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PREDICTING TRANSIENT INSTABILITY OF POWER SYSTEMS BASED ON HYBRID SYSTEM REACHABILITY ANALYSIS [1]

Yoshihiko Susuki * **Hiroaki Ebina** *
Takashi Hikihara *

* *Department of Electrical Engineering, Kyoto University, Katsura, Nishikyo, Kyoto, 615-8510, Japan*

Abstract: This paper shows a novel method for predicting transient instability of power systems based on reachability analysis of hybrid systems. The analysis is performed by computing reachable sets of unsafe sets in nonlinear hybrid automata that represent both continuous electro-mechanical dynamics of generators and discrete operations by relay devices. The unsafe sets here are subsets of system states in which power systems show unacceptable operations such as stepping-out of generators. Then the transient instability of power systems can be estimated by investigating whether a system state exists in the reachable sets or not. The estimation is possible at any onset of accidental faults such as line and plant trips. This paper demonstrates the proposed method through an analysis of single-machine infinite bus system.

Keywords: power systems; models; transient stability analysis; hybrid modes; reachability

1. INTRODUCTION

There is growing recognition that the dynamics of electric power systems become complicated as the current change of technological bases and economic environment (Fairley 2004, Gellings and Yeager 2004). As a technological aspect, various power apparatuses including HVDC systems and FACTS are applied to conventional ac transmission systems. As an economic change, regulatory reforms of electricity markets require a substantial modification of conventional system operation. These technical and non-technical trends possibly cause the dynamics of power systems to be complicated. It is by now widely recognized that we cannot fully analyze and control such complex dynamics using conventional framework of power engineering. A comprehensive approach to the analysis and control has been therefore strongly required (Talukdar *et al.* 2003, Dobson *et al.* 2004).

Hybrid dynamical system and control are active research subjects in computer science and control engineering (Domenica *et al.* 2001, Alur and Pappas 2004). A hybrid automaton (Henzinger 1996) is a well-known mathematical formulation of hybrid systems. The formulation is applicable to modeling and analysis of complex engineered systems that involve both continuous and discrete dynamics. Reachability analysis of hybrid automata is here of paramount importance for safety specifications of engineered systems: for examples, steam boiler and flight management systems (Lygeros *et al.* 1999, Tomlin *et al.* 2003). The analysis is usually performed by computing reachable sets of the hybrid automata and therefore urges us to develop several numerical schemes

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such as level set methods (Mitchell and Tomlin 2000, Tomlