,\\ d\tilde{Y}_t^L=(\pi_{11}^L-\delta\tilde{Y}_t^L)dt, & \tau_2^L\le t<\infty,\end{cases}$$

where τ^F is the (generally unknown) time period in which the developing country F adopts the policy derived in Subsection 3.1. As in Subsection 3.3, we obtain the developed country's problem as follows:

$$V_2^L(x,y^L)=\begin{cases} \dfrac{a^L x\pi_{00}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^L xy^L}{r^L-\mu+\delta}+\left(\dfrac{x}{x_2^L}\right)^{\beta^L}\left[-\dfrac{a^L x_2^L(\pi_{01}^L-\pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)}+K^L\right] & \\ \quad \underbrace{-\left(\dfrac{x}{x_2^F}\right)^{\beta^F}\left[\dfrac{a^L x_2^F(\pi_{00}^L-\pi_{01}^L)}{(r^L-\mu)(r^L-\mu+\delta)}-k\right]}_{F(k)}, & x<x_2^F,\\ \dfrac{a^L x\pi_{01}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^L xy^L}{r^L-\mu+\delta}+k+\left(\dfrac{x}{x_2^L}\right)^{\beta^L}\left[-\dfrac{a^L x_2^L(\pi_{01}^L-\pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)}+K^L\right], & x_2^F\le x\le x_2^L,\\ \dfrac{a^L x\pi_{11}^L}{(r^L-\mu)(r^L-\mu+\delta)}+\dfrac{a^L xy^L}{r^L-\mu+\delta}+(K^L+k), & x\ge x_2^L.\end{cases} \tag{10}$$

The optimal stopping time is

$$\tau_2^L=\inf\{t>0\ ;\ X_t\ge x_2^L\},$$

where the threshold x_2^L is determined by the value-matching and smooth-pasting

conditions:

$$x_2^L = \frac{\beta^L}{\beta^L - 1}\left(\frac{(r^L - \mu)(r^L - \mu + \delta)K^L}{a^L(\pi_{01}^L - \pi_{11}^L)}\right). \tag{11}$$

Note that the trigger x_2^L does not depend on the amount of direct aid k. To clarify the terms in the value function that are controlled by the developed country, we define the following equation:

$$F(k) := \left(\frac{x}{x_2^F}\right)^{\beta^F}\left[\frac{a^L x_2^F(\pi_{00}^L - \pi_{01}^L)}{(r^L - \mu)(r^L - \mu + \delta)} - k\right]. \tag{12}$$

***Lemma* 3.2.** *Suppose that assumptions* (*AS*.1) *and* (AS.2) *hold. Moreover, we assume that*

$$\pi_{00}^L - \pi_{10}^L = \pi_{01}^L - \pi_{11}^L. \tag{AS.3}$$

Then, we obtain

$$x_1^L = x_2^L.$$

Furthermore, this implies

$$G(k) = F(k).$$

Proof. These results follow from equations (8) and (11), and equations (9) and (12). □

This lemma implies that the timing of the developing country's implementation is not influenced by the country particular that implements the environmental policy first. However, this does significantly affect the shape of the value function of the developed country. In the remainder of this paper, we assume that (AS.3) holds, and we let x^L, τ^L and $H(k)$ denote $x_1^L = x_2^L$, $\tau_1^L = \tau_2^L$ and $G(k) = F(k)$, respectively.

4. Optimizing Direct Aid

In this section, we assume that the developed country chooses the optimal direct aid that maximizes its value function. First, we consider the maximization problem without incorporating strategic interactions:

$$\sup_{0 \le k \le \frac{K^F}{b}} H(k).$$

Because

$$\frac{dH(k)}{dk}=\begin{cases}>0, & k<\dfrac{QK^F}{1+bQ},\\ <0, & \dfrac{QK^F}{a+bQ}<k\leq\dfrac{K^F}{b},\end{cases}$$

we obtain the following optimal solution:

$$k^*=\begin{cases}\dfrac{QK^F}{1+bQ}, & Q>0,\\ 0, & -\dfrac{1}{\beta^F b}\leq Q\leq 0,\end{cases}$$

where

$$\begin{aligned}Q&=\frac{a^L(\pi^L_{10}-\pi^L_{11})\,(r^F-\mu)(r^F-\mu+\delta)}{a^F(\pi^F_{01}-\pi^F_{11})\,(r^L-\mu)(r^L-\mu+\delta)}-\frac{1}{\beta^F b}\\ &=\frac{a^L(\pi^L_{00}-\pi^L_{01})\,(r^F-\mu)(r^F-\mu+\delta)}{a^F(\pi^F_{00}-\pi^F_{10})\,(r^L-\mu)(r^L-\mu+\delta)}-\frac{1}{\beta^F b}.\end{aligned}$$

If there is no external effect, i. e., if $\pi^L_{10}-\pi^L_{11}=\pi^L_{00}-\pi^L_{01}=0$, then $Q=-\dfrac{1}{\beta^F b}$. Without external effect, the behavior of the developing country is independent of the behavior of the developed country. Consequently, the developed country does not provide any direct aid to the developing country.

In this expression, $x^F(k^*)$ is the critical value for the developing country that provides direct aid of k^*:

$$\begin{aligned}x^F(k^*)&=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk^*)}{a^F(\pi^F_{01}-\pi^F_{11})}\right)\\ &=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk^*)}{a^F(\pi^F_{00}-\pi^F_{10})}\right).\end{aligned}$$

$x^F(0)$ is the critical value for the developing country that receives no direct aid:

$$x^F(0)=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)K^F}{a^F(\pi^F_{01}-\pi^F_{11})}\right)=\frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)K^F}{a^F(\pi^F_{00}-\pi^F_{10})}\right).$$

We now consider the optimal level of direct aid for a developed country that knows that it will implement the environmental policy before the other country does. In this case, the developed country minimizes its value function, equation (7), by changing $k\in[0,\ K^F/b]$. When the value of x is large enough to satisfy

