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Lumping a Physically-based Distributed Sediment Runoff Model with Embedding River Channel Sediment Transport Mechanism

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Synopsis

The recent development of one-dimensional model from this study is lumped representation of a distributed sediment runoff model with embedding hillslopes and river channels sediment transport mechanisms. Based on the assumption of steady state conditions, the relationship between outflow discharge and water storage in hillslope and river channel can be derived. Then a lumped sediment runoff model is developed. The maximum sediment storage in both areas was mathematically derived as functions of sediment transport capacity and total storage of water at each grid-cell. Soil detachment and redeposition represented by the balance between the actual sediment storage and the maximum sediment storage. The performance of lumped model is examined in the Lesti River, Indonesia.

Keywords: lumping, sediment runoff model, erosion, hillslope, river channel, Lesti River basin

1. Introduction

Sediment runoff models are extensively used for hydrological investigations in engineering and environmental science. They are applied to evaluate the effectiveness of various control strategies such as source control includes soil conservation techniques, reducing runoff risk or protecting against erosion and as load models linked to water quality investigations.

In addition, the processes controlling sediment runoff are complex and interactive. This complexity results in the term “erosion runoff processes” internal catchment area. Sediment transport is highly dependent on topography, land use, and soil type; thus to develop a physically based distributed rainfall-sediment-runoff model is crucial. Another fact, the difficulty in the observation and measurement of the erosion and sediment transport processes during a runoff and erosion event due to small temporal and spatial scales makes necessary the use of a physically-based distributed sediment runoff model for the spatio-temporal predictions of runoff, erosion, and deposition at the internal locations and catchment outlet.

Complex distributed sediment runoff models have been developed. KINEROS, LISEM, WEPP, and EUROSEM (Morgan et al., 1998) are some of the examples. These models are normally designed to estimate sediment runoff during a single rainfall event.

The common problem of those fully distributed models have limitation in application because those models generally require much computation time to conduct a simulation especially for large catchments and long-term simulations. In the past, lumped and semi-distributed models have been widely preferred for representing the rainfall-sediment-runoff process at scales ranging from few to several hundreds square kilometers. Unfortunately, given lack of direct physical interpretation of their model parameters, long term observed data records are needed for their calibration, which may not always be available.

From the above considerations, the need is obvious for creating a new generation of a lumped model with parameters directly related to physically meaningful quantities derived from appropriate distributed scales. A concept for lumping a distributed rainfall runoff model using digital topographic information to reduce computational burden required in runoff simulation was proposed (Ichikawa and Shiiba, 2002). The governing equations and the lumped model parameters are derived from a distributed rainfall runoff model by assuming that a rainfall runoff process of the system reaches a steady state condition.

As an extension of the lumping method for a physically-based distributed rainfall-runoff model (Ichikawa and Shiiba, 2002), the authors extended the method with adding a new lumping method for sediment transport processes incorporating surface and subsurface flow in unsaturated and saturated zones in hillslope area (Apip et al., 2008).

Total runoff and sediment loads from hillslopes to and storage in river channel reaches can disrupt aquatic habitats, impact river hydromorphology, and water quality. Therefore it is often convenient to visualize a catchment as consisting of the channel network and the contributing areas that can be described as hillslopes. The reasonableness of this characterization varies, depending upon the hydrologic systems under consideration and upon the scale of consideration (small vs. large scale).

Total sediment is made available for removal from a river catchment by physical, chemical, and biological processes operating on both hillslopes and river channels. However, it might be expected that the sediment yield becomes more directly influenced by riil, interril, and sheet flow processes operating on the hillslope as the scale of the catchment diminishes. In the fact, the source area of sediment for large catchment scale is primarily the river channel bed and banks, hillslope processes only feeding material for subsequent storage lower down the slope or in the river channel. To accommodate this, the recent developments of one-dimensional model from this study are physically-based distributed sediment runoff model and its lumping with embedding hillslopes and river channels sediment transport mechanisms.

The advantage to lump a distributed model is to produce a new lumped sediment runoff model version as interest in sediment runoff modeling extends to large catchments scale, to derive lumped model parameters by keeping the physical meanings of an original distributed model, which are obtained from integration of distributed equation and information from grid-cell based scale to the catchment scale, new lumped sediment runoff model version is run without any additional calibration, and to reduce computational time respectively. Those models can be used as a modeling tool for simulating the time-dependent response of runoff and sediment transport processes at the catchment scale which facilitates the analysis: (1) total runoff and sediment loads in both hillslopes and river channel processes; (2) interacting processes of erosion sources and deposition; and (3) internal catchment behaviors. The lumped model is applied in the Lesti River located in the upper Brantas River basin, Indonesia.

2. Physically-Based Distributed Rainfall-Sediment-Runoff Model

Effective Rainfall Model

This study considers that the kinetic energy of a raindrop as important key in determining soil erosion rate due to rainfall detachment or splash detachment, herein effective rainfall prediction is included in sediment runoff model structure. Effective rainfall is estimated from gross rainfall (Rg) on an event based basis. Effective rainfall is defined as difference between gross rainfall and canopy interception. When the canopy of the land use type intercept water, the rainfall is divided into two parts, as direct throughfall (DT) and intercepted rainfall (IC). The fraction of Rg contribute to DT is effected by the proportion of the land surface covered with vegetation (COV). The gross rainfall which is intercepted is stored on the leaves and branches of the vegetation as interception store (ICstore), it will become the source of evaporation. Interception store changes depending to the total gross rainfall for each time event and maximum capacity of the vegetation cover (ICmax). If the depth of IC is higher than ICstore, the remain of IC-ICstore is not held in the ICstore, this is termed the temporarily intercepted throughfall (TIF). The source of stemflow (SF) and leaf drainage (LD) is come from TIF. The depth of SF for each time is predicted as a function of land use type and the average of acute angle (PA). The difference between TIF-SF for each time defines as LD. The effective rainfall to the ground, which is available for runoff and soil erosion modeling, is generated by the summation of the direct throughfall, stemflow, and leaf drainage. Conceptually all the processes above are simplified in Fig. 1.

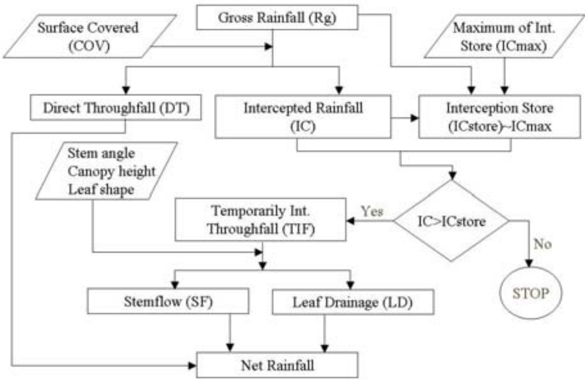


Fig. 1 Schematic diagram of rainfall interception process.

