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Quantification of Parameter Uncertainty in Distributed Rainfall-Runoff Modeling

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Synopsis

In general, hydrological models have several (or a lot of) parameters that cannot be directly measured, which only are inferred by calibration procedure against a historical input-output data record. While the applications of automatic parameter estimation techniques have received considerable attention over the last decades, such classical methods have received criticism for their lack of rigor in handling with uncertainty in the parameter estimates. This work addresses the calibration of the distributed rainfall-runoff model KsEdgeFC2D, the quantification of parameter uncertainty and its effect on the prediction of streamflow for Kamishiiba catchment (211km^2). In this study, to analyze the propagation of parameter uncertainty into prediction, we employ the Shuffled Complex Evolution Metropolis (SCEM-UA) global optimization algorithm. Moreover, we compare SCEM-UA derived optimal parameter values to those estimated using deterministic SCE-UA method with three different objective functions to account for the structural stability of KsEdgFC2D model and to demonstrate the capability of the SCEM-UA algorithm to efficiently evolve to parameter posterior distribution.

Keywords: Automatic parameter estimation, Parameter uncertainty, SCEM-UA

1. Introduction

Rainfall-runoff process in heterogeneous real world is commonly simplified and represented by various hydrological models (Wagener *et al*., 2004). These models are conversion and simplification of reality, thus no matter how spatially sophisticated and accurate they may be those models only represent aspects of conceptualization or empiricism of modelers (or hydrologists). Accordingly, their outputs are as reliable as hypothesis, structure of models, and quantity and quality of input data, and parameter estimates (Gupta *et al*., 1999; Muletha and Nicklow, 2005). Model parameters, in general, are classified into two (Kuczera and Franks, 2002). First is physical parameter which is physically measurable property of the watershed and the other is conceptual parameter that can only be inferred by some matching process, often called calibration, between the simulated responses and the observed ones (Sorooshian and Gupta, 1995). From above definition, virtually even so-called ‘physically-based’ models, be they lumped or distributed, would be regarded as being conceptual.

The conceptual parameter values are adjusted between each run of the model, either manually by modelers or automatically by some computer-based optimization algorithm and the corresponding observation until some optimal parameter set has been found. Especially, the application of automatic parameter optimization algorithms has received considerable attention and has improved over the last decades (e.g., Sorooshian and Dracup, 1980; Duan *et al*., 1992; Vrugt *et al*., 2003). Until the early 1990’s, the available automated optimization techniques practically were not able to find the global optimal values in a prescribed objective function because of the presence of multiple local optima, parameter interaction, and discontinuous and non-convex response surface defined by the selected objective function. These insights in the response surface led to the development of global optimization algorithm, namely Shuffled Complex Evolution (SCE-UA) optimization algorithm (Duan *et al*., 1992; 1993; 1994). However, while remarkable progress has been made in the development and application of automated calibration procedures, such methods have not been free from criticism for their insufficiency handling with parameter uncertainty (Vrugt *et al*., 2003; 2005).

As many hydrologists pointed out in their literatures, estimates of hydrologic model parameters are generally error-prone because used data (e.g., rainfall, streamflow) during calibration contain measurement errors and model structural imperfection never explicitly represents the system or fits the observed data (Schaake, 2003). Manifold studies have been conducted to quantify or assess parameter uncertainty and its propagation into subsequent prediction results. For example, multinormal approximations (Kuczera and Mroczkowski, 1998), simple uniform random sampling over the feasible parameter space (Uhlenbrook *et al*., 1999), and Markov Cahin Monte Carlo (MCMC) methods (Kuczera and Parent, 1998; Vrugt *et al*., 2003) are developed to analyze parameter uncertainty. In comparison with traditional statistical theory based on first-order approximations and multinormal distributions, MCMC methods have become increasingly popular as one of the general purpose approximation methods for complex inference, search and optimization problem. Shuffled Complex Evolution Metropolis (SCEM-UA) algorithm is an effective and efficient evolutionary MCMC sampler which has enhanced search capability. This method, modification of the SCE-UA algorithm by developed by Duan *et al*., (1992), operates by merging the strengths of the Metropolis algorithm, controlled random search, competitive evolution, and complex shuffling. The stochastic characteristic of Metropolis scheme makes it possible to avoid the tendency of the SCE-UA algorithm to collapse to the global minimum on response surface. This powerful algorithm can provide not only optimal parameter set but also its underlying posterior distribution within a single optimization run (Vrugt *et al*., 2003).

In this study, we employ the SCEM-UA method to estimate confidence limits for model parameters. Especially, the aim of this work is to explore the capacity of SCEM-UA to identify the posterior parameter distribution for the complex KsEdgeFC2D model applied to the Kamishiiba catchment (211 ㎢), and to evaluate the effect of parameter uncertainty on hydrograph simulation. Moreover, we compare SCEM-UA derived parameter values to those estimated using deterministic SCE-UA method with three different objective functions, Simple Least Square estimation criterion (SLS), Heteroscedastic Maximum Likelihood Estimator (HMLE) and Modified Index of Agreement (MIA) to account for the structural stability of KsEdgFC2D model and to demonstrate the capability of the SCEM-UA algorithm to efficiently evolve to parameter posterior distribution. This paper is organized as follows. In section 2, we describe the distributed hydrologic model, KsEdgeFC2D. Section 3 presents details on the SCEM-UA algorithm used to infer the posterior model parameter distributions as compared with SCE-UA. In section 4 and 5 the result of case study is illustrated. Finally, we summarize methodology and analyzed results in section 6.