$x \geq x^F(0)$, $x \geq x^F(k)$ holds for all $k \in [0, K^F/b]$. This implies that the developed country chooses the optimal level $k^{**}=0$ to minimize the third line of equation (7). On the other hand, when the value of x is small enough to satisfy $x < x^F(0)$, the direct aid k that is consistent with $x > x^F(k)$ is not optimal because of the value matching condition at $x = x^F(k)$ in equation (7). Hence, in order to minimize the first and second lines of equation (7), the developed country solves

$$\sup_{\{k \in [0, K^F/b] \mid x^F(k) \geq x\}} H(k) \tag{13}$$

The optimal solution for k^{**} of equation (13) is expressed as

$$k^{**} = \begin{cases} k^*, & 0 < x \leq x^F(k^*), \\ \dfrac{K^F}{b} - \dfrac{x(\beta_1 - 1)}{\beta_1} \dfrac{a^F(\pi_{01}^F - \pi_{11}^F)}{b(r^F - \mu)(r^F - \mu + \delta)}, & x^F(k^*) < x < x^F(0). \end{cases}$$

Knowing that it will implement its environmental policy after the other country does so, the developed country minimizes its value function (10) by changing $k \in [0, K^F/b]$. The optimal level of direct aid is k^{**} because $H(k) = G(k) = F(k)$. Thus, the optimal level of direct aid is independent of whether the developed country implements its environmental policy first.

Thus, in a nonstrategic situation, the developed country gives k^{**} and implements its environmental policy at $\tau^L = \inf\{t > 0 ; X_t \geq x^L\}$, whereas the developing country implements its environmental policy at $\tau^F = \inf\{t > 0 ; X_t \geq x^{F(k)}\}$. This result applies to a strategic situation wherein the developed country may change the threshold and the amount of direct aid, in order to reverse the order of the implementation of the policies, by comparing V_1^L with V_2^L (or V_1^F with V_2^F in a strategic situation for a developing country).

First, we consider the developing country's strategic decision. We assume that $x^L \leq x^F(k^{**})$, and we let $\tilde{V}_1^F(x, y^F, z)$ and $\tilde{V}_1^F(x, y^F, z)$ denote the right-hand side of equation (3) and equation (5), with x^F having been replaced by z, respectively. Then, for $\tilde{x}^F < x^L$, on the basis of assumptions (AS. 2) and (AS. 3), we have

$$V_1^F(x, y^F) = \tilde{V}_1^F(x, y^F, x^F(k^{**})) \leq \tilde{V}_1^F(x, y^F, x^L) = \tilde{V}_2^F(x, y^F, x^L) \leq \tilde{V}_2^F(x, y^F, \tilde{x}^F) \tag{14}$$

Equation (14) shows that the developing country does not change the order of $x^L \leq x^F(k^{**})$ to $\tilde{x}^F < x^L$. Similarly, we can show that the developing country does not reverse the order when $x^L > x^F(k^{**})$.

Next, we investigate whether the developed country changes the order. We assume that $x^L \leq x^F(k^{**})$ and let $\tilde{V}_1^L(x, y^L, w, k)$ and $\tilde{V}_1^L(x, y^L, w, k)$ denote the right-hand side of equation (7) and equation (10), with x^L having been replaced by w, respectively. Consequently, on the basis of assumptions (AS.2) and (AS.3) for values of $(\tilde{x}^L, k)$ that satisfy $x^F(k) < \tilde{x}^L$, we have

$$V_1^L(x, y^L, x^L, k^{**}) \leq \tilde{V}_1^L(x, y^L, x^F(k), k) = \tilde{V}_2^L(x, y^L, x^F(k), k) \leq \tilde{V}_2^L(x, y^L, \tilde{x}^L, k) \quad (15)$$

Equation (15) shows that the developed country does not change the order of $x^L \leq x^F(k^{**})$ to $x^F(k) < \tilde{x}^L$. We obtain similar results when $x^L > x^F(k^{**})$. Therefore, strategic countries make the same decisions as myopic countries that ignore the possibility of changing the order in which the policies are implemented, when assumptions (AS.1) and (AS.2) hold.

Proposition* 4.1.** *If* $x \leq x^F(k^*)$, ***the optimal level of direct aid is k^*. ***If*** $x^F(k^*) < x < x^F(0)$, ***the developed country adjusts its optimal level of direct aid such that the critical value of the developing country is equal to*** x, ***as is indicated by*** $k = \frac{K^F}{b} - \frac{x(\beta_1 - 1)}{\beta_1} \frac{a^F(\pi_{01}^F - \pi_{11}^F)}{b(r^F - \mu)(r^F - \mu + \delta)}$. ***If*** $x^F(0) \leq x$, ***the developed country does not offer any direct aid to the developing country.***

What happens if assumptions (AS.1) and (AS.2) are not made? The developing country could attempt to change the order in which policies are implemented. Consequently, expecting the developing country to renege, the developed country is likely to reconsider the timing and the amount of direct aid. The developing country will consider acting strategically. These actions will be repeated until both countries no longer have an incentive to change their decisions; i. e., the strategies will generate an equilibrium. However, it is generally difficult to see how such an equilibrium could be reached.

The next proposition concerns the order of policy implementation.

***Proposition* 4.2.** *If*

$$K^L\left(\frac{\beta^L}{\beta^L - 1}\right)\frac{(r^L - \mu)(r^L - \mu + \delta)}{a^L(\pi_{00}^L - \pi_{10}^L)}\left\{\left(1 - \frac{1}{\beta^F}\right) + \frac{a^L b(\pi_{10}^L - \pi_{11}^L)(r^F - \mu)(r^F - \mu + \delta)}{a^F(\pi_{01}^F - \pi_{11}^F)(r^L - \mu)(r^L - \mu + \delta)}\right\} > K^F\left(\frac{\beta^F}{\beta^F - 1}\right)\frac{(r^F - \mu)(r^F - \mu + \delta)}{a^F(\pi_{01}^F - \pi_{11}^F)},$$

then $x^F(k^*) < x^L$.

Proof. This result is obtained by comparing the value of x^L from Lemma 2.2 with the value of $x^F(k^*)$ implied by Definition 3.1.

This inequality depends on the parameters a^i, b, K^i, and r^i and the external effect. According to the proposition, if this inequality holds and $x < x^L$, the developing country implements its environmental policy first.