For modeling process, this study used the same algorithm of rainfall interception with KINEROS or EUROSEM model (Morgan et al., 1998) with several reasons. The algorithm retains a strong physically base, equations developed in laboratory experiment and validated by field observation as well as applicable for tropical region.

Cell Distributed Rainfall Runoff Model

Since soil erosion and sediment transport by water is closely related to rainfall and runoff processes, erosion and sediment transport modelling cannot be separated from the procedures to use runoff generation model by using a hydrological model. Cell Distributed Rainfall Runoff Model Version 3 (CDRMV3) a physically-based model developed at Innovative Disaster Prevention Technology and Policy Research Laboratory DPRI-Kyoto University (Kojima et al., 2003) is used as a base of a distributed sediment transport model. The model includes a stage-discharge, *q-h*, relationship for both surface and subsurface (Tachikawa et al., 2004):

$$q = \begin{cases} v_m d_m (h/d_m)^{\beta}, & 0 \le h \le d_m \\ v_m d_m + v_a (h - d_m), & d_m \le h \le d_a \\ v_m d_m + v_a (h - d_m) + \alpha (h - d_a)^m, & d_a \le h \end{cases} \quad (1)$$

$$v_m = k_m i, \quad v_a = k_a i, \quad k_m = k_a / \beta, \quad \alpha = \sqrt{i} / n$$

where q is discharge per unit width; h is water depth; i is the slope gradient; k_m is the saturated hydraulic conductivity of the capillary soil layer; k_a is the hydraulic conductivity of the non-capillary soil layer; d_m is the depth of the capillary soil layer; d_a is the depths of capillary and non-capillary soil layer; and n is the roughness coefficient based on the land cover classes.

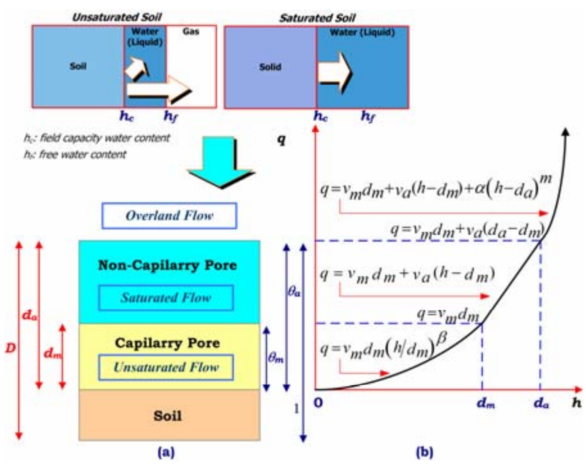


Fig. 2 (a) Concept of hillslope soil layer structure and (b) stage-discharge relationship of CDRMV3.

The model incorporating soil layer model structure which consists of a capillary pore which unsaturated flow occurs inside and a non-capillary pore in which saturated flow occurs. In the soil layer, slow flow and quick flow are modeled as unsaturated Darcy flow with variable hydraulic conductivity and saturated Darcy flow. According to this mechanism, surface flow will occurs if the water levels are higher than the total soil depth. In the CDRMV3, the horizontal sub-surface and surface flows, *q* (discharge with unit width), are calculated by the approximation equation (1) corresponding to stage-discharge relationship (Fig. 2).

The model assumes that the flow lines are parallel to the slope, the hydraulic gradient is equal to the slope. The kinematic wave of model does not consider the vertical water flow like infiltration effects. The input rainfall data *r(t)* is directly added to subsurface flow or surface flow according to the water depth on the area where the rainfall dropped.

The model parameters to be determined are n ($m^{-1/3}s$), k_a (m/s), d_a (m), d_m (m), and β. For river flow routing, surface flow with rectangular cross section is assumed for kinematic wave approximation.

The Lax-Wendroff finite difference scheme is used to solves the one-dimensional kinematic wave equation with the stage-discharge equation to simulate runoff generation and routing. The simulation area is divided into an orthogonal matrix of square grid-cells (250 m x 250 m). The continuity equation takes into account flow rate of each grid-cell or slope element as:

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r(t) \qquad (2)$$

where t and x are time and distance along water flow, respectively; and r is the effective rainfall intensity.

Distributed Sediment Runoff Model

Catchment process is described in terms of processes occurring on hillslopes which defined as the sources of surface erosion and overland flow, and in river channels as the source of river bed erosion, deposition and as a transfer component. Considered together, these two elements form the catchment.

a. Hillslope Erosion Model

The concept of spatially distributed sediment runoff modeling for hillslopes is shown in Fig. 3. A sediment transport algorithm is newly added to the CDRMV3. Runoff generation, soil erosion and deposition are computed for each grid-cell and are routed between grid-cells following water flow direction. The sediment transport algorithm includes multiple sources of sediment transport, which are soil detachment by raindrop (*DR*) and hydraulic detachment or deposition driven by overland flow (*DF*). The basic assumption of this model is that the sediment is yielded when overland flow occurs. The eroded sediment is transported by overland flow to river channel.

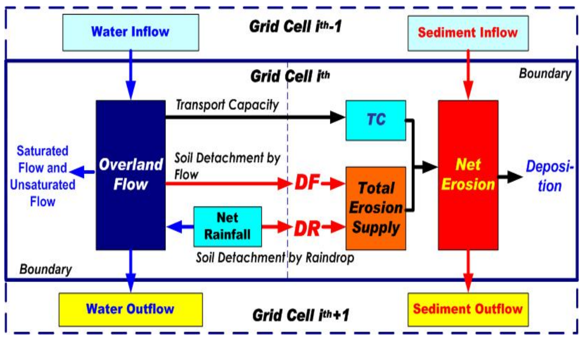


Fig. 3 Schematic diagram of the physically based sediment runoff model at a grid-cell scale.

Soil detachment and transport is handled with the continuity equation representing *DR* and *DF* as:

$$\frac{\partial(h_s c)}{\partial t}+\frac{\partial(q_s c)}{\partial x}=e(x,t) \qquad (3)$$
$$e(x,t)=DR+DF$$

where C is the sediment concentration in the overland flow (kg/m^3); h_s is the depth of overland flow (m); q_s is the discharge of overland flow (m^3/s); and e is the net erosion ($kg/m^2/hr$).

Soil detachment by raindrop is given by an empirical equation in which the rate is proportional to the kinetic energy of effective rainfall and decreases with increasing h_s. From the observation of rainfall characteristic in the study area (Oishi et al., 2005) and dampening soil detachment rate by h_s (Morgan et al., 1998) the empirical equation for *DR* for the i^{th}

grid-cell is expressed as:

$$DR_i = k\ KE\ e^{-b*h_{si}} = k\ 56.48\ \ r_i\ \mathrm{e}^{-b*h_{si}} \qquad (4)$$

where k is the soil detachability (kg/J); KE is the total kinetic energy of the net rainfall (J/m^2); and b is an exponent to be tuned.

