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Estimation of Sub-hourly and Hourly IDF Curves Using Scaling Properties of Rainfall at Gauged Site in Asian Pacific Region

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Synopsis

In urban drainage systems, knowledge of short-duration rainfall events can be considered as one of the most critical elements when their hydrological behavior wants to be investigated. The proposed method is based on the scale-invariance theory whose concepts imply that statistical properties of the extreme rainfall processes for different temporal scales are self-related by a scale-changing operator involving only the scale ratio. The methodology is applied to extreme rainfall data obtained at the rain gauges in Asian Pacific countries characterized with different climate conditions. This study indicates that rainfall intensity in time is scaling for extreme events, and hence this concept is shown to provide a useful practical approach to the evaluation of the design rainfall.

Keywords: Rainfall IDF, Scaling invariance, Rainfall extremes, Design rainfall.

1. Introduction

In urban drainage systems, knowledge of short duration rainfall events can be considered as one of the most critical elements when their hydrological behavior wants to be investigated. For planning and design of various hydraulic structures, extreme rainfall for a given return period is required. In particular, rainfall extremes with high temporal resolution (one hour or shorter) are necessary for the design of drainage systems in urban catchments usually characterized by fast response. The temporal resolution of rainfall data usually available for practical applications is often lower than the data requested for the design procedures or mathematical models application, greatly affecting their reliability. Design rainfall, which is a maximum amount of rainfall for a given duration and for a given return period, is always required for design of various hydraulic structures. At a site where adequate annual extreme (AE) rainfall intensity data records are available, frequency analysis is commonly used to estimate the design rainfall for a given duration and for a selected return period. Results from the frequency analysis are usually presented in form of the intensity duration frequency (IDF) curves. Obviously, for a given return period, the IDF curves decrease with increasing time interval. Minor attention has been paid in the past to improve current techniques of data analysis. Actually, in most cases design practice is based on unproved or unrealistic assumptions concerning the structure of rainfall in space and time.

The traditional method to construct IDF curves has three main steps (Nhat *et al.*, 2006). Based on the raw data, the first step is to obtain annual maximum intensity series for each time interval length. Then, for each time interval a statistical analysis has to be done to compute the quantiles for different return periods. Lastly, the IDF curves are usually determined by fitting a specified parametric equation for each return period to the quantiles estimates, using regression techniques.

This traditional method, however, has certain limitations. For example, a high number of parameters are involved, which makes it non-parsimonious from the statistical point of view (Baghirathan *et al.*, 1978; Bell, 1969; Sherman, 1931; Bernard, 1932; Chen, 1983; Garcia-Bartual, 2001; Takara, 2005). Traditional methods cannot take into consideration characteristics of rainfall for different durations (the time scaling problem); and it is based on the AE data available at local site only (the spatial scaling problem). When high resolution rain gages are available in the catchment, the registration period can be not sufficiently long for obtaining practically usable statistical analyses (partially gauged site).

This would suggest that short duration rainfall should be estimated apart long duration data, i.e. 1, 3, 6, 12, and 24 hours whose values are usually provided by the national hydrological services. Nguyen *et al.* (2002) proposed a method for estimating the distribution of short-duration extreme rainfalls at partially gauged sites based on the property of scale invariance of rainfall.

Therefore, one of the main objectives of this study is to reduce the number of parameters to be estimated in order to increase their reliability. The other main objective is to reduce the estimation process to one single step. Some regularity in hydrological observations, such as scale invariance, has been detected on storm records in the past. The present study deals with the estimation of the IDF curves, using the scaling properties observed on data of extreme storm intensities with a few number of parameters. The present study proposes a method for estimating the distribution of sub-hourly extreme rainfalls at sites where data for time interval of interest do not exist, but rainfall data for longer duration are available. The proposed method is based on the scale-invariance (or scaling) theory whose concepts imply that statistical properties of the extreme rainfall processes for different temporal scales are self-related by a scale-changing operator involving only the scale ratio.

The methodology is applied to extreme rainfall data obtained at the rain gauges in Asian Pacific countries characterized with different climate conditions. This study indicates that rainfall intensity in time is scaling for extreme events, and hence this concept is shown to provide a useful practical approach to the evaluation of the design rainfall. The paper is organized in five sections, the first being this introduction. The data used in this analysis given in section 2. We give the methodology for estimation sub-hour and hourly rainfall extremes based on the scale invariance properties in section 3. Section 4 demonstrates the proposed procedures with applications and evaluations using real data in Pacific region. Conclusions are drawn in Section 5.