2. Applied Hydrologic Model

KsEdgeFC2D is a conceptual distributed hydrologic model developed by Ichikawa *et al*. (2001) including discharge-stage relationship with saturated-unsaturated flow (Tachikawa *et al*., 2004). The model solves the one-dimensional kinematic wave equation with the discharge-stage equation using the Lax-Wendroff finite difference scheme according to the flow direction map (see Figure 1). All geomorphologic information is extracted from 250m based DEM.

The model assumes that a permeable soil layer covers the hillslope as illustrated in Figure 1(d). The soil layer consists of a capillary layer in which unsaturated flow occurs and a non-capillary layer where saturated flow occurs. According to this mechanism, if the depth of water is higher than the soil depth, then overland flow occurs. The discharge-stage relationship is expressed by equation (2) corresponding to water levels (see Figure 1(d)) defined as:

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r(t) \tag{1}$$

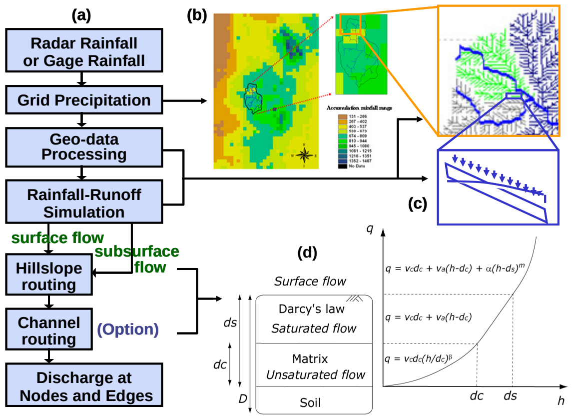

Fig. 1 Schematic representation of KsEdgeFC2D (a) Modular structure of KsEdgeFC2D (b) Distributed grid rainfall data (c) Slope and channel components extracted from DEM (d) Model structure for the hillslope soil layer and discharge -stage relationship

$$q = \begin{cases} v_c d_c (h/d_c)^{\beta}, & 0 \le h \le d_c \\ v_c d_c + v_a (h - d_c), & d_c \le h \le d_s \\ v_c d_c + v_a (h - d_c) + \alpha (h - d_s)^m, & d_s \le h \end{cases} \tag{2}$$

Flow rate of each slope segment are calculated by above governing equations combined with the continuity equation like equation (1). where, $v_c = k_c i$; $v_a = k_a i$; $k_c = k_a / \beta$; $\alpha = \sqrt{i}/n$; i is slope gradient, k_c is saturated hydraulic conductivity of the capillary soil layer, k_a is hydraulic conductivity of the non-capillary soil layer, n is roughness coefficient, d_c is the depth of the capillary soil layer and d_s is soil depth. Detailed explanations of model structure appear in Tachikawa *et al*., (2004).

3. Methodology

Estimation of the reliability of model output is important to hydrologic engineering and water resources planning since model output reliability can be used to assess model verification and the selection of suitable models (Melching *et al.*, 1990). Beven (1989) has reviewed the limitations of the current generation of distributed, physically-based models of watershed hydrology and has suggested that a possible way forward requires a realistic estimation of prediction uncertainty. Only recently methods for realistic assessment of parameter uncertainty have began to appear in the literature. These include, as stated in the introduction part, the classical use of first-order approximations to parameter uncertainty, evaluation of likelihood ratios, and parametric bootsrapping or MCMC methods. SCEM-UA is one of the popular and efficient tools to evaluate parameter uncertainty and predictability of hydrological models (Feyen *et al.*, 2006).

In this study, we examine the applicability or capability of SCEM-UA to assess hydrologic model parameter uncertainty and to search for best-fit parameter values through comparing with those optimized by deterministic automated global search algorithm, SCE-UA. Moreover, the influence of objective functions (e.g., SLS, HMLE, MIA) on optimal parameter and model performance is demonstrated to account for the model structural uncertainty (Lee *et al.*, 2007). In the following subsections 3.1, 3.2, and 3.3 we describe the SCE-UA based on three different objective functions to search for global optimal parameter set and the SCEM-UA based on MCMC sampler using Metropolis Hasting (MH) strategy respectively.

3.1 Shuffled Complex Evolution (SCE-UA) Algorithm

The SCE-UA is used to identify the best fitted parameter set, which is a single-objective global optimization method designed to handle with high -parameter dimensionality encountered in calibration of a nonlinear hydrologic simulation models. This evolutionary approach method has been performed by a number of researchers on a variety of models with outstanding positive results and has proved to be an efficient, powerful method for the automatic optimization. SCE algorithm is basically synthesized by following three concepts: (1) combination of a simplex procedure with the concepts of controlled random search approaches; (2) competitive evolution; and (3) complex shuffling. The integration of these steps above mentioned makes the SCE method effective and flexible.

3.2 Applied Objective Functions for SCE_UA Calibration

The aim of computer-based automatic calibration is to find the values of the model parameters that minimize or maximize the numerical value of the objective functions. In general, the most commonly utilized objective functions in hydrological modeling are variations of the Simple Least Squares (SLS) function defined as:

$$SLS = \sum_{t=1}^{N} (q_t^{obs} - q_t(\theta))^2 \quad (3)$$

where q_t^{obs} is observed stream flow value at time t; $q_t(\theta)$ is model simulated stream flow value at time t using parameter set θ; N is the number of flow values available. SLS has a feature that large discharge is emphasized due to squared errors while low flows are neglected, thus the parameter set fitting around peak discharge value is likely to obtain.