Following the theoretical work of EUROSEM (Morgan et al., 1998), the concept of transport capacity is used to determine sediment transport rates in overland flow. Sediment transport capacity of overland flow (TC) is defined as the maximum value of sediment concentration to transport, which is estimated for each grid-cell. Then for the i^{th} grid-cell, DF is simulated as a result of overland flow and function of TC as follows:

$$DF_i = \alpha\ (TC_i / 1000 - C_i) h_{s_i} \qquad (5)$$

where α is the detachment/deposition efficiency factor. Detachment or deposition by flow is assumed to be proportional to the TC deficit. Following the TC approach; if actual suspended sediment from upper grid-cells is lower than this capacity, detachment or erosion occurs, otherwise soil deposition excess.

Many, mostly empirical, equations have been developed to predict sediment transport capacity of flow as function flow characteristics, slope, and material characteristics. These equations often use a threshold value of stream power, shear stress, or discharge. In this study, the transportation capacity is calculated based on the Unit Stream Power (USP) theory that can be applied for sediment transport in open channels and surface land erosion (Yang, 1973). The USP theory stems from a general concept in physics that the rate of energy dissipation used in transporting sediment materials should be related to the rate of material being transported. Sediment concentration in the water flow must be directly related to USP. The USP theory contributing to TC is defined as a product of the overland flow velocity, v, and slope, i, in the i^{th} grid-cell (see equation(6)). Small particles such as clay and silt move mostly in suspension and easily carried by the flow while the sand fraction moves as bed-material and more difficult to move by flow. This is accomplished that TC depends on the particle settling velocity, shear velocity, grain size, kinematic viscosity of the water, and water density. A relationship between USP and the upper limit to the sediment concentration in the overland flow, C_t (ppm), can be derived (Yang, 1973). Hence TC is the product of C_t as:

$$TC = \log C_t = I + J\ \log((vi - v_{critical} i) / \omega) \qquad (6)$$

in which:

$$I = 5.435 - 0.386 \log(\omega D_{50} / NU) - 0.457 \log(U^* / \omega)$$
$$J = 1.799 - 0.409 \log(\omega D_{50} / NU) - 0.314 \log(U^* / \omega)$$
$$\omega = \sqrt{\frac{2}{3} + \frac{36}{\left(\frac{\rho_s}{\rho_w} - 1\right) g \left(\frac{D_{50}}{1000}\right)^{2/NU}}} - \sqrt{\frac{36}{\left(\frac{\rho_s}{\rho_w} - 1\right) g \left(\frac{D_{50}}{1000}\right)^{2/NU}}}$$

where vi is the unit stream power, m/s (v is flow velocity in m/s and i is the slope gradient m/m); $v_{critical}i$ is the critical unit stream power ($v_{critical}$ is the critical flow velocity); ω is the sediment fall velocity (m/s) calculated by Rubey's equation; ρ_s is the sediment particle density (kg/m^3); ρ_w is the water density (kg/m^3); g is the specific gravity (m/s^2); D_{50} is the median of grain size (mm); and NU is the kinematic viscosity of the water (m/s^2). $U^*(= \sqrt{g\ i\ h_s}$) is the average shear velocity (m/s).

This model does not explicitly separate rill and interrill erosion. Gullying, river bank erosion and lateral inflow of sediment to river channel are not considered.

b. River Channel Erosion Model

As interest in erosion and sediment yield extend to progressively larger catchment areas, the relative importance of river channels increases. The eroded soils from hillslopes defined as wash load provided to river channels flow with sediment transport mechanism and routing process. The total sediment load within river channels consists of the sum of the bed-material load and wash load. Normally, sediment transport mechanism of river channels incorporates sources of bed materials load for both suspended and bed load material, which are composed of grains found in the stream bed.

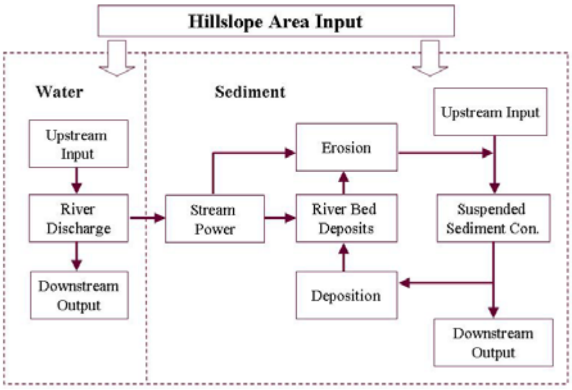


Fig. 4 River channel erosion and deposition mechanism for fine sediment particle.

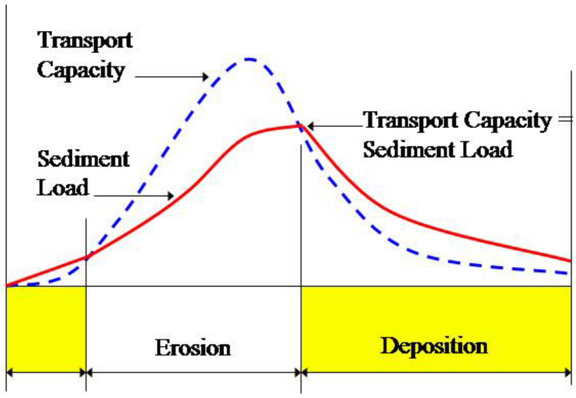


Fig. 5 Relation of deposition to transport capacity and sediment production on hillslopes or river channels.

As first stage of model development for soil erosion from river bed, channel flow and erosion are simulated in model using general approach adopted for rill erosion with kinematic wave model as tool for routing process. The main difference with hillslopes erosion mechanism is that soil detachment by raindrop impact within the channel area is neglected. Furthermore, only finer sediment particle is modeled as suspended bed material, bed load is unconsidered.

The principal sediment transport mechanism controlling model behavior in the simulations are the transport capacity of river channels flow generation specified in terms of stream power, current sediment concentration, and release of the water and sediment concentration fractions. River flow transports most of the eroded soil particles, while under certain conditions; the sediment load in flow can be limited by the flow's transport capacity. If sediment load exceeds the transport capacity, deposition occurs (see Fig. 4). Temporal representation of dynamic sediment transport mechanism, erosion or deposition, at a rate dependent on the flow's transport capacity for both areas, hillslope and river channel, in which representing at catchment scale process is shown by Fig. 5.

Nonequilibrium Concentration Sediment Transport

The sediment transport function within river channel has been intended for the estimation of sediment transport rate or concentration at a nonequilibrium condition with deposition process. When the wash load and concentration of fine material is high, nonequilibrium bed-material sediment transport may occur, and its amount is a function of wash load. Wash load which depends on the supply from hillslopes has been assumed is high enough to significantly affect the fall velocity of sediment particles, flow viscosity, relative density of sediment and water.