5. The Social Welfare

In this section, we compare the social welfare level implied by this model with those obtained using two other models: namely the cooperative two-agent model and the noncooperative twoagent model in which there is no direct aid. In the cooperative model, the two agents act as if there is a central commanding agent, as both agents share the same value function. We highlight two possible scenarios that are realistic and that generate interesting insights with regard to improving social welfare. Under Scenario 1, the developed country is the first to implement

its policy in all three models. Under Scenario 2, the developing country first implements its policy in the cooperative model and the noncooperative model that does not account for direct aid. We assume that social welfare in this model is represented by the sum of the value functions of the developing and the developed country. In addition to (AS.1)–(AS.3), for the sake of simplicity, we assume that

$$b=1, \quad r^F=r^L. \tag{AS.4}$$

5. 1 Scenario 1

In this scenario, the developed country is the first to implement its policy in regard to all three models.

5. 1. 1 The cooperative model

When the developing country and the developed country cooperate, social welfare is as follows:

$$SW^C(x, y^L, y^F)=\begin{cases} \dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x_c^F}\right)^\beta\left[\dfrac{a^L x_c^F(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^F\right] \\ \quad -\left(\dfrac{x}{x_c^L}\right)^\beta\left[\dfrac{a^L x_c^L(\pi_{00}^L-\pi_{10}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^L(\pi_{00}^F-\pi_{01}^F)}{\rho_1\rho_2}-K^L\right], & x<x_c^L, \\ \dfrac{a^L x\pi_{10}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{01}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L \\ \quad -\left(\dfrac{x}{x_c^F}\right)^\beta\left[\dfrac{a^L x_c^F(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^F\right], & x_c^L\leq x\leq x_c^F, \\ \dfrac{a^L x\pi_{11}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{11}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L+K^F, & x\geq x_c^F, \end{cases} \tag{16}$$

where $\rho_1=r-\mu$ and $\rho_2=r+\delta-\mu$. Given the value-matching and smooth-pasting conditions to eq.(12), we obtain

$$x_c^F=\frac{\beta}{\beta-1}\frac{K^F}{\dfrac{a^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}+\dfrac{a^L(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}},$$

$$x_c^L=\frac{\beta}{\beta-1}\frac{K^L}{\dfrac{a^L(\pi_{00}^L-\pi_{10}^L)}{\rho_1\rho_2}+\dfrac{a^F(\pi_{00}^F-\pi_{01}^F)}{\rho_1\rho_2}}.$$

Because there is no external effect in this cooperative model, social welfare is optimal. In other words, these thresholds achieve the optimal social welfare. However, the cooperative model is unrealistic because it means that all countries

agree with the central commanding agent.

5. 1. 2 The noncooperative model with direct aid

When direct aid is introduced into the noncooperative model, social welfare is as follows:

$$SW^{Aid}(x, y^L, y^F)=\begin{cases} \dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x^F(k^*)}\right)^{\beta}\left[\dfrac{a^L x^F(k^*)(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k^*)(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}-(K^F-k^*)-k^*\right] \\ \quad -\left(\dfrac{x}{x^L}\right)^{\beta}\left[\dfrac{a^L x^L(\pi_{00}^L-\pi_{10}^L)}{\rho_1\rho_2}+\dfrac{a^F x^L(\pi_{00}^F-\pi_{01}^F)}{\rho_1\rho_2}-K^L\right], \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x<x^L, \\ \dfrac{a^L x\pi_{10}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{01}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L \\ \quad -\left(\dfrac{x}{x^F(k^*)}\right)^{\beta}\left[a^L\dfrac{x^F(k^*)(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k^*)(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}-(K^F-k^*)-k^*\right], \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x^L\le x\le x^F(k^*), \\ \dfrac{a^L x\pi_{10}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{01}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L \\ \quad -\left(\dfrac{x}{x^F(k)}\right)^{\beta}\left[\dfrac{a^L x^F(k)(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k)(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}-(K^F-k)-k\right], \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x^F(k^*)\le x\le x^F(0), \\ \dfrac{a^L x\pi_{11}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{11}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L+K^F, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x\ge x^F(0). \end{cases} \tag{17}$$

In the region of $x^F(k^*)\le x\le x^F(0)$, the value of k is consistent with Proposition 4.1.

Therefore,

$$k=K^F-\frac{x(\beta-1)}{\beta}\frac{a^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2}.$$

The developed country can appropriately adjust its direct aid with respect to x, so that the developing country implements it's policy immediately. Therefore, SW^{Aid} is equal to SW^C in the region of $x^F(k^*)\le x\le x^F(0)$.

$$x^F(k)=\begin{cases}\underbrace{\dfrac{\beta}{\beta-1}\left(\dfrac{\rho_1\rho_2(K^F-k^*)}{a^F(\pi^F_{01}-\pi^F_{11})}\right)}_{=x^F(k^*)}, & x<x^F(k^*),\\ \dfrac{\beta}{\beta-1}\left(\dfrac{\rho_1\rho_2(K^F-k)}{a^F(\pi^F_{01}-\pi^F_{11})}\right), & x^F(k^*)\le x\le x^F(0),\\ \underbrace{\dfrac{\beta}{\beta-1}\left(\dfrac{\rho_1\rho_2K^F}{a^F(\pi^F_{01}-\pi^F_{11})}\right)}_{=x^F(0)} & x\ge x^F(0).\end{cases}$$

Note that the critical value in the developing country depends on the level of direct aid. It implies that direct aid plays an important role in improving the social welfare. Regardless of the amount of direct aid, the critical value in the developed country is given by

$$x^L=\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2K^L}{a^L(\pi^L_{00}-\pi^L_{10})}\right).$$