2. Data used in the analysis

Numerical analysis was performed on annual extreme (AE) rainfall series from the Asian Pacific countries for storm durations of 6, 10, 15, and 30 minutes (the typical time of concentration for small urban catchments) and 1, 2, 6, 12, and 24 hours (the typical times of concentration for larger rural watersheds). The data were supplied by participating countries attending the APFRIEND Workshop in Kuala Lumpur, Malaysia in June 2005. A total of four stations from each countries were chosen for the analysis and are shown in Table 1. The four stations were selected from Asian Pacific region, based on record length, and availability of current data sets. The record lengths ranged from 30 to 89 years and data sets ranged from 1915 up to 2005, respectively. Record lengths vary per storm duration for a given site.

3. Methodology

The space-time structure and variability of rainfall is very complex and despite much research effort in dynamic (numerical) modelling and coupling of dynamic and statistical descriptions of rainfall, accurate and practical results have not been achieved. In order to solve the physical equations, many parameters and simplifying assumptions are required making these models too complicated for practical applications and they are not consistent at different time scales.

Table 1 List of recording rain gauges used in the analyses.

Country	Station	Latitude	Longitude	Elevation	Length of data (years)	Duration
Japan	Nagoya	35° 10.0’	136° 57.9’	51.00	65	10’,30’,1..24-hr
Korea	Daegu	35° 53.0’	128° 37.0’	57.60	89	10’,30’,1..24-hr
Australia	Geraldton airport	28°.80.0’	114°.70.0’	33.00	48	6’,12’,30’,1..24-hr
Malaysia	31170070	03° 14.0’	101° 45.1’	-	34	15’,30,1..24-hr

On the other hand, recently developed scaling concepts (Gupta and Waymire, 1990) offer a new and better possibility to represent rainfall over different scales based on the empirical evidence that rainfall exhibits scale-invariance symmetry. Scale invariance symmetry implies that the statistical properties of rainfall at different scales are related to each other by a scale changing operator involving only the scale ratio.

Scale models of rain have evolved from fractal geometry, mono-fractal fields, multi-fractals, generalized scale invariant models, and universal multi-fractals Unfortunately, these models were found to be insufficient to describe all features of rainfall fields, such as anisotropy and stratification. Recently, Burlando and Rosso (1996) provided a practical scaling model that can be used to derive Depth-Duration-Frequency relationships using lognormal probability distribution.

In this study, scaling properties of extreme rainfall are examined based on the scaling behaviour of the statistical moments over different durations using a simple scaling approach (Menabde, 1999; Nguyen, 2000, Nhat et al., 2007). The approach consists of examining the scale invariance properties of extreme rainfall time series based on the scaling behaviour of the statistical moments over different durations employing the Extreme Value type 1 (EV1) distribution and L-moments technique of parameter estimation.

Scaling characteristics of extremes rainfall

The scaling or scale-invariant models enable us to transform hydrologic information from one temporal or spatial scale to another one, and thus, help overcome the difficulty of inadequate hydrologic data.

A natural process fulfills the simple scaling property if the underlying probability distribution of some physical measurements at one scale is identical to the distribution at another scale. The basic theoretical development of scaling has been investigated by many authors, including Gupta and Waymire (1990) and Kuzuha *et al.,*(2002).

Rainfall intensity *I*(*d*) with duration d*,* exhibits a simple scale invariance behavior if

$$I(\lambda d) \overset{dist}{=} \lambda^{H} I(d) \qquad (1)$$

holds. The equality “ $\overset{dist}{=}$ ” refers to identical probability distributions in both sides of the equations; λ denotes a scale factor and H is a scaling exponent. From equation (1), it leads to a simple scaling law in a wide sense

$$E[\{I(\lambda d)\}^{q}] = \lambda^{K(q)} E[\{I(d)\}^{q}] \qquad (2)$$

where *E*[] is the expected value operator and q is the moment order. The random variable *I*(*d*) exhibits a simple scale invariance in a wide sense if Equation (2) holds. If *H* is a non-linear function of *q*, the *I*(d) is a general case of multi-scaling.