For flow in river channel and at a nonequilibrium, transport capacity concentration (TC) of flow is modeled as a function of modified Yang's unit stream power (Yang, et al., 1979), which is an expression for the total load with high concentration of fine sediment particle. Regarding equation(6) when sediment concentration is not too low, the incipient motion criteria, called critical stream power, can be neglected. To apply equation(6) to a river with a high concentration of fine materials and wash load, the values of viscosity, fall velocity, and relative density have to modified to consider the influence of high concentration of fine material on those values. Herein, the modified unit stream power formula proposed by Yang et al. (1979) is expressed as:

$$TC = LogC_t = 5.165 - 0.153\frac{\omega_m D_{50}}{NU_m} - 0.297\log\frac{U_*}{\omega_m} + (1.780 - 0.360\log\frac{\omega_m D_{50}}{NU_m} - 0.480\log\frac{U_*}{\omega_m})\log(\frac{\gamma_m}{\gamma_s - \gamma_m}\frac{vi}{\omega_m}) \quad (7)$$

the coefficients in equation(7) are identical to those in equation(6). However, the values of ω, γ_s, and NU are modified for sediment transport in sediment-laden

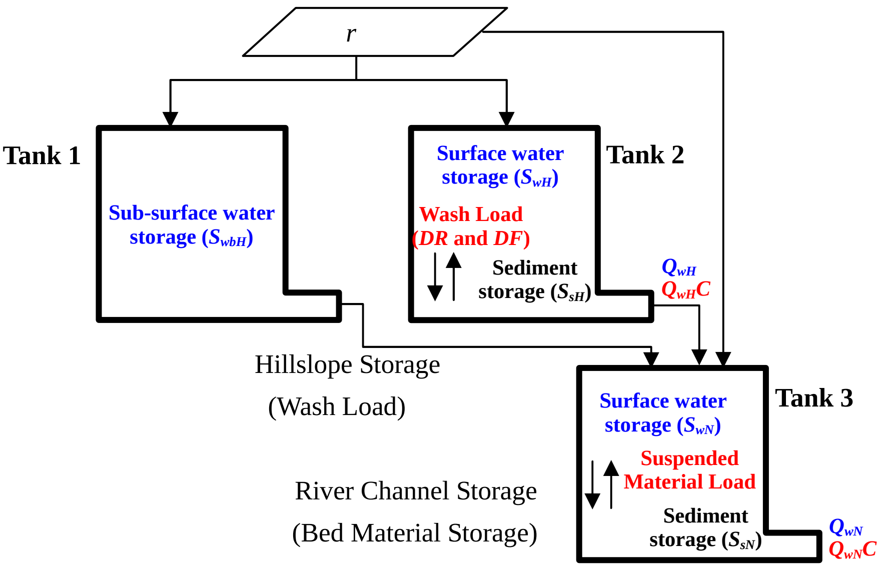


Fig. 6 Schematic diagram of the lumped sediment runoff model at catchment scale.

flows with high concentrations of fine suspended materials. Fall velocity, viscosity, and relative density are modified following these equations as:

$$\omega_m = \omega(1-C)^{7.0}$$

$$NU_m = \frac{\gamma}{\gamma_m}\mu_r NU$$

$$\gamma_m = \gamma_w + (\gamma_s - \gamma_w)C$$

$$\mu_r = e^{5.06C}$$

where C is the suspended sediment concentration including wash load, ω_m is the sediment particle fall velocity of sediment-laden flow, NU_m is the kinematic viscosity of sediment laden, γ_m is the specific weight of sediment laden, and μ_r is the relative dynamic viscosity.

3. Lumping of Physically-based Distributed Sediment Runoff Model Structure

Lumped Sediment Runoff Model Based On Traditional Method

A simple model of catchment response by separating hillslope process and river channel process (Sivapalan et al., 2002) is adopted and extended to incorporate sediment transport processes. This study uses the same principle to explore how sediment yield is related to hydrological response, erosion source, transport mechanism and depositional processes.

On a rainfall event basis, the sediment runoff processes are assumed only affected by surface runoff without consider the effect of sediment load from subsurface layer. According with storage-type concept, the model consists of three water stores, it called rainfall runoff model, and two sediment stores, it called sediment runoff model (Fig. 6).

When the water depth larger than the maximum subsurface flow depth of Tank 1, surface runoff occurs and is added to Tank 2, outflow discharge (Q_w) from Tank 3 as a function of water storage amount (S_w) from each Tanks. After overland flow occurs, sediment transport mechanism on hillslope is computed (Tank 2). The sediment storage (S_s) in (Tank 2) is supplied by the balance between hillslope soil erosion rate, redeposition rate, and sediment discharge released to river channel. Herein, soil erosion by effective rainfall (DR), soil erosion or redeposition by overland flow (DF) are calculated.

Similarly, at a given time t, the river channel store in Tank 3 is supplied with sediment material from the hillslope plus the river bed erosion, and only suspended bed material load is consider. Some of total sediment load, as amount of wash load plus suspended material load, in river channel store is redeposited back into river bed, while another fraction is transported to catchment outlet. Similarly with the hillslope process, the mass of sediment stored in the river channel is determined by the balance between hydraulic erosion rate, redeposition rate, and the release of sediment discharge to the catchment outlet. The rate of erosion or redeposition is depending on the transport capacity of flow and current sediment concentration carried by flow.

The continuity equation of runoff and sediment models is represented as follows:
hillslope process:

$$\frac{dS_{wH}}{dt} = r_H \; A_H - Q_H \tag{8}$$

$$\frac{dS_{sH}}{dt} = DR + DF_H - Q_H C_H \tag{9}$$

$$= k\, 56.48\, r_H + \left(\alpha \left(S_{sH}^{\max} - S_{sH}\right) - Q_H C_H\right) / A_H$$

river channel process:

$$\frac{dS_{wN}}{dt} = r_N \; A_N + Q_H - Q_N \tag{10}$$

$$\frac{dS_{sN}}{dt} = Y_H + DF_N - Q_N C_N \tag{11}$$

$$= Q_H C_H + \left(\alpha \left(S_{sN}^{\max} - S_{sN}\right) - Q_N C_N\right) / A_N$$

where Y_H is the hillslope sediment yield, α is the erosion/deposition efficiency factor, r is the effective rainfall intensity, S_s^{max} is the maximum storage amount of sediment concentration, A is the total area, and the subscript of H and N show hillslope and river channel section, respectively.

If the effective rainfall intensity is known, equations(8,9,10,11) cannot be solved directly to obtain the outflow hydrograph and sedimentgraph from hillslope or river channel, because other variables are unknown. A second relationship is needed to relate Q, S_w, S_{ws}, $h_{s\text{-}avr}$, and S_s^{max} for hillslope and river channel.

To create a second relationship between those variables, lumped traditional models need many model parameters which have to be tuned before time simulation. Then, for calibration and validation, the models require long historical observed data to get the best model performance and parameter. Based on this consideration, the traditional lumped models have been widely used because practically is easier to used and need more efficient computation time, other wise its have limitation. First, the values of model parameters are derived based on time-series of catchment output analysis, generally estimated using statistical techniques, the problem that the observed data may not always be available. Second, parameters do not represent physical interpretation of sediment runoff processes. Third, input and parameter values for the area as a whole are obtained by area-weighting individual values, the effect of spatially distributed information for those values are not considered in governing model parameters.