5. 1. 3 The noncooperative model

Social welfare in the noncooperative model is as follows:

$$SW^N(x,y^L,y^F)=\begin{cases}\dfrac{a^Lx\pi^L_{00}}{\rho_1\rho_2}+\dfrac{a^Lxy^L}{\rho_2}+\dfrac{a^Fx\pi^F_{00}}{\rho_1\rho_2}+\dfrac{a^Fxy^F}{\rho_2}\\ \quad-\left(\dfrac{x}{x^F(0)}\right)^\beta\left[\dfrac{a^Lx^F(0)(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2}+\dfrac{a^Fx^F(0)(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2}-K^F\right]\\ \quad-\left(\dfrac{x}{x^L}\right)^\beta\left[\dfrac{a^Lx^L(\pi^L_{00}-\pi^L_{10})}{\rho_1\rho_2}+\dfrac{a^Fx^L(\pi^F_{00}-\pi^F_{01})}{\rho_1\rho_2}-K^L\right], & x<x^L,\\ \dfrac{a^Lx\pi^L_{10}}{\rho_1\rho_2}+\dfrac{a^Lxy^L}{\rho_2}+\dfrac{a^Fx\pi^F_{01}}{\rho_1\rho_2}+\dfrac{a^Fxy^F}{\rho_2}+K^L\\ \quad-\left(\dfrac{x}{x^F(0)}\right)^\beta\left[\dfrac{a^Lx^F(0)(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2}+\dfrac{a^Fx^F(0)(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2}-K^F\right], & x^L\le x\le x^F(0),\\ \dfrac{a^Lx\pi^L_{11}}{\rho_1\rho_2}+\dfrac{a^Lxy^L}{\rho_2}+\dfrac{a^Fx\pi^F_{11}}{\rho_1\rho_2}+\dfrac{a^Fxy^F}{\rho_2}+K^L+K^F, & x\ge x^F(0),\end{cases}\tag{18}$$

where

$$x^F(0)=\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2K^F}{a^F(\pi^F_{01}-\pi^F_{11})}\right)=\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2K^F}{a^F(\pi^F_{00}-\pi^F_{10})}\right),$$
$$x^L=\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2K^L}{a^L(\pi^L_{00}-\pi^L_{10})}\right).$$

5. 1. 4 Comparative results and numerical examples

We obtain the following proposition by comparing the social welfare levels implied by the three models: i. e., the cooperative model, the noncooperative model, and the noncooperative model in which there is direct aid.

***Proposition* 5.1.** *Suppose that* (AS.1)–(AS.3) *hold in this scenario. Then, for* $0<x<\infty$, *we obtain* $SW^C \leq SW^{Aid} \leq SW^N$. *Furthermore,* $SW^C = SW^{Aid}$ *is achieved in the region of* $x^F(0) > x \geq x^F(k^*)$. $SW^C = SW^{Aid} = SW^N$ *is achieved in the region of* $x \geq x^F(0)$.

Proof. The results follow from equations (16), (17), and (18). □

In this subsection, we present numerical examples. Table 1 presents the base-case parameter values used in this scenario. We calculate the thresholds, x_c^L, x^L, $x^F(k^*)$, and $x^F(0)$ (see Table 2). These results imply that if the current value of x_0 is \$30 per ton, only the developing country does not implement the environmental policy in regard to the noncooperative model. The developing country in the noncooperative model should adopt the environmental policy when x reaches \$40 per ton.

Figure 1 shows the optimal amount of direct aid with respect to the variable x. In addition, as we explained in the previous section, for $x \in [0, x^F(k^*)]$, the amount

Table 1 Parameter values

Parameter		Value
a^F	Developing country's sensitivity to pollution	1
a^L	Developing country's sensitivity to pollution	3
r	Discount rate	0. 04
δ	Natural decay rate	0. 02
μ	Expected percentage rate of growth of X_t	0
σ	Volatility parameters of X_t	0. 2
K^F	Cost of implementing the policy for the developing country	\$2. 5 billion
K^L	Cost of implementing the policy for the developed country	\$3 billion
π_{00}^F	Initial pollutant flow in the developing country	\$800, 000 tons/ yr
π_{01}^F	Pollutant flow in the developing country	\$700, 000 tons/ yr
π_{11}^F	Pollutant flow in the developing country	\$400, 000 tons/ yr
π_{00}^L	Initial pollutant flow in the developed country	\$500, 000 tons/ yr
π_{10}^L	Pollutant flow in the developed country	\$200, 000 tons/ yr
π_{11}^L	Pollutant flow in the developing country	\$100, 000 tons/ yr

Table 2: Values of some thresholds

x_c^L	x^L	x_c^F	$x^F(k^*)$	$x^F(0)$
14. 4	16	20	26. 667	40

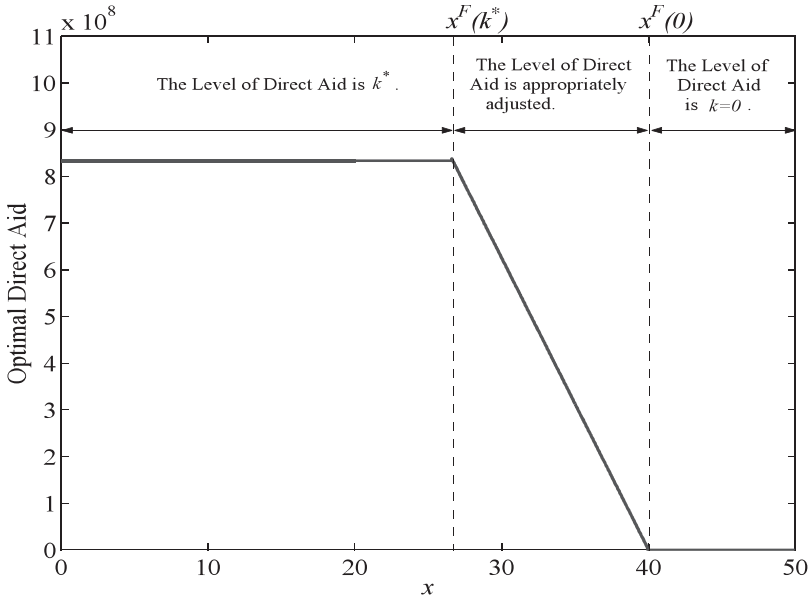

Figure 1 Amount of direct aid.

of optimal direct aid is k^*, i. e., \$0.83 billion. Notice that this amount of direct aid is paid to the developing country just when the developing country implements the policy. It does not imply that the developed country has to pay for the amount of aid immediately. If the value of x is small enough, then the developing country does not adopt the policy. Consequently, the developed country does not need to pay for it for the time being. That is, since r discounts the amount of optimal aid in Figure 1, the present value of optimal aid for a sufficiently small x is nearly equal to 0. This amount of direct aid is refered to as a kind of commitment aid. For $x \in (x^F(k^*), x^F(0)]$, the optimal direct aid is appropriately adjusted. In this region, the developing country does not voluntarily implement the environmental policy. However, the developed country can improve its own value function by the immediate implementation of the developing country, even if the developed country pays the adjusted aid to the developing country. Therefore, the developed country is ready to provide the aid to the developing country, such that the developing country implements the policy immediately. For $x \in (x^F(0), \infty)$, the developing country implements the policy immediately without receiving direct aid. Therefore, the developed country need not provide direct aid to the developing country.