The moments *E*[] are plotted on the logarithmic chart versus the scale λ for different moment order *q*. The slope function of the order moment *K*(*q*) is plotted on the linear chart versus the moment order *q*. If the plotted results are on a straight line, the random variable shows simple scaling, while in other cases, the multi-scaling approach has to be considered.

Estimation of IDF rainfall extremes

The IDF curves are often fitted to the extreme value type I (EVI) distribution developed by Gumbel and it is still the most often used distribution by many national meteorological services in the world to describe rainfall extremes. It will also be used in this study along with the method of moments. The annual maximum rainfall intensity *I*(*d*) has a cumulative probability distribution CDF (Gumbel, 1958), which is given by

$$\begin{aligned} &F\left[I(d)\right]=1-\frac{1}{T}=\exp[-\exp\{-\left[I(d)-\mu\right]/\sigma\}] \\ &I(d)=\mu-\sigma\ln(-\ln T) \end{aligned} \quad (3)$$

where the location parameter μ and scale parameter σ to be calculated from data series based on L moment method .

According to the scaling theory, the IDF formula can be derived (Nhat *et al.*, 2007) with

$$\begin{cases} I_{d,T}=\dfrac{\mu^{*}+\sigma^{*}[-\ln(-\ln(1-1/T)]}{d^{-H}} \\ \mu^{*}=\mu_{(\lambda d)}.(\lambda)^{-H};\sigma=\sigma_{(\lambda d)}.(\lambda)^{-H} \end{cases} \quad (4)$$

It is worthwhile to note that the simple scaling hypothesis leads to the equality between the scale factor and the exponent in the expression relating rainfall intensity and duration. The IDF relationship can be derived from longer duration data series based on three parameters: scale exponent, the location and scale parameters of EVI distribution.

An application of the scale invariance concept in hydrology is presented for disaggregation (or downscaling) of rainfall intensity from low resolution (e.g., 1-day or 1-hour) down to high resolution (e.g., 1-hour or 10-min). In the other words, statistical properties of rainfall for short durations can be inferred form those of rainfall data available for longer duration. In principle, the statistical properties (e.g., moments) of the scaling relationship could be obtained from large scales and then used to estimate the process properties at smaller scales as will be illustrated in the derivation of scaling IDF curves.

Model Performance

As an indication of goodness of fit between the observed and predicted values the coefficient of determination (R^2), the root mean square error (RMSE), and the mean absolute percent error (MAPE) were calculated. The RMSE and MAPE are defined as follows:

- Root Mean Square Error (RMSE)

$$RMSE=\sum_{i=1}^{N}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_{obs,i}-X_{Est,i}\right)^{2}} \quad (5)$$

- Mean Absolute Percent Error (MAPE)

$$MAPE=100\%.\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_{obs,i}-X_{Est,i}}{X_{Obs,i}}\right)^{2}} \quad (6)$$

Where: $X_{Obs,i}$: the i th observed data point,
$X_{Ests,i}$: the i th estimated value,
n : the number of data point.

4. Results and Discussions

Scale invariance properties of rainfall

The annual maximum rainfall intensity series were measures for 8 durations: 10 min, 30 min, 1 hour, 3 hour, 6 hour, 12 hour, and 24 hour. The scaling behavior of AE rainfall series investigated by computing the moment of extreme rainfall for each duration, and then by examining the log-log plots of these estimated moments against their duration.

Figure 1 illustrates the log-log plots of the moment versus durations for 4 stations in 4 countries of Asian Pacific region. By observing the log-log plots of the moment versus duration of four stations, it can be seem that the relationship between the moments and duration were linear with two different slopes for two different sections: the first one is from 10 min to 1 hour, and the second form 1 hour to 1 day. The break point at the location of 1-hour duration could imply a transition in the rainfall dynamic from a steep slope for short duration rainfall, which could indicate a high variability of rainfall intensities in convective cells, to a milder slope for long duration rainfalls that could be due to the smaller variability of frontal rainfall systems. However, there is no physical basis to the rainfall generating mechanism (a characteristic duration below with frontal systems are the most influential and above which individual storms dominate the scaling behavior) or the rainfall may have an artificial break related to the resolution of the measuring device at rain gauge stations.

In addition, according to Equation (1) and (2), the linear relationship between the moment of order q and the duration in log-log scale could indicate the AE for sub-hourly series are scaling. The slopes of straight lines in the log-log scale are the scaling exponent, $K(q)$, in the scaling relationships Equation (2). These scaling relationships can be identified for two different scaling exponents. The values of the scaling exponents for each interval were computed.