Lumping distributed model creating a new generation of a lumped model, in which model parameters directly related to physically meaningful quantities derived from appropriate distributed scales. Herein, lumped model parameters are described by a discrete relationships of Q, S_w, S_{ws}, $h_{s\text{-}avr}$, and S_s^{max} for hillslope and river channel, which are derived from the result of lump a distributed approach for rainfall runoff model and sediment runoff model. Those relationships are produced under spatially distributed topographic data, land use data, climate data, and hydrological process.

Spatially Distributed Effective Rainfall

Lumped of distributed approach and new lumped model parameters are obtained with spatially uniform gross rainfall data input but spatially distributed effective rainfall. Different pattern of land use type for each grid-cell causes the variation of effective rainfall depth over hillslope area and river channels.

Total effective rainfall depth for areas, hillslope and river channel, according to the fraction when the rainfall on reaching the vegetation canopy is quantified as bellows:

$$DT = \frac{\sum_{i=1}^{N} a_i DT_i}{\sum_{i=1}^{N} a_i} \quad a_i \approx R_G \neq 0.0 \tag{12}$$

$$LD = \frac{\sum_{i=1}^{N} a_i LD_i}{\sum_{i=1}^{N} a_i} \qquad a_i \approx R_G \neq 0.0 \tag{13}$$

$$SF = \frac{\sum_{i=1}^{N} a_i SF_i}{\sum_{i=1}^{N} a_i} \qquad a_i \approx R_G \neq 0.0 \tag{14}$$

total effective rainfall depth is expressed as follows:

$$r = DT + LD + SF \tag{15}$$

where DT is the direct throughfall, LD is the leaf drainage, SF is the stemflow, a is the grid-cell area with $R_g \neq 0.0$, N is the total grid, and R_g is the gross rainfall depth.

Hillslope Sediment Runoff Model

a. Lumping Distributed Rainfall Runoff Model

The lumping method of physically-based distributed rainfall runoff model (Ichikwa and Shiiba, 2002) was used and extended. In hillslope area, the lumping process is intended for the distributed rainfall runoff model incorporating soil layer model structure which consists of a capillary pore which unsaturated flow occurs inside, a non-capillary pore in which saturated flow occurs, and surface flow.

To obtain the lumped sediment runoff model, the process equation, namely the kinematic wave equation, is integrated over the entire system of grid-cells describing the hillslope. This is done first by computing the total water stored amount in the soil and on the surface by adding up the single grid-cell volumes as a function of the geomorphology and topology of the catchment under the assumption of steady state conditions of rainfall-runoff by spatially uniform rainfall input.

The relationship between total storage of water in the i^{th} grid-cell (s_{wi}) and the outflow discharge in the i^{th} grid-cell (Q_i) is theoretically derived. Flux Q_i is expressed as the product of hypothesis rainfall intensity ($\bar{r}$) and the upslope contributing areas (U_i):

$$Q_i(x) = Q_i(0) + \bar{r}\int_0^x w_i(x)\ dx = \bar{r}\,U_i + \bar{r}\,x_i\ w(x)$$

$$q_i(x) = Q_i(x)/w_i(x) = \bar{r}U_i / w_i(x) + \bar{r}x_i \tag{16}$$

where x is the horizontal distance from the up- stream end of a grid-cell and w is the width of grid- cell. The upslope contributing area can be generated from water flow accumulation data in each grid-cell.

In the case unsaturated flow ($q(x)$ less than or equal q_m):

$$F(q(x)) = q(x)h(x) - \int_0^{h(x)} g(h)\,dh = q(x)h(x) - \frac{V_m h^{\beta+1}}{\beta+1\,d_m^{\beta-1}}$$

In the case saturated flow ($q(x)$ less than or equal q_a):

$$F(q(x)) = q(x)h(x) - \int_0^{d_m} V_m d_m\left(\frac{h}{d_m}\right)^{\beta} dh - \int_{d_m}^{h(x)} \left(V_m d_m + V_a(h-d_m)\right) dh$$

$$= q(x)h(x) - \frac{V_m d_m^{\ 2}}{\beta+1} - 0.5V_a\left(h^2 - d_m^{\ 2}\right) - d_m(h-d_m)(V_m - V_a)$$

In the case surface flow ($q(x)$ great than q_a):

$$F(q(x)) = q(x)h(x) - \int_0^{d_m} V_m d_m\left(\frac{h}{d_m}\right)^{\beta} dh - \int_{d_m}^{d_a} \left(V_m d_m + V_a(h-d_m)\right) dh - \int_{d_a}^{h(x)} \left(V_m d_m + V_a(h-d_m) + \alpha(h-d_m)^m\right) dh$$

$$= q(x)h(x) - \frac{V_m d_m^{\ 2}}{\beta+1} - 0.5V_a\left(h^2 - d_m^{\ 2}\right) - \frac{\alpha}{m+1}(h-d_a)^{m+1} - d_m(h-d_m)(V_m - V_a)$$

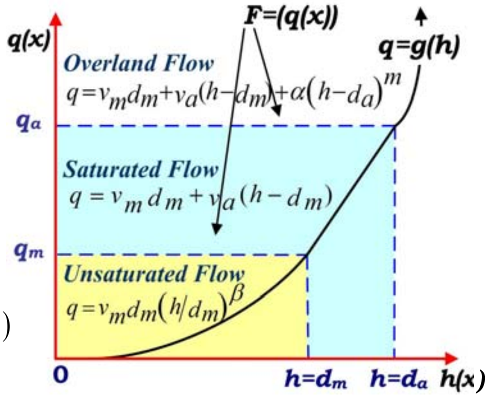


Fig. 7 Schematic drawing of q, h, and F relationship.

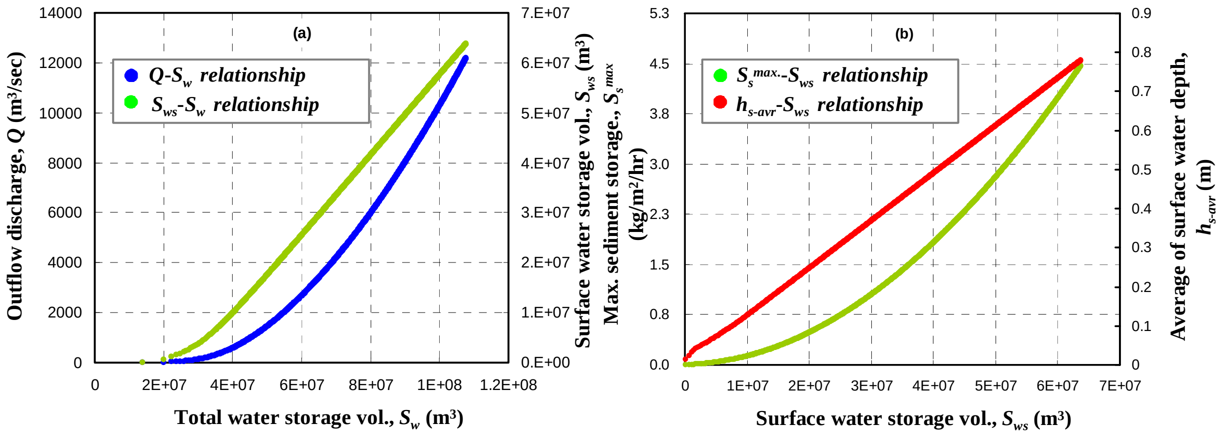


Fig. 8 (a) Plots of discrete relationships between Q-S_w and S_{ws}-S_w; and (b) discrete relationships between S_s^{max}-S_{ws} and $h_{s\text{-}avr}$-S_{ws}.