A newly proposed measure is the Modified Index of Agreement (MIA) to reduce the influence of the squared term during high flows as putting a weight on flow values. This objective function is calculated as:

$$MIA = 1 - \frac{\sum_{t=1}^{n} \left(\frac{q_t^{obs} - q_t(\theta)}{q_t^{obs}} \right)^2}{\sum_{t=1}^{n} \left(\frac{\left| q_t(\theta) - q_t^{mean} \right| + \left| q_t^{obs} - q_t^{mean} \right|}{q_t^{mean}} \right)^2} \quad (4)$$

where q_t^{mean} is mean value of observed time series.

Sorooshian and Dracup (1980) proposed a different objective function to consider entire behavior of hydrograph, the Heteroscedastic Maximum Likelihood Estimator (HMLE), which enables to estimate the most likely weights through the use of the maximum estimation theory. This new measure can eliminate some of the subjectivity involved in the selection of transformation and / or a weighting scheme by handling heteroscedastic error, so that it yields a more balanced performance over the entire hydrograph. It is calculated as:

$$\min_{\theta,\lambda} \quad HMLE = \frac{1}{N} \frac{\sum_{t=1}^{N} w_t \varepsilon_t}{\prod_{t=1}^{N} w_t} \quad (5)$$

where $\varepsilon_t = q_t^{obs} - q_t(\theta)$ is the model residual at time t; w_t is the weight assigned to time t, computed as $w_t = f_t^{2(\lambda-1)}$; $f_t = q_t^{true}$ is the expected true flow at time t; λ is the transformation parameter which stabilizes the variance.

3.3 Shuffled Complex Evolution Metropolis (SCEM-UA) Algorithm

The Markov Chain Monte Carlo sampler, entitled in Shuffled Complex Evolution Metropolis algorithm is well suited for the practical assessment of parameter uncertainty in hydrological models. This sampler incorporate effective characteristics of SCE-UA such as controlled random search, competitive evolution, and complex shuffling with the strengths of the Metropolis-Hasting algorithm to evolve a population of sampled points to an estimation of the stationary posterior distribution of the parameters. Two big revisions prevent the convergence to a small attraction region, instead, and facilitate convergence to a stationary posterior target distribution of parameters. The first modification is replacement of the downhill simplex method by a Metropolis annealing covariance-based offspring approach, thereby avoiding a deterministic drift toward a single mode. Second, the SCEM-UA does not divide the complex into subcomplexes during the generation of the offspring and uses a different replacement way, to terminate occupations having lower posterior density on the parameter space. General steps for SCEM-UA implementation are outlined below.

3.3.1 SCEM-UA process

1. Initialize the process including the selection of the population size s and the number of complexes q.
2. Generate s samples $\{\theta_1, \theta_2, \cdots, \theta_s\}$ from the predefined prior distribution and compute the posterior density $\{p(\theta_1|y), p(\theta_2|y), \cdots, p(\theta_s|y)\}$ of each point, where y = known observed data.
3. Sort the points in order of decreasing posterior density and store them in array $D[1:s, 1:n+1]$, where n is the number of parameters.
4. Initialize the starting points of the parallel sequences $S^1, S^2, \cdots, S^q$, such that S^k is $D[k, 1:n+1]$, where $k = 1, 2, \cdots, q$.
5. Partition the s points of D into q complexes $C^1, C^2, \cdots, C^q$, each containing m points.
6. Evolve each sequence. Evolve each of the parallel sequences according to the Sequence Evolution Metropolis (SEM) algorithm described in 3.3.2.

7. Unpack all complexes *C* back into *D* and rank the points in order of decreasing posterior density.
8. Check the convergence criteria, Gelman and Rubin (GR) statistic. If GR are satisfied, stop; otherwise, return to step 5.

3.3.2 SEM algorithm, Key component of the SCEM-UA

This algorithm produces new candidate points in each of the parallel sequences, S^k by generating draws from an adaptive proposal distribution by using the information induced in the *m* samples of C^k.

1. Compute the mean μ^k and covariance structure Σ^k of the parameters of C^k. Sort the *m* point in complex C^k in order of decreasing posterior density.
2. Compute α^k, the ratio of the mean posterior density of the *m* points in C^k to the mean posterior density of the last *m* generated points in S^k.
3. If α^k is smaller than a predefined likelihood ratio, *T*, generate a candidate point, $\theta^{(t+1)}$, by using a multinormal distribution centered on the last draw, $\theta^{(t)}$, of the sequence S^k, and covariance structure $c_n^2\Sigma^k$, where, c_n is a jumprate. Otherwise, go to step 4.
4. Generate offspring $\theta^{(t+1)}$, by using multinormal distribution with mean μ^k and covariance structure $c_n^2\Sigma^k$.
5. Compute the posterior density, $p\left(\theta^{(t+1)}\middle|y\right)$, of $\theta^{(t+1)}$. If $\theta^{(t+1)}$ is outside the feasible space, set $p\left(\theta^{(t+1)}\middle|y\right)$ to zero.
6. Compute the ratio $r = p\left(\theta^{(t+1)}\middle|y\right)/\,p\left(\theta^{(t)}\middle|y\right)$ and randomly sample a uniform label *Z* over interval [0, 1].
7. If *Z* is smaller than or equal to *r*, accept the new candidate point. Otherwise, reject the candidate point and then remain at the current position in the sequence, it means $\theta^{(t+1)} = \theta^{(t)}$.
8. Add the point $\theta^{(t+1)}$ to the sequence S^k.
9. If the candidate point is proper, replace the best member of C^k with $\theta^{(t+1)}$, and go to 10; otherwise replace the worst member of C^k with $\theta^{(t+1)}$.
10. Compute AR^k, the ratio of the posterior density of the best to the posterior density of the worst member of C^k.
11. Repeat the steps 1-10 until predefined number of iteration before complexes are shuffled.