To evaluate the types of scaling: simple or multiple, the relationships between the scaling exponents $K(q)$ and the order of moment q were established for each section of the scaling regimes. The plots of $K(q)$ versus q of every scaling relationship were made in ordinary scale for all four stations. Figure 2 show the relationship for four representative stations. These plots showed that the relationships between $K(q)$ and q were strongly linear as indicated by the very high values of the coefficient of determination (R^2) for the fitted linear regression lines. The values R^2 were greater than 0.99. The almost perfect linearity of the scaling exponents plotted against the order of the moments from data for these stations strongly supports the simple scaling properties of the five moments versus durations. The scale exponent for four stations presented as Table 2.

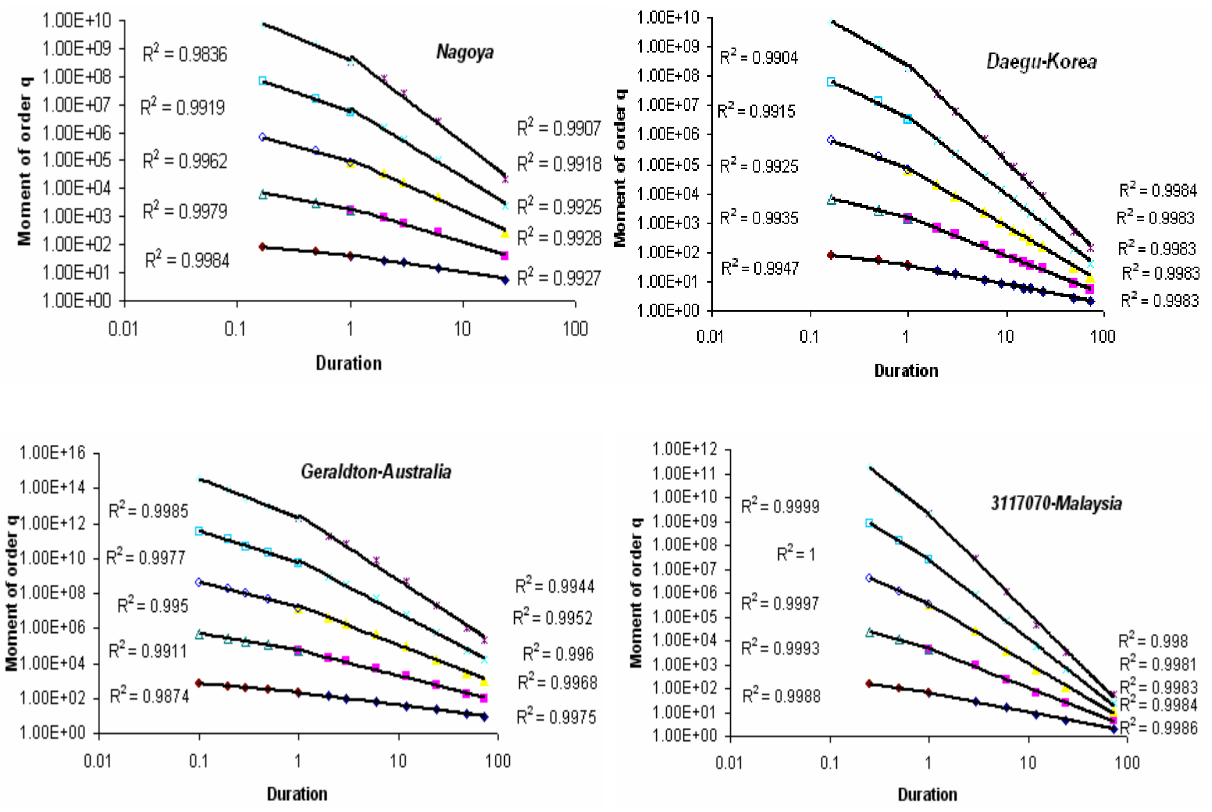

Figure 1 Log-Log plots of the moment order q versus durations.