The storage volume of overland flow for the i^{th} grid-cell (s_{wi}) as a function of h, w, and x is given as:

$$s_{wi} = \int_0^x h_i(x)\, w_i(x)\ dx \qquad (17)$$

by substituting the variable of integration from x to q using the relationship given by (17):

$$s_{wi} = \frac{w}{r} \int_{q(0)}^{q(X)} f(q)\ dq = \frac{w}{r}\left[F(q(x)) - F(q(0))\right] \qquad (18)$$

Figure 7 schematically shows a relationship between q, h, and F. $F(q(x))$ can be further estimated depending on the surface soil condition and q-h relationship for three layers. It is assumed that q is a function of h and can be analytically integrated with h. If the values of $q_i(x)$ using equation(16) is known, and $h_i(x)$ numerically is obtained using equation(1), then $F(q(x))$ can be calculated. The storage of water at the hillslope area, S_w, can be calculated by adding up s_{wi} from each grid-cell.

Lumped Rainfall Runoff Model Parameter

Finally, Q of the outlet is linked to S_w as a function of the topographic and physical characteristics of each grid-cell, as well as effective rainfall intensity. To relate Q of the hillslope output and S_w, the Q-S_w relationship was established. Through variable hypothesis rainfall intensities, the nonlinear Q-S_w discrete relationship at the hillslope area was obtained (Fig. 8a). In order to obtain the dynamic distribution of overland flow in the hillslope area, a relationship between the surface water storage amount, S_{ws}, and S_w was developed (Fig. 8a).

b. Lumping Distributed Sediment Runoff Model

The sediment runoff processes in this study are affected by dynamic spatial distribution of overland flow. The relationship between detachment and redeposition represented by equation(9) depends on the balance between S_s and S_s^{max}, the depth of overland flow as well as. Those variables are produced from lumping distributed approach as:

The Maximum Sediment Storage (S_S^{max})

The maximum sediment storage is defined as the total sediment transport capacity of overland flow in a whole of the hillslopes for each time step calculation. Therefore, we expressed the maximum sediment storage as the function of TC from the i^{th} grid-cell, surface water storage amount in the i^{th} grid-cell (s_{wsi}), and S_{ws}. The maximum sediment storage at hillslope area is calculated by adding up TC_i (see equation (6)) multiplied to s_{wsi} for all grid-cells as:

$$S_s^{\max} = \sum TC_i\, s_{ws_i} / (S_{ws} 1000) \qquad (19)$$

TC (ppm) for the i^{th} grid-cell has been estimated by:

$$TC_i = C_{ti} = 10^{5.0105 + 1.363 \log((USP_i - USP_{critical})/\omega)} \qquad (20)$$

where v in the i^{th} grid-cell is calculated if $h > d_a$ based on the q-h relationship as:

$$v_i = k_a i_i + \frac{\sqrt{i_i}}{n_i}\left(h_i - d_a\right)^{m-1} \qquad (21)$$

Sediment Concentration (C)

Based on the relationship between the current sediment storage (S_s) (kg/m^3/hr) and S_w for each time step calculation, the value of C from hillslope area can be solved as:

$$C = \frac{S_s}{S_{ws}} \qquad (22)$$

for each time-step calculation C is assumed to be uniform over the hillslope area and this is the variable of sediment continuity (see equation(9)).

Lumped Sediment Runoff Model Parameter

To relate the sediment transport variables and the rainfall runoff variables at the hillslope scale, a discrete relationship between $S_s^{\max}$, S_{ws}, and the average of overland flow water depth over hillslope area, $h_{s\text{-}avr}$, was resulted as shown in Fig. 8b using the above procedures.

All the discrete second relationships, which expressed new lumped model parameters for both rainfall runoff and sediment runoff models, are transformed and stored into table form as a “look-up table” before time-varying simulation. New lumped sediment runoff model carries out the calculation during the time-varying simulation and provides information on the temporal variation in rainfall as input for rainfall runoff model, which is changed to total discharge based on steady state condition. Furthermore, the temporal and spatial dynamics of the water stored amount, outflow water discharge, sediment stored amount, erosion or deposition rate, and outflow sediment discharge are passed to the linked between the continuity equations (8,9,10,11) and the discrete relationship for each variable through the look up table.

River Channel Sediment Runoff Model

a. Lumping Distributed Rainfall Runoff Model

Lumping distributed sediment runoff model for river channel area is intended regarding the lumping method for a distributed rainfall runoff model by one layer. River channel section is saturated area, therefore soil depth can be set-up as zero depth with no unsaturated and saturated layers. The process is expressed by surface layer, in which the rainfall directly reaches to the total runoff.

A method to lump a distributed sediment runoff model for one layer, in case the surface layer, was derived (Apip et al., 2007) as an extension from the lumping method proposed by Ichikawa et al., (2000). Herein, lumped rainfall runoff model derived from lumping distributed rainfall runoff model is expressed by a non-linear reservoir, the storage is non-linearly related to outlet water discharge by storage constants K and p as follows:

$$S_w = K\,Q^p \qquad (23)$$

by substituting equation (23) into equation (10) becomes:

$$\frac{dS_{wN}}{dt} = r_N\, A_N + Q_H - \left(\frac{S_w}{K}\right)^{1/p} \qquad (24)$$

K is the model parameter having a physical meaning, can be interpreted as the time of concentration for a kinematic wave to travel across the system.