A detailed description and explanation of the SCEM-UA method appears in Vrugt *et al.* (2003).

4. Case Study

The study site is the Kamishiiba catchment, upstream area of Kamishiiba dam, which lies within Kyushu region in Japan and covers an area of 211km^2. The topography of this area is hilly with the elevation varying from 400m to 1700m. Land use type is forest, thereby enabling to regard the drainage as a typical Japanese mountainous area. In the model, the catchment is based on 250 by 250m grid blocks. Observed discharge data converted from water level of dam inflow having 10min temporal resolution are available for Kamishiiba dam gauging station (Kyushu Electric Power Co., Inc). Distributed rainfall (1km by 1km, see figure 1(b)) is gauged on Eshiroyama radar station. Table 1 shows historical short-term events for this study.

Table 1 Historical storm events

objective of Event	*Date of occurrence*	*Rainfall duration (hr)*	*Peak discharge (m^3/s)*
Calibration	1999 / 09 / 22	96	590
Validation	1997 / 09 / 15	84	1203

Although recent hydrologic models are based on physics to a certain extent, some processes are only represented in a lumped conceptual way (Feyen *et al.*, 2006). Consequently, even some parameters of physically based model lack physical basis and cannot be directly observed from field surveying. In the KsEdgeFC2D, there remain five parameters that need to be estimated by calibration procedure against observed streamflow data. Parameters to be optimized and a range of prior distribution are illustrated in Table 2.

The SCEM-UA algorithm contains two algorithmic parameters as well as the SCE-UA method that need to be specified manually by the user: 1) the number of complexes and sequences, *q*; 2) the population size, *s*, which also determine the number of points within each complex. Vrugt *et al.* (2003) recommended the use of larger population sizes and larger number of parallel sequences to be able to precisely capture the complex shape of the covariance structure. Moreover, the SEM, sub-process of SCEM-UA algorithm also has three algorithmic parameters to be chosen carefully: 1) the number of evolution steps defined as L=m/10; 2) predefined likelihood ratio T; 3) predefined jumprate, $c_n = 2.4/\sqrt{n}$. Essential algorithmic parameters for running SCEM-UA are tabulated in Table 3.

Table 2 Calibration parameters of the KsEdgeFC2D model with upper and lower bounds of the predefined parameter space

Parameters optimized (KsEdgeFC2D)	*Lower bound*	*Upper bound*
n	0.1	0.5
k_a	0.01	0.05
d_s (m)	0	1.0
d_c (m)	0.0	0.6
$\beta = k_a / k_c$ (beta)	0	20

Table 3 Algorithmic parameter in SCEM-UA method

Parameter	*Description*
q	the number of complexes $q = 10$
s	the population size $s = 200$
L	the number of evolution steps $L = m/10 = (s/q)/10 = (200/10)/10 = 2$
T	likelihood ratio T=10^6
c_n	jump rate $c_n = 2.4/\sqrt{n} = 2.4/\sqrt{5} \approx 1.07$

5. Results

This case study illustrates the efficiency and effectiveness of the Shuffled Complex Evolution Metropolis algorithm for realistic assessment of prediction uncertainty on hydrologic responses provided by the complex hydrological modeling like distributed modeling. In addition to analysis of parameter uncertainty, we compare the optimal parameter values estimated using SCEM-UA with the ones derived using the original SCE-UA global optimization algorithm with different objective functions to identify the structural inadequacy of used hydrologic model.

The SCEM-UA algorithm was implemented with a population size s=200 and q=10 complexes, 20 points in each complex. An important issue in MCMC sampling is convergence of the sampler to the stationary posterior distribution. Practically, Scale Reduction score ($\sqrt{SR}$) developed by Gelman and Rubin (1992) has been used to be a criterion of convergence. If the $\sqrt{SR}$ is less than 1.2, the Markov chain is considered to be converged into the target posterior distribution; otherwise, the evaluation steps are repeated until obtaining a suitable value. Figure 2 illustrates the calculated $\sqrt{SR}$ against the number of MCMC iterations. All parameters become stable after approximately 5,000 iterations (*i.e.*, $\sqrt{SR}$ <1.2).