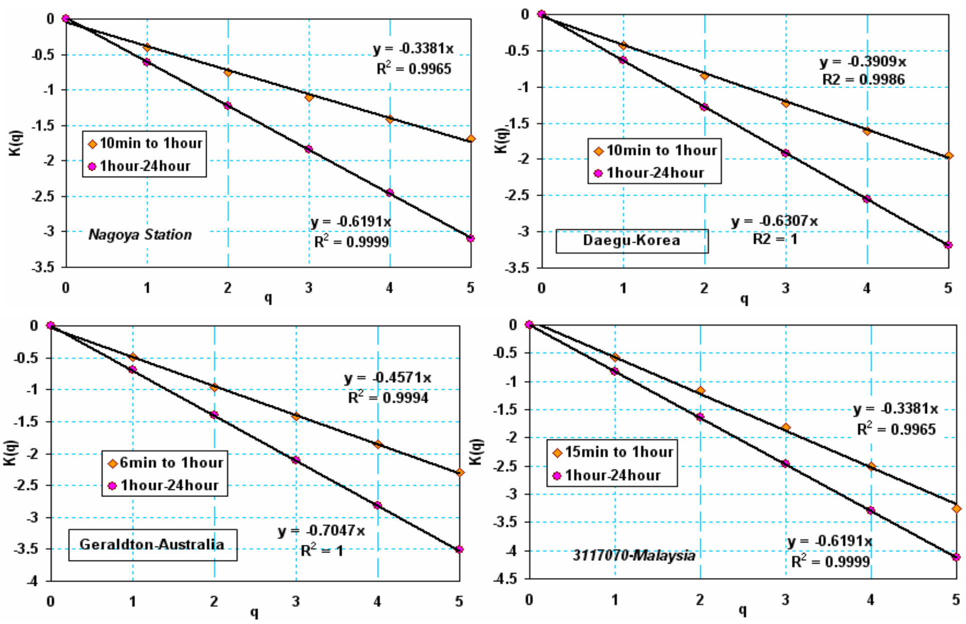

Figure 2 Relationship between $K(q)$ and order of moment q .

Table 2 The scaling exponent factors for 4 gauged sites.

Name of the station	*10min to 1 hour*		*1hour to 1day*	
	H_d	R^2	H_d	R^2
Nagoya-Japan	-0.338	0.996	-0.619	0.999
Daegu-Korea	-0.390	0.998	-0.630	1
Geraldton A.-Australia	-0.457	0.999	-0.705	1
31170070-Malaysia	-0.3338	0.999	-0.619	0.999

This important property enables the derivation of the moments of the AE for a short durations from the AE for other longer durations for which AE are available. Such a derivation is feasible using a suitable scaling regime and its scaling exponents. In conclusion, analysis of the relationships between the moments and the durations showed that these relationships can be described by power-form functions, indicating the scaling behavior of these moments with durations. Furthermore, the scaling properties of the rainfall in time series are simple scaling and composed of two different regimes for two distinct intervals: from 10-min to 1-hour and from 1-hour to 1-day durations.

Estimation of annual extremes rainfall

Graphical evaluation

The simple scaling properties of the moments of annual extreme (AE) rainfall intensity to durations found in the previous section enables the application of the estimation methods developed in section 3. These methods were employed to estimate AE 1-hour rainfall from AE 24-hour rainfall and AE 10-min rainfalls from AE 1-hour rainfalls. The estimates were compared to the observation values by graphical and numerical means. The graphical comparisons were based on: (i) the quantiles plots in which the ordinate is annual extreme (AE) rainfall estimation and observation AE rainfalls, and the abscissa is the empirical probability; (ii) as well as on the quantiles-quantiles (Q-Q) plots in which the ordinate is the AE rainfalls estimated by scaling method and the abscissa is the corresponding observed AE rainfalls.

Figure 3 illustrates the scaled annual extreme and observed annual extreme rainfall versus probability for the sites. Figure 3 indicates that the scaled annual maximum estimates are similar to the observed. This result was typical for all stations analyzed in this paper. The scaling procedure does well to predict the observed series. Figure 4 illustrates the plots of the scaled (estimated extreme rainfall) versus observed rainfall performed for short-duration (e.g. 6 min, 15 min and 10-min) at the sites.