The value of K is derived from the lumping distributed rainfall runoff approach (Apip et al., 2007). K is influenced by spatially distributed of slope length (L), slope gradient (i), roughness coefficient (n), upper contributing area (U), and total area (A). It proves that K can be derived from the integration of distributed equation. In new lumped model, K is dimensional parameter (m$^{6/5}$s$^{3/5}$) is defined as:

$$K = \sum_{j=1}^{N} \frac{w}{(A)^p}\frac{k_j}{p+1}\left(\left(L_j + \frac{U_j}{w}\right)^{p+1} - \left(\frac{U_j}{w}\right)^{p+1}\right) \qquad (25)$$

in which $p = \dfrac{1}{m}$, and $\left(\dfrac{1}{\alpha}\right)^{\frac{1}{m}} = k$

$$\alpha = \frac{\sqrt{i}}{n} = \frac{\sqrt{\sin\theta}}{n}$$

where i is the slope gradient (m/m), n is the roughness coefficient, m is the exponent constant, which can be shown to be 5/3 from manning’s equation, and j is the number of grid-cell.

b. Lumping Distributed Sediment Runoff Model

The concept of lumping method for hillslope area is applied for river channel area. The maximum sediment storage of the river channel can be calculated from each grid-cell based on water stored, s_{wi}, and TC_i. Sediment transport capacity at each grid-cell (TC_i) is function of topographic variables and hydrological responses. For each time-step sediment concentration is assumed to be uniform over the river channel and this is the variable of sediment continuity.

Flow velocity derived from lumping distributed rainfall runoff model by one layer is:

$$v_i(x) = \frac{q_i(x)}{h_i(x)}$$

$$= \frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\left(\frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\alpha_i}\right)^p}$$

$$v_i = \left(\frac{Q}{A}\left(L_i + \frac{U_i}{w}\right)\right)^{1-p} k_i^{-1} \qquad (26)$$

where v_i is independent variable of Unit Stream Power at each grid-cell (USP_i) as follows:

$$USP_i = v_i \sin\theta_i = v_i\, i_i$$

$$USP_i = \left(\frac{Q_w}{A}\left(L_i + \frac{U_i}{w}\right)\right)^{1-p} k_i^{-1}\, i_i \qquad (27)$$

Transport capacity concentration of flow for each grid-cell inside river channel area is predicted using equation(7) with variable unit stream power is substituted by equation(27).

The maximum sediment storage at the river channel area is calculated by adding up TC_i multiply to s_{wi} for all grid-cells as follows:

$$S_s^{\max} = \sum TC_i\, s_{wi}$$

$$= \sum_{i=1}^{N} w\left(\frac{Q}{A}\right)^p k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right) TC_i$$

(28)

4. Numerical Experiment and Model Evaluation

Study Area

The lumped model derived from lumping a distributed model was applied to the Lesti River catchment (351.3 km^2) (see Fig. 9), a tributary catchment in the Upper Brantas River basin (11,800 km^2), in East Java, Indonesia. This area represents a tropical volcanic area, where land use types are largely dominated by agriculture lands. Most of urban lands, dense forests, and paddy fields are relatively small and concentrated in several spot areas. At the confluence point of the Lesti River and the Brantas main reach, the Sengguruh dam was constructed in 1998. Unexpectedly, most of the gross storage (21.5 million m^3) has been already filled with the large amount of sedimentation from the Lesti River

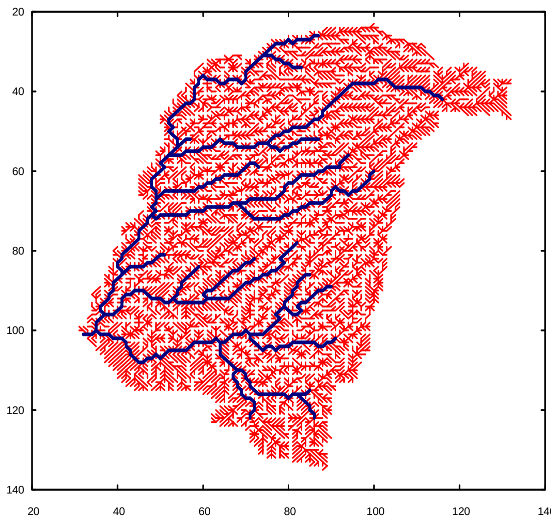
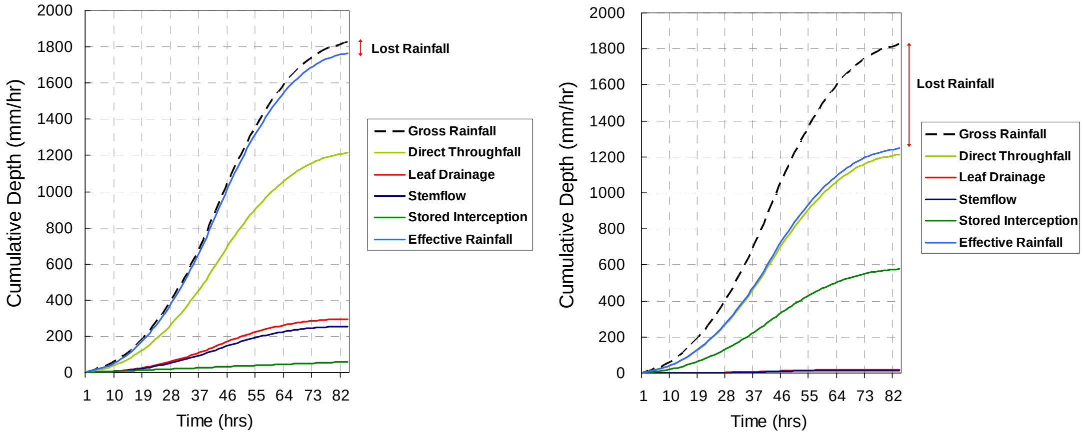


Fig. 9 Flow direction image with river channel network in Lesti River catchment.

Impact of Interception to Effective Rainfall

As explained before, the lumped model parameters which are derived based on lumping distributed model use the spatially distributed effective rainfall intensity as function of land use type. Figure 10 illustrates the dynamic modeling of the interception process under the dominant of vegetation; (a) low interception store, and (b) high interception store. The relative importance of leaf drainage and stemflow are seen to increase with low interception store, and otherwise.



(a) (b)

Fig. 10 Simulation the division of cumulative gross rainfall into leaf drip, direct throughfall, stem flow, and effective rainfall under; (a) low interception storage, and (b) high interception storage.