Figure 2 has certain interesting implications. For $0 < x < x^F(k^*)$, $SW^C < SW^{Aid}$ holds for two reasons. First, there is the full external effect of the developed country because the critical value of the developed country is not

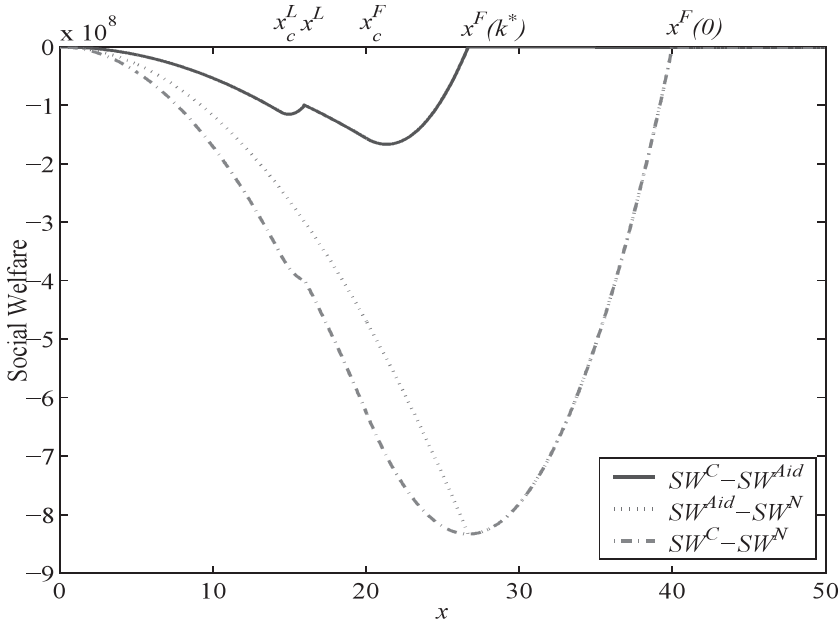

Figure 2 Each social welfare comparison.

influenced by direct aid. This implies that the developed country does not implement the environmental policy with the timing implied by the cooperative model. Therefore, around x_c^L, the gap between SW^C and SW^{Aid} increases. Second, the developed country does not voluntarily give unlimited direct aid to the developing country. This means that $x_c^F = x^F(k^*)$ does not always hold. Therefore, around the point x_c^F, the gap of $SW^C - SW^{Aid}$ increases. For $x^F(k^*) < x < x^F(0)$, the developed country appropriately adjusts the level of direct aid with respect to x. Figure 2 makes it clear that for $x^F(k^*) < x$, $SW^C = SW^{Aid}$ holds. This is because the developing country is induced to implement the policy by means of direct aid. That is, direct aid has the role to divide the benefit into two agents. Thus, the social welfare level in the noncooperative model that incorporates direct aid is optimal for this region. Note that social welfare in the noncooperative model that incorporates direct aid is higher than the level implied by the noncooperative model.

$SW^{Aid} - SW^N$ appears in Figure 3. The line in the upper part of the figure represents the benefit obtained by the developed country from providing direct aid. The gap between the two lines indicates that some of the benefit is shared with the developing country. The point is that the developed country benefits as much as does the developing country.

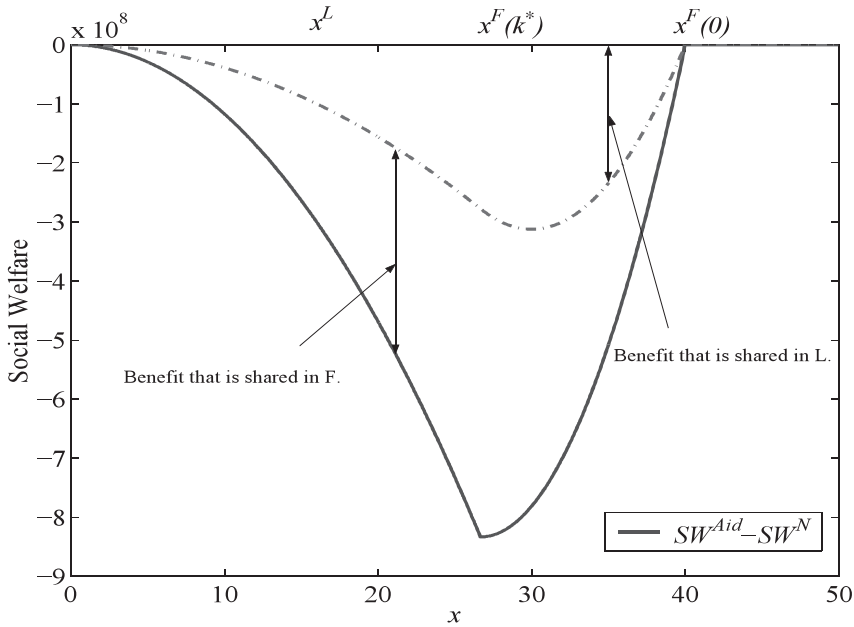

Figure 3 The difference between the social welfare levels SW^{Aid} and SW^{N}. (In addition, this indicates the benefit that is shared between the countries)

5. 2 Scenario 2

In this scenario, the developing country first implements its policy in the cooperative model and next, the noncooperative model that incorporates direct aid. In practice, there are certain probabilities for the developed country to induce the developing country to implement the policy before the developed country. For example, when the targetted emission reduction of the developing country is much higher than that of the developed country, despite costs being the same, the developed country benefits from external effect well. Therefore, the developed country offers the aid actively. As a result, the order to implement the policy may change.