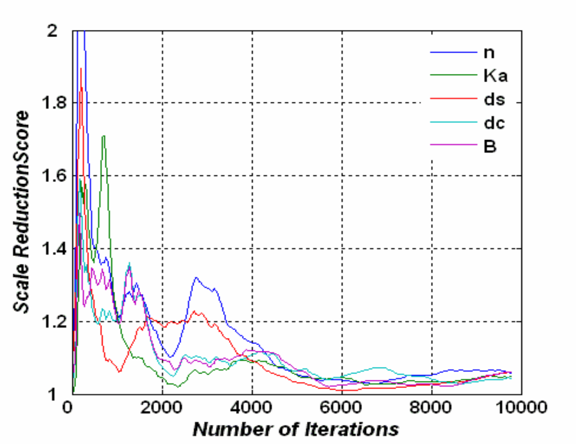

Fig. 2 Evaluation of the Gelman and Rubin Scale Reduction score for KsEdgeFC2D parameters

Figure 3 presents the marginal posterior probability distributions for the KsEdgeFC2D parameters obtained using 5,000 sample parameter sets generated after convergence of the SCEM-UA algorithm. The moments of the posterior parameter distributions and the most likely parameter combination are presented in Table 4. Here, the first 5,000 simulations of each parallel sequence were discarded (*i.e.*, $\sqrt{SR}$ >1.2). The limit of the X-axis in Figure 3 corresponds to the range specified for the estimated parameter uncertainty through the stochastic optimization procedure. The histograms for the k_a, d_s and β approximate a normal distribution centered around the optimal parameter values and then the posterior mean value is close to the optimal parameter value while other histograms such as n and d_c are skewed to the upper value of prior range. The encompassed points around the maximum value of the prior parameter range presented in Figure 4 (see the plot for correlation between n and β) infers that pre-specified prior parameter range for calibration is not suitable. Although the initial guess (*i.e.*, prior uncertainty range) for model parameters is physically reasonable or practically appropriate, unknown huge parameter interaction leads to the numerically acceptable values, which are too much different from hydrologist's concern.

Figure 4 describes scatter plots in two dimensions of the parameter space of the 5000 parameter sets sampled from the posterior distribution after convergence to stationary posterior distribution. Due to no contribution of the surface roughness coefficient, n to subsurface flow simulation, this parameter shows no correlation with the other calibration parameters which is confirmed by the summarizing correlation coefficients presented in Table 4. On the other hand, dominant parameters of subsurface flow such as k_a, d_s, β show a positive or negative correlation (see the Figure 4 and Table 4).

Moreover, the optimal parameter set estimated by the SCE-UA method is included in Table 4. Gupta *et al*. (1998) introduced a multi objective analysis framework to investigate deficiencies in the model structure, which are reflected in a structural inability to simultaneously reproduce different aspects of the system response with a single set of parameters. Different parameter combinations are required to fit different response modes such as low and high flow (Bastidas, 1998). However, in KsEdgeFC2D model case, we did not detect a big difference of hydrological responses or a distinguished improvement for different modes of hydrographs according to objective functions. As shown in Table 4, optimal parameter values of not only the SCEM-UA but also the SCE-UA algorithm are approximately similar regardless of three objective functions. It means that the KsEdgeFC2D model has stable and appropriate structure to simulate rainfall-runoff processes. Also, this result indicates that the Pareto solutions estimated by Multi objective optimization algorithm for KsEdgeFC2D model might have a very small and narrow range. Hydrographs reproduced by two algorithms using each optimal parameter set are presented in Figure 5. Because the residuals between the observed streamflow and the simulated ones are assumed to be mutually independent, Gaussian distributed, with constant variance, all best-fit parameters estimated by SCEM-UA are much closer to the calibrated parameters using SLS objective function than others from HMLE, MIA (See Figure 3).

Finally, probabilistic predictions of the hydrograph were obtained from the ensemble simulation of the KsEdgeFC2D model for 5000 parameter combinations sampled from the posterior parameter distribution. Figures 6 and 7 illustrate how the results of the SCEM-UA algorithm can be translated into estimates of hydrograph prediction uncertainty. In these figures, the black line indicates the observed streamflow data, the blue line, the simulated hydrograph using the most likely parameter set having the highest posterior probability and the grey shaded region, 90% hydrograph prediction uncertainty associated with the posterior distribution of the parameter estimates. Here, parameter uncertainty boundary estimated by the posterior distribution is narrow and failed to bracket the observations for the most calibration period; particularly, the recession part of simulated hydrograph is not matched to the observed one. This result may come from the imperfect model structure and/or input data. Figure 7 shows that the parameter sets evaluated from calibration event reproduce the observed discharges reasonably well during validation period.

6. Summary

Identifiability process including a selection of suitable model and an estimation of proper parameter values is difficult due to a range of uncertainties involved in a modeling process that are also unavoidably propagated into the model output. Even with the robust global automatic optimization algorithms, classical parameter estimation approaches are not able to treat parameter uncertainty and its influence on hydrograph simulation. However, the SCEM-UA, an automatic Bayesian parameter inference algorithm based on Markov Chain Monte Carlo methods, has been proven to be very efficient for estimation of target posterior parameter distribution. In this study, we demonstrated the capability of the SCE M-UA algorithm to calibrate the KsEdgeFC2D model for the Kamishiiba catchment. Summarized results from this study are as follows.