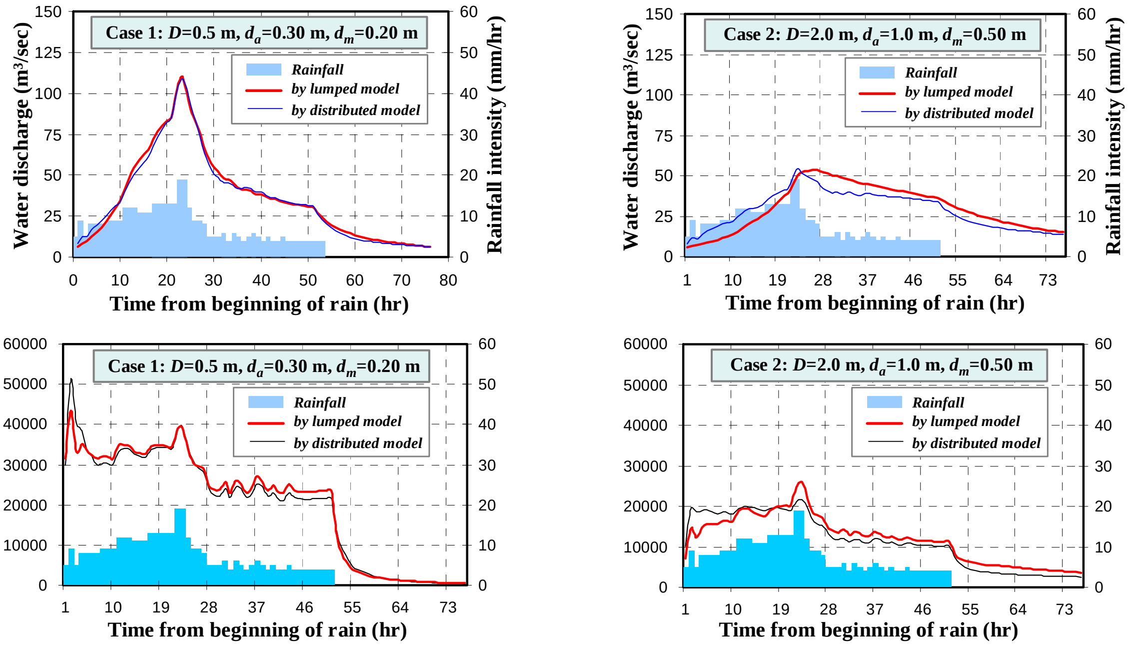


Fig. 11 Computed water discharges and sediment concentrations with different combinations soil thickness (*D*) by using original distributed sediment runoff model and it lumped model version.

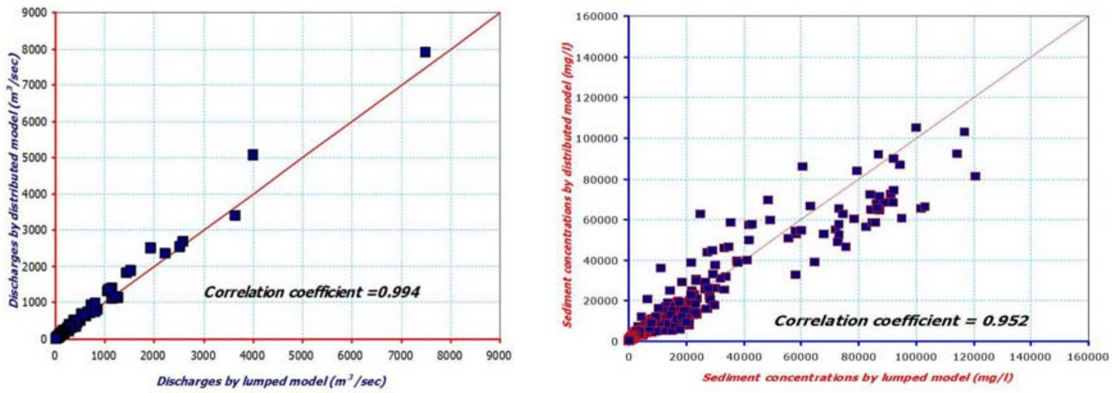


Fig. 12 Comparison of simulated outflow discharges and sediment concentrations by lumped model and distributed model. The solid line is 1 to 1.

Hillslope Model Evaluation

Figure 11 shows comparisons of simulated water and sediment concentration at the outlet for two cases of soil thickness which calculated by the distributed and the lumped models. The blue and black lines are simulated water and sediment concentration by the distributed model and red lines are simulated water and sediment concentration by the lumped model. In the case when soil thickness is shallow (Case 1), discrepancy between the results for both the water discharge and sediment concentration by the lumped and distributed is generally less than in the case surface soil thickness is more thick (Case 2). The difference increases in the case when soil thickness is very thick, where the lumped model generally tends to under estimate water discharge and sediment concentration at the rising limb, and over estimate at the falling limb of hydrograph and sedimentgraph. This difference is due to the assumed steady state condition in deriving the lumped model. In Case 2, the hillslope system did not reach a steady state due to slow movement of water for deep soil layer. Thus the Q-S_w relationship for the lumped model shows some difference from distributed one.

Within the range of possible soil depths, the difference is small and it does not cause severe problems for applications. Table 1 shows the differences value in cumulative total runoff and sediment yield calculated by lumped and distributed models, which are less than 7%.

Table 1 Simulated cumulative runoff and sediment yield

Variables	Cases	Distributed Mod.	Lumped Mod.	Difference (%)
Runoff (m^3)	Case 1	2724.20	2776.63	1.56
	Case 2	2192.48	2293.90	4.63
Sediment yield (kg)	Case 1	1517.05	1548.86	2.10
	Case 2	856.45	909.13	6.15

River Channel Model Evaluation

The model which was lumped was applied to study site and we evaluate the performance of the lumped model by comparing to simulation results computed by distributed model. The numerical experiment was run for the river channel area.

The lumped model performance, qualitative and statistic, for outflow discharges and sediment concentrations during varying-time simulation with several events are given in Fig 12. The figure shows the simulation results for outflow discharge and sediment concentration computed by lumped model basically approximated the simulation results computed by distributed model in the spatial and temporal changes. The values of correlation coefficient more than 0.90, its mean that the lumped river channel erosion model is in an acceptable way to reproduce the discharge and sediment concentration which calculated by distributed model.

4 Conclusions

New version of lumped sediment runoff model for hillslope and river channel was developed. The main advantage of lumping distributed model lies in the capability to obtain new generation of lumped model without losing model and parameter physical interpretation. The lumped model runs efficiently in terms of calibrating and running time.

Within the range of possible cacthment area and soil depth, simulation results computed by the lumped model for hillslope area and river channel agree well with the simulation results computed by the original distributed model. For hillslope model, the discrepancy of the lumped model and distributed model increases when the soil thicknesses increase, low accumulative rainfall amount, and/or spatial temporal variation of rainfall are large.

The analyses spatial scale dependency of a lumped sediment runoff model derived from a physically-based distributed sediment runoff model and it application for large catchments are important areas of further research.

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河川部の土砂輸送過程を考慮する物理分布型土砂流出モデルの集中化手法

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要　旨

斜面部の土砂生産と河道部の土砂輸送を再現する分布型土砂流出モデルの集中化手法を提案する。本手法は，降雨流出過程の定常性を仮定することにより，流域下端の流量と空間分布する斜面部および河道部の貯水量との関係を導出する。つぎに，その関係をもとに，斜面部・河道部の輸送可能土砂量を各グリッドセルの貯水量の関数として算定する。各グリッドセルにおける土砂の侵食・堆積過程は，その輸送可能土砂量と上流からの土砂供給とのバランスをもとに計算する。提案するモデルをインドネシアのレスティ川流域に適用することにより，集中化手法の妥当性を検証した。

キーワード:集中化，分布型降雨土砂流出モデル，土壌侵食，斜面，河道，レスティ川流域