5. 2. 1 The cooperative model

When the developing country and the developed country cooperate, social welfare is given as follows:

$$SW^C(x, y^L, y^F)=\begin{cases} \dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x_c^F}\right)^\beta\left[a^L\dfrac{x_c^F(\pi_{00}^L-\pi_{01}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^F(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}-K^F\right] \\ \quad -\left(\dfrac{x}{x_c^L}\right)^\beta\left[\dfrac{a^L x_c^L(\pi_{01}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^L(\pi_{10}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^L\right], & x<x_c^L \\ \dfrac{a^L x\pi_{01}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{10}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L \\ \quad -\left(\dfrac{x}{x_c^L}\right)^\beta\left[\dfrac{a^L x_c^L(\pi_{01}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x_c^L(\pi_{10}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^L\right], & x<x_c^L, \\ \dfrac{a^L x\pi_{11}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{11}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L+K^F, & x\geq x_c^F. \end{cases} \tag{19}$$

The value-matching and smooth-pasting conditions imply

$$x_c^F=\frac{\beta}{\beta-1}\frac{K^F}{\dfrac{a^F(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}+\dfrac{a^L(\pi_{00}^L-\pi_{01}^L)}{\rho_1\rho_2}},$$

$$x_c^L=\frac{\beta}{\beta-1}\frac{K^L}{\dfrac{a^L(\pi_{01}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F(\pi_{10}^F-\pi_{11}^F)}{\rho_1\rho_2}}.$$

Because there is no external effect in the cooperative model, social welfare is optimal. However, the cooperative model is unrealistic because it requires that there be an agreement between all countries.

5. 2. 2 The noncooperative model with direct aid

When direct aid is introduced to the noncooperative model, social welfare is as follows:

$$SW^{Aid}(x, y^L, y^F)=\begin{cases}\dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x^F(k^*)}\right)^{\beta}\left[\dfrac{a^L x^F(k^*)(\pi_{00}^L-\pi_{01}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k^*)(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}-(K^F-k^*)-k^*\right] \\ \quad -\left(\dfrac{x}{x^L}\right)^{\beta}\left[\dfrac{a^L x^L(\pi_{01}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x^L(\pi_{10}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^L\right], \\ \qquad\qquad x<x^F(k^*), \\ \dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x^F(k)}\right)^{\beta}\left[\dfrac{a^L x^F(k)(\pi_{00}^L-\pi_{01}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k)(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}-(K^F-k)-k\right] \\ \quad -\left(\dfrac{x}{x^L}\right)^{\beta}\left[\dfrac{a^L x^L(\pi_{01}^L-\pi_{11}^L)}{\rho_1\rho_2}+\dfrac{a^F x^L(\pi_{10}^F-\pi_{11}^F)}{\rho_1\rho_2}-K^L\right], \\ \qquad\qquad x^F(k^*)\le x<x^L, \\ \dfrac{a^L x\pi_{00}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{00}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L \\ \quad -\left(\dfrac{x}{x^F(k)}\right)^{\beta}\left[\dfrac{a^L x^F(k)(\pi_{00}^L-\pi_{01}^L)}{\rho_1\rho_2}+\dfrac{a^F x^F(k)(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}-(K^F-k)-k\right], \\ \qquad\qquad x^L\le x<x^F(0), \\ \dfrac{a^L x\pi_{11}^L}{\rho_1\rho_2}+\dfrac{a^L xy^L}{\rho_2}+\dfrac{a^F x\pi_{11}^F}{\rho_1\rho_2}+\dfrac{a^F xy^F}{\rho_2}+K^L+K^F, \\ \qquad\qquad x\ge x^F(0).\end{cases} \tag{20}$$

In the region of $x^F(k^*)\le x\le x^F(0)$, the value of k is consistent with Proposition 4.1. Therefore,

$$k=K^F-\frac{x(\beta-1)}{\beta}\frac{a^F(\pi_{00}^F-\pi_{10}^F)}{\rho_1\rho_2}$$

As in Section 5.1, SW^{Aid} is equal to SW^C in the region of $x^L\le x\le x^F(0)$. However, even if the developed country can adjust its direct aid appropriately, SW^{Aid} is not equal to SW^C in the region of $x^F(k^*)\le x\le x^L$.

$$x^F(k)=\begin{cases}\underbrace{\dfrac{\beta}{\beta-1}\left(\dfrac{a\rho_1\rho_2(K^F-k^*)}{\pi_{00}^F-\pi_{10}^F}\right)}_{=x^F(k^*)}, & x<x^F(k^*), \\ \dfrac{\beta}{\beta-1}\left(\dfrac{a\rho_1\rho_2(K^F-k)}{\pi_{00}^F-\pi_{10}^F}\right), & x^F(k^*)\le x\le x^F(0), \\ \underbrace{\dfrac{\beta}{\beta-1}\left(\dfrac{a\rho_1\rho_2K^F}{\pi_{00}^F-\pi_{10}^F}\right)}_{=x^F(0)} & x\ge x^F(0).\end{cases}$$

Irrespective of the amount of direct aid, the critical value in the developed country is given by

$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{\pi^L_{00}-\pi^L_{10}}\right)$$