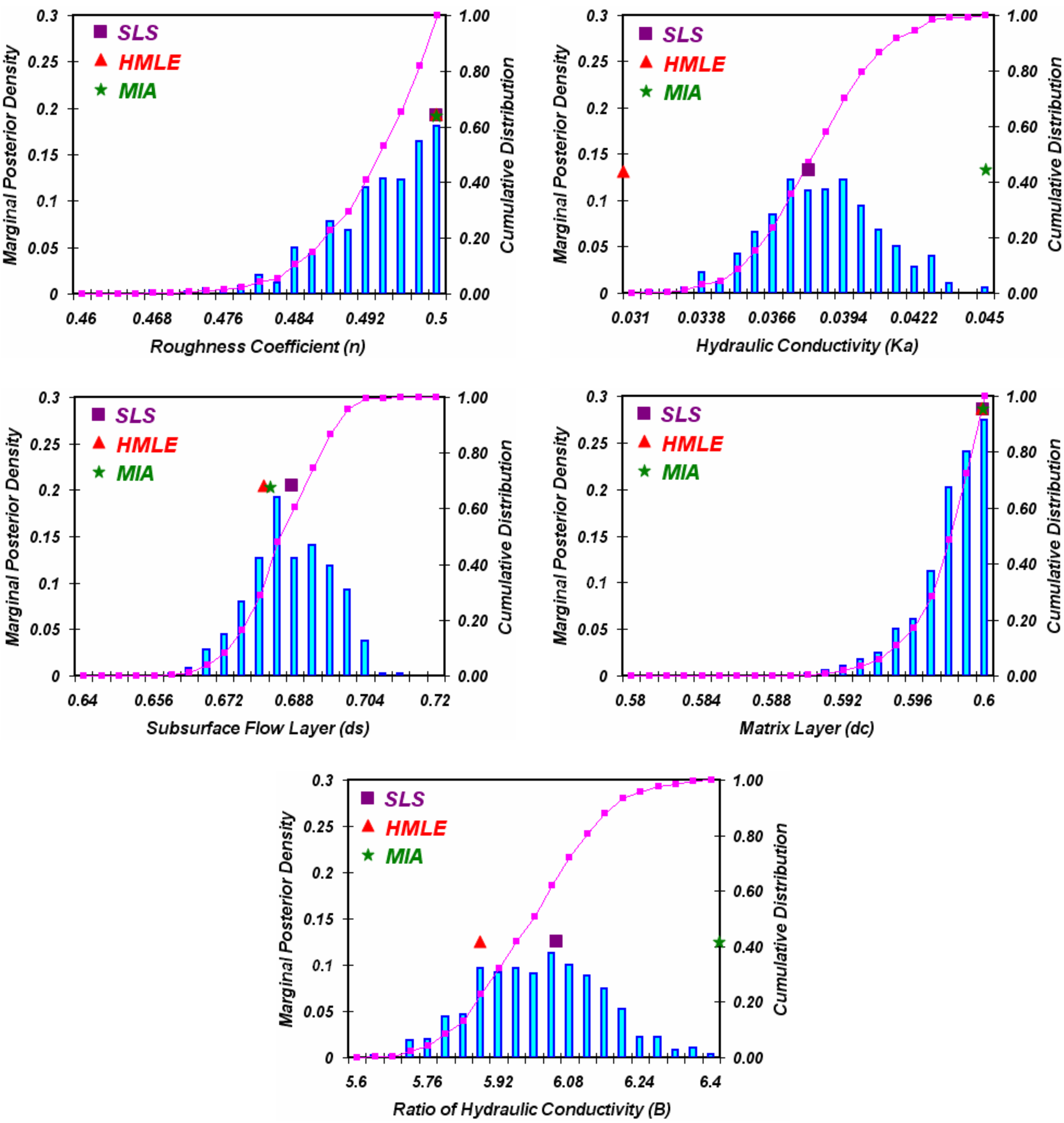

Fig. 3 Marginal posterior probability distribution of the KsEdgeFC2D model parameters

Table 4 Shuffled Complex Evolution Metropolis Posterior Mean (Mean), Standard Deviation (STDV) and Correlation Coefficients between the generated samples for the KsEdgeFC2D parameters

Parameter	Mean	STDV	Correlation coefficients					Optimal parameter values			
			n	k_a	d_s	d_c	β	SCEM	SCE_SLS	SCE_HMLE	SCE_MIA
n	0.497	0.052	1	-0.033	-0.13	0.05	-0.06	0.499	0.5	0.5	0.5
k_a	0.037	0.004	-	1	-0.61	0.09	-0.69	0.037	0.038	0.03	0.047
d_s	0.684	0.045	-	-	1	0.19	0.87	0.691	0.688	0.682	0.683
d_c	0.589	0.050	-	-	-	1	0.14	0.599	0.6	0.6	0.6
β	6.12	1.058	-	-	-		1	6.082	6.046	5.882	6.642