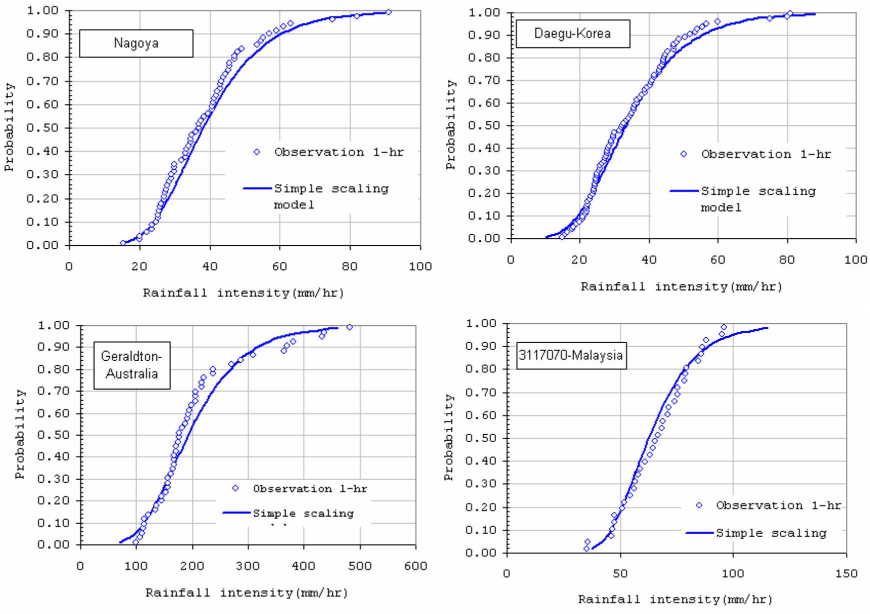

Figure 3 Scaling estimates compared to observation.
Circle dot are observation hourly rainfall, the lines are scaling method.

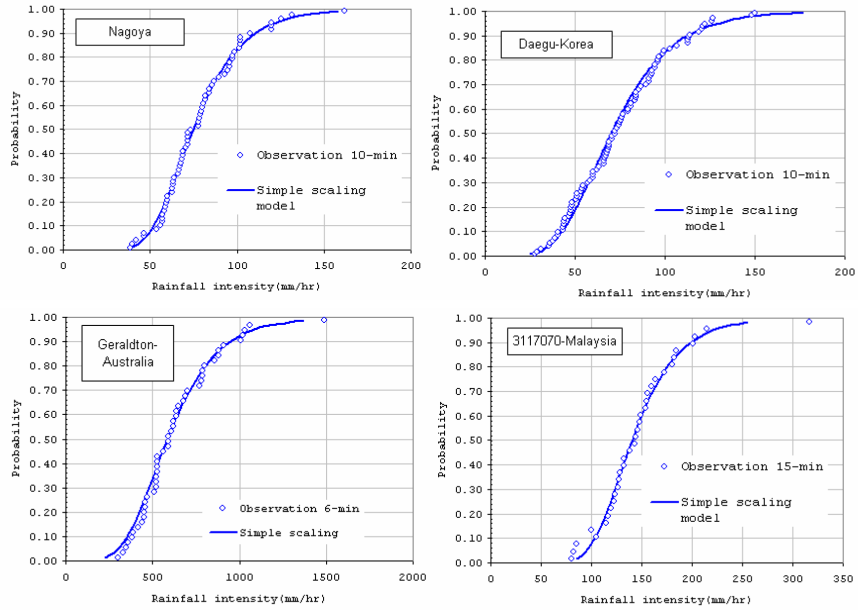

Figure 4 Scaling estimates compared to observation.
Circle dot are observation sub-hourly rainfall, the lines are scaling method.