5. 2. 3 The noncooperative model

Social welfare in the noncooperative model is as follows:

$$SW^N(x, y^L, y^F) = \begin{cases} \dfrac{a^L x\pi^L_{00}}{\rho_1\rho_2} + \dfrac{a^L xy^L}{\rho_2} + \dfrac{a^F x\pi^F_{00}}{\rho_1\rho_2} + \dfrac{a^F xy^F}{\rho_2} \\ \quad -\left(\dfrac{x}{x^F(0)}\right)^\beta \left[\dfrac{a^L x^F(0)(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2} + \dfrac{a^F x^F(0)(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2} - K^F\right] \\ \quad -\left(\dfrac{x}{x^L}\right)^\beta \left[\dfrac{a^L x^L(\pi^L_{00}-\pi^L_{10})}{\rho_1\rho_2} + \dfrac{a^F x^L(\pi^F_{00}-\pi^F_{01})}{\rho_1\rho_2} - K^L\right], & x < x^L, \\ \dfrac{a^L x\pi^L_{10}}{\rho_1\rho_2} + \dfrac{a^L xy^L}{\rho_2} + \dfrac{a^F x\pi^F_{01}}{\rho_1\rho_2} + \dfrac{a^F xy^F}{\rho_2} + K^L \\ \quad -\left(\dfrac{x}{x^F(0)}\right)^\beta \left[\dfrac{a^L x^F(0)(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2} + \dfrac{a^F x^F(0)(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2} - K^F\right], \\ & x^L \le x \le x^F(0), \\ \dfrac{a^L x\pi^L_{11}}{\rho_1\rho_2} + \dfrac{a^L xy^L}{\rho_2} + \dfrac{a^F x\pi^F_{11}}{\rho_1\rho_2} + \dfrac{a^F xy^F}{\rho_2} + K^L + K^F, & x \ge x^F(0), \end{cases} \tag{21}$$

where

$$x^F(0) = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi^F_{01}-\pi^F_{11})}\right) = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi^F_{00}-\pi^F_{10})}\right),$$
$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{a^L(\pi^L_{00}-\pi^L_{10})}\right).$$

5. 2. 4 Comparative results and Numerical example

We obtain the following proposition by comparing the levels of social welfare implied by the three models, the cooperative model, the noncooperative model, and the noncooperative model that incorporates direct aid.

***Proposition* 5.2.** *Suppose that* (AS.1)–(AS.3) *hold in this scenario. Then, for* $0 < x < \infty$, *we obtain* $SW^C \le SW^{Aid} \le SW^N$. $SW^C = SW^{Aid} = SW^N$ *is achieved in the region of* $x \ge x^F(0)$. $SW^C = SW^{Aid}$ *is achieved in the region of* $x^F(0) > x \ge x^L$.

Proof. The results follow from equations (19), (20), and (21). □

As in Section 5.1, SW^{Aid} is equal to SW^C in the region of $x^L \le x \le x^F(0)$. However, even if the developed country can adjust its direct aid appropriately, SW^{Aid} is not equal to SW^C in the region of $x^F(k^*) \le x \le x^L$. The reason is that the timing of the developed country's policy implementation is not influenced by direct

aid. Therefore, the developed country's external effect cannot be dealt with by using direct aid.

As in Subsection 5.1.4, we present numerical examples, but by using different parameter values to those used in Table 1 (see Table 3). We calculate the thresholds, x_c^L, x^L, $x^F(k^*)$, and $x^F(0)$ (see Table 4). These imply that if the current value of x_0 is \$23 per ton, the countries that do not implement the environmental policy are the developed and developing countries in the noncooperative model, the developed and developing countries in the noncooperative model, and the developing country in the noncooperative model that incorporates direct aid. When x reaches \$24 per ton, the countries that implement the environmental policy tend to be the developed and developing countries in regard to both the noncooperative model and the developed and developing countries in the noncooperative model that incorporates direct aid.

Figure 4 shows the optimal amount of direct aid with respect to the variable x. In addition, for $x \in [0, x^F(k^*)]$, the optimal amount of direct aid is k^*, i. e., \$1.25 billion. For $x \in (x^F(k^*), x^F(0)]$, the optimal level of direct aid is appropriately adjusted. For $x \in (x^F(0), \infty)$, the developed country need not provide direct aid to the developing country.

Figure 5 illustrates $SW^C - SW^{Aid}$. For $0 < x < x^L$, $SW^C < SW^{Aid}$ holds for two reasons. First, the developed country does not voluntarily give unlimited direct aid to the developing country. This implies that $x_c^F = x^F(k^*)$ does not necessarily hold. Therefore, around the point of x_c^F, the difference between SW^C and SW^{Aid} increases. Second, there is a full external effect from the developed country because the critical value of the developed country is not influenced by

Table 3 Parameter values.

Parameter Value		Value
π_{00}^F	Initial pollutant flow in the developing country	\$1, 000, 000 tons/ yr
π_{10}^F	Pollutant flow in the developing country	\$600, 000 tons/ yr
π_{11}^F	Pollutant flow in the developing country	\$500, 000 tons/ yr
π_{00}^L	Initial pollutant flow in the developed country	\$500, 000 tons/ yr
π_{01}^L	Pollutant flow in the developed country	\$300, 000 tons/ yr
π_{11}^L	Pollutant flow in the developing country	\$100, 000 tons/ yr
π_{10}^L	Pollutant flow in the developed country	\$300, 000 tons/ yr
π_{01}^F	Pollutant flow in the developing country	\$900, 000 tons/ yr

Table 4 Thresholds.

x_c^F	$x^F(k^*)$	x_c^L	x^L	$x^F(0)$
12	15	20. 571	24	30

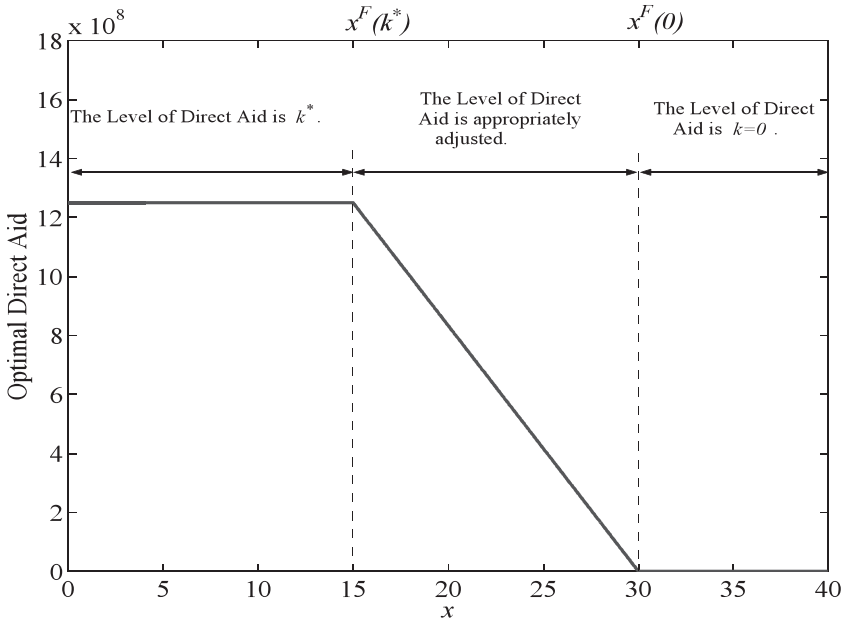

Figure 4 Optimal amount of direct aid.