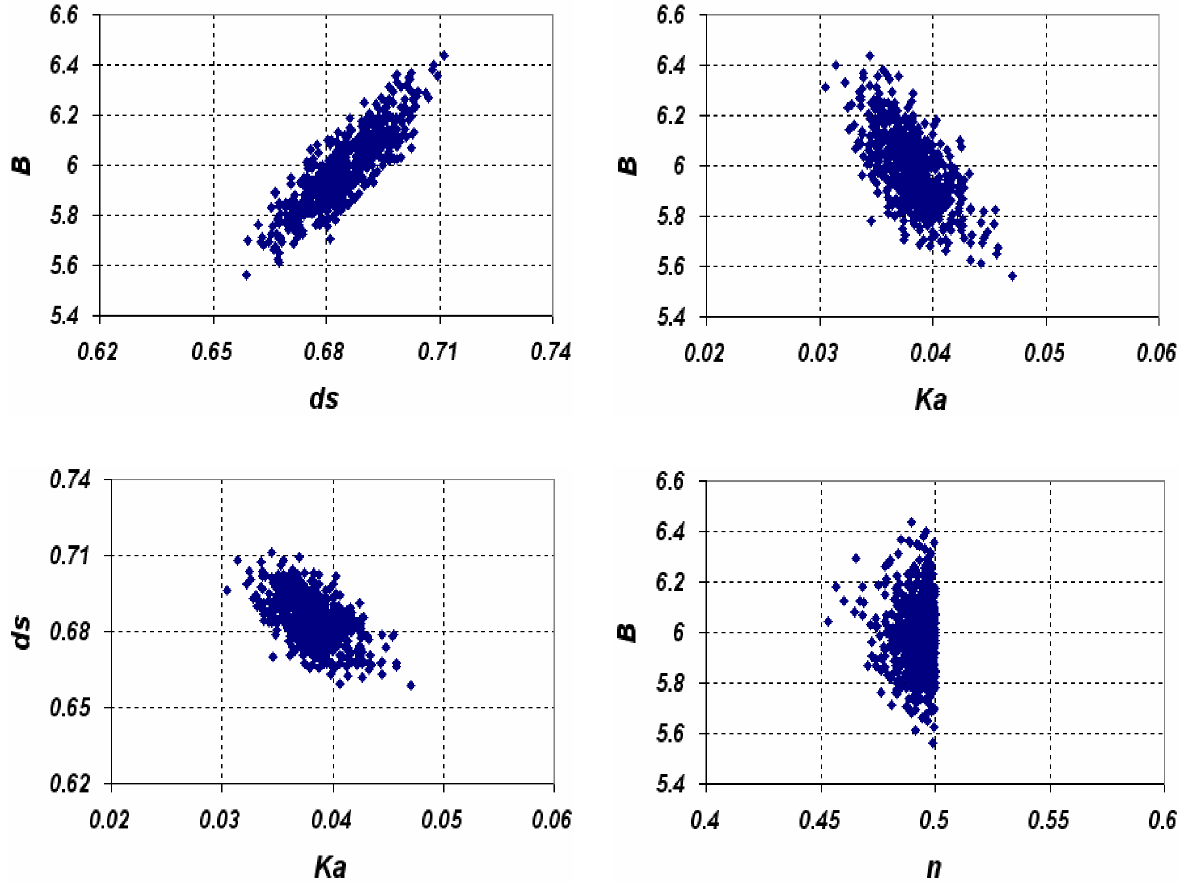

Fig. 4 Scatter plots in two dimensional parameter space of 5000 sampled parameter sets showing the correlation of different combinations of parameters (Correlation Coefficients for all combinations are illustrated in Table 4)

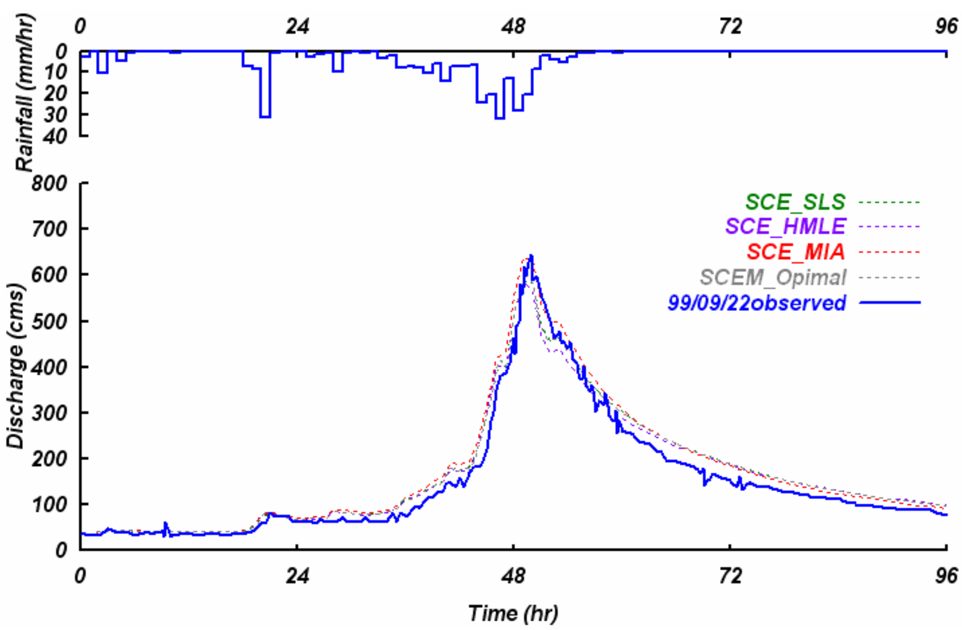

Fig. 5 Simulated Hydrographs using two (SCEM-UA, SCE-UA with 3 different objective functions) algorithms