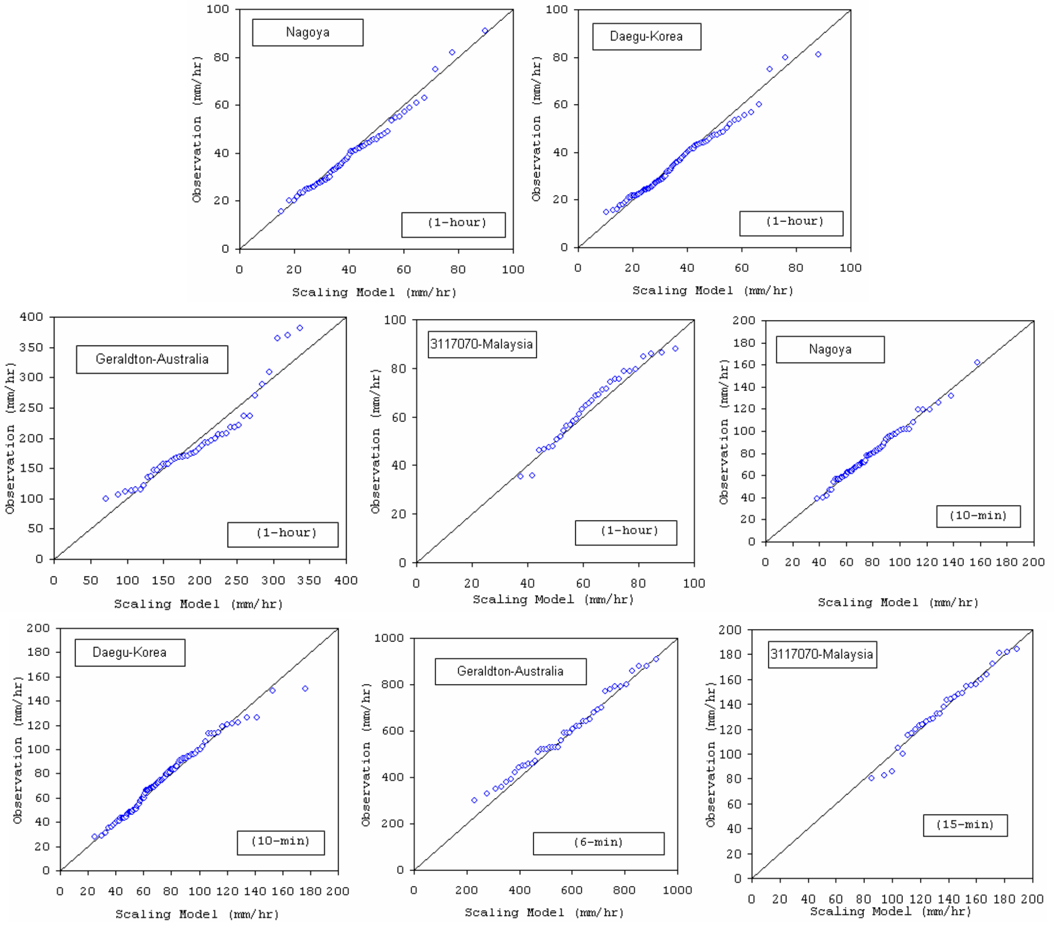

Figure 5 Quantile-Quantile plots of annual extreme rainfalls for hourly and sub-hourly duration

The model performance variables (RMSE) and the best distribution which most accurately fits the observed data were performed for all sites and storm durations. From the figure above, it can see that the scaling methods provided estimates that appear to be close to the observations. The quantiles plots for these stations show good fit between estimated AE rainfalls and the observed ones. The Q-Q plots also show that the estimated values are very close to the observations (see Figure 5). Similar results were found for the remaining stations. The scaling method uses only 3 parameters, therefore, produced more accurate estimates.

Numerical evaluation

The evaluation statistics, RMSE and MAPE, were computed for AE rainfall estimated by scaling methods. For illustration, the values of the RMSE and MAPE for four stations shown in Table 4. It can be clearly seen from Table 4 that scaling methods more accurate estimated and the values of RMSE and MAPE for scaling method for all stations are small.

In summary, scaling method employ the simple scaling properties of moment with duration for AE rainfalls for other durations. The simple scaling model provides a straightforward and quick estimation using scaling factor. Consequently, scaling method produces more accurate extremes rainfall estimates than conventional methods.

Estimation of IDF Rainfall

The graphical results of IDF estimates are shown in Figure 5 for the Nagoya (Japan) station. It can be seen that the scaled estimates are relatively close to observed estimates for short duration storms. Similar observations can be made for other stations used in this study.

The RMSE and MAPE computed for scaling method.