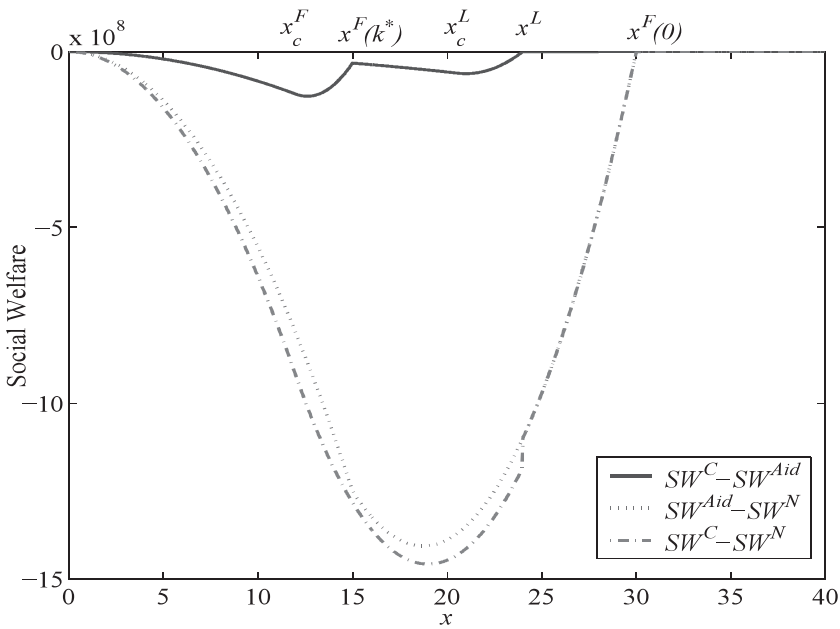

Figure 5 Comparing social welfare.

direct aid. This implies that the developed country does not implement the environmental policy with the timing implied by the cooperative model. Therefore, around x_c^L, the gap $SW^C - SW^{Aid}$ increases. For $x^F(k^*) < x < x^F(0)$, the developed country appropriately adjusts the level of direct aid with respect to x. However, for this region, Scenario 2 differs from Scenario 1 because there is a social welfare loss. Figure 5 shows that, for $x^L < x$, $SW^C = SW^{Aid}$ holds. Thus, the level of social welfare implied by the noncooperative model that incorporates direct aid is optimal for welfare in this region. If the developed country implements the policy on x_c^L (e. g., by using an emission trading system or by setting an environmental tax for a group of developed countries), then, as in Scenario 1, for $x^F(k^*) < x$, $SW^C = SW^{Aid}$ holds. However, social welfare in the noncooperative model that incorporates direct aid is higher than in the noncooperative model.

Figure 6 illustrates $SW^{Aid} - SW^N$. The line in the upper part of the figure denotes the benefit acquired by the developed country by providing direct aid. The gap between the two lines indicates that the benefit is shared with the developing country.

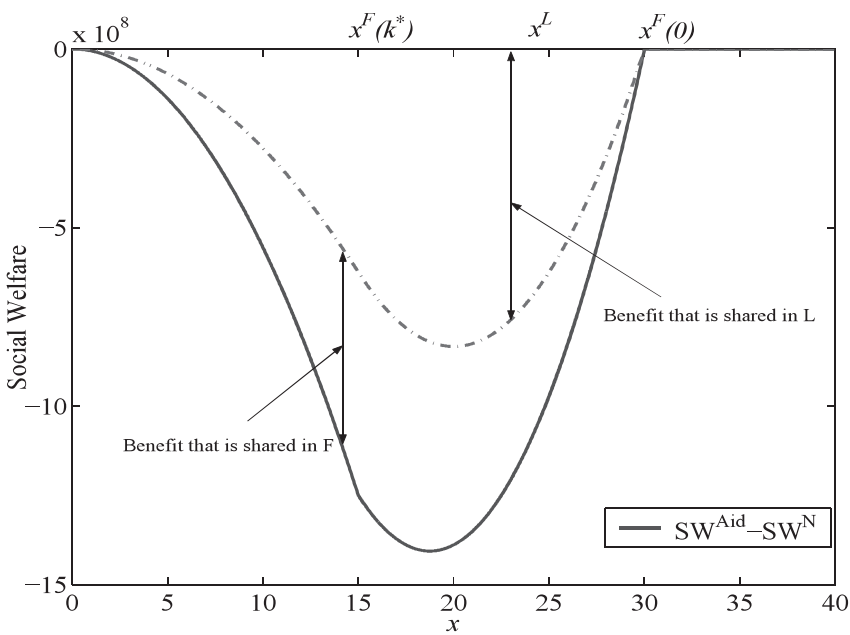

Figure 6 The difference between the social welfare levels SW^{Aid} and SW^N (in addition, this shows that the benefit is shared between the countries).

6. Conclusion

Our analysis has yielded two important findings. First, we derived the closed solution for the optimal level direct aid. This optimal level of aid depends on the stochastic variable that governs the effect of pollution stocks on the social costs of environmental damage over time. Second, the social welfare level in a noncooperative model that incorporates direct aid differs from the level implied by a cooperative model. There are two reasons for this. First, the external effect of the developed country cannot be reduced by providing direct aid. Second, the developed country does not voluntarily give unlimited direct aid to the developing country. However, social welfare is higher in a noncooperative model that incorporates direct aid than in a noncooperative model that does not. From the standpoint of the developed country, the financial-aid framework is self-motivated, rather than forced. Therefore, we suggest that direct aid is a viable solution to the problems caused by the external effects of environmental policy.

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