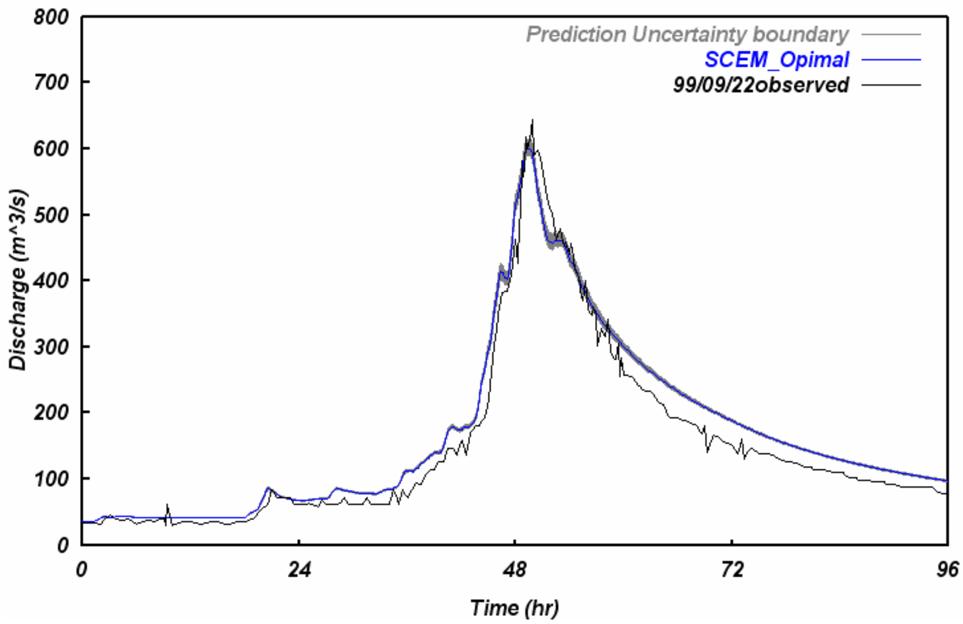

Fig. 6 Hydrograph prediction uncertainty associated with the most probable set derived using the SCEM-UA algorithm for the calibration period (9/22/1999).

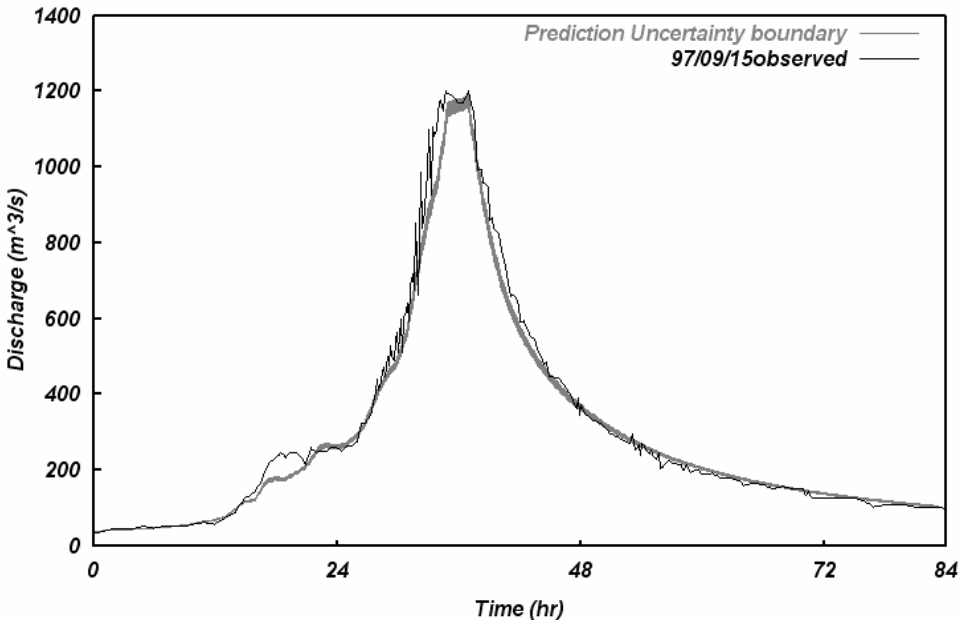

Fig. 7 Hydrograph prediction uncertainty associated with the most probable set of calibration event derived using the SCEM-UA algorithm for the validation period (9/15/1997).

1) The SCEM-UA algorithm was able to successfully explore the feasible parameter space and to converge into the target posterior parameter distributions after approximate 5000 iterations.

2) When comparing with the optimal parameter set calibrated by the original SCE-UA algorithm, the most likely parameters having the highest frequency were very similar as those values.

3) The subjectively selected three different objective functions provided approximately constant parameter sets and the simulated hydrographs based on those optimal parameter sets are quite similar.

4) Posterior density such as k_a, d_s and β followed to a normal distribution while other evaluated parameter were excessively skewed to the upper value of prior parameter range.

5) Hydrographs predictions based on the posterior parameter distributions demonstrated that KsEdgeFC2D model was able to reproduce the observed discharges with reasonable accuracy for the Kamishiiba catchment.

6) The parameter uncertainty bounds were narrow and did not cover all observations. It means that improvement in model structure or input data may result in more accurate predictions.

The analysis of reasons why equifinality happen in hydrologic modeling, including the comparison of posterior parameter distributions in different hydrologic models and its translation to hydrograph simulation, is on going.

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分布型降雨流出モデルにおけるパラメータの不確実性の定量化

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要　旨

一般的に水文モデルには直接計測することのできないモデルパラメータが含まれ，それらの値は過去の水文データに適合するように決定される。これまで数十年に渡って，モデルパラメータの自動推定に関する研究が多数なされてきたが，それらの手法はパラメータ推定に伴う不確かを考慮できないという欠点を有する。本研究では，分布型降雨流出モデルKsedgeFC2Dのモデルパラメータ推定について，パラメータ推定の不確かさとその河川流量予測への影響を，上椎葉流域(211㎢)を対象に分析する。パラメータの不確かさが予測値にどのように伝達するかを分析するために，Shuffled Complex Evolution Metropolis アルゴリズム(SCEM-UA)を採用した。SCEM-UAを用いることにより，効率的にモデルパラメータの事後分布が得られることがわかった。また，SCEM-UAによって求められたパラメータの値と3つの異なる目的関数を設定したSCE-UA法によって得た値とを比較し，SCEM-UAの適用性を検討した。

キーワード：自動パラメータ推定，パラメータの不確かさ，SCEM-UA