Stations	10-min		1-hour	
	RMSE(mm/hr)	*MAPE (%)*	*RMSE(mm/hr)*	*MAPE (%)*
Nagoya	2.226	2.822	2.413	3.061
Okazaki	3.687	4.962	3.277	4.411
Ohkusa	7.657	7.969	2.312	2.407
Toyohashi	4.175	4.688	2.606	2.926
Taguchi	3.930	4.947	2.916	3.672

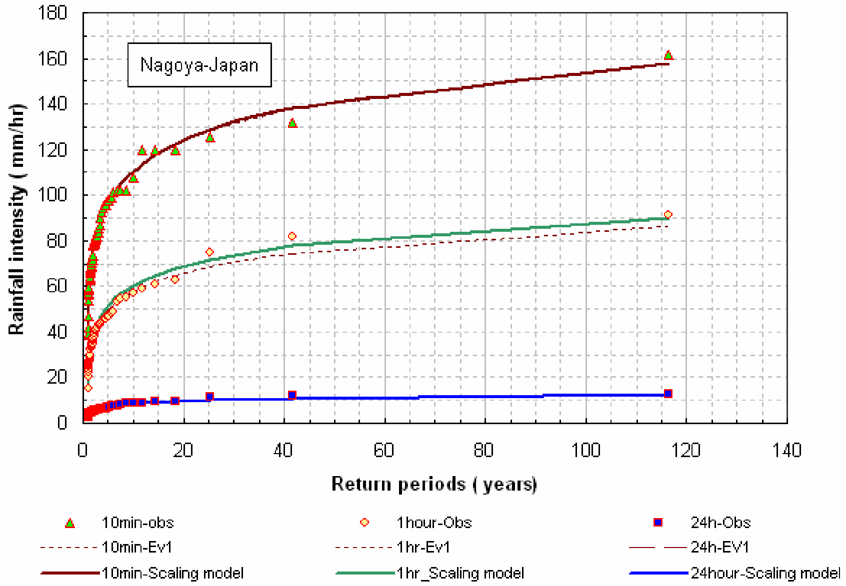

Figure 6 The IDF relationship comparisons between scaling method and conventional methods.

5. Conclusions

The results of this study show that rainfall follows a simple scaling process with two different scaling regimes: 6 minute to 1 hour and 1 hour to 24 hour. Results found from scaling estimates are very similar to observed data for short duration and low return periods.

Based on the scale invariance of rainfall properties shown a simple scaling behavior within two regimes: from 10-min to 1-hour and 1-hour to 1-day, the statistical properties of the annual extreme rainfalls for a given duration can be inferred from those for other durations. These importance findings enable the development of the methods for estimating annual extreme rainfalls for these stations based on the statistical moment of the selected rainfall series. The benefit of using the principles of scaling is that it reduces the amount of parameters required to compute the quantiles. If data is missing from a station, then the first order moment of the duration in question is the only parameter required to compute the quantiles.

The graphical and numerical comparisons of the performance of the scaling methods have indicated that the scaling method could provide more accurate extreme rainfall estimates than those given by scaling model. The IDF curves for annual extreme rainfall for the 4 stations in Asian Pacific region were constructed using the scaling approach and the traditional method. The IDF curves constructed by both methods were similar in both the shape and the magnitude. However, the IDF curves developed by the scaling approach relate the statistical properties of the annual extreme rainfalls for different durations using the scaling relationships. This feature of the proposed scaling procedure enables to derive IDF curves for rainfall durations that have not been observed or measured. The traditional method can not be used for the cases without data.

Results of this study are of significant practical importance because statistical rainfall inferences can be made from a higher aggregation model (ie. observed daily data) to a finer resolution model (ie. less than one hour that might not have been observed). This is important since daily data are more widely available from standard rain gauge measurements, but data for short durations are often not available for the required site. Further studies should be carried out to study the relationship between extreme rainfall characteristics with other climate variables (e.g., temperature, humidity, wind) in order to develop extreme rainfall models that could be used for evaluating the impact of climate change and variability on the magnitude and frequency of occurrence of extreme precipitation events.

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アジア太平洋域における地上観測雨量のスケーリング特性を用いたIDF曲線の推定

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要　旨

都市域における雨水排水システムにおいては，短時間の降水に関する情報が最も重要である。通常，利用することができる降雨データの時間分解能は，治水計画で必要となる時間分解能よりも粗いことが多い。そこで本研究では，時間分解能以下の極値降水量を，それよりも時間分解能が粗い降雨データから推定することを目的とする。提案する手法は，異なる時間間隔の極値降水量の統計的特性は，あるスケール比率で関係付けられるというスケール不変性を基本としている。この手法を，気候条件の異なるアジア太平洋域での極値降水量の推定に応用した。その結果，これらの地域において，本手法は計画降雨を評価する上で実用的な方法であることが示された。

キーワード：降雨IDF関係，スケール不変性，降水極値，計画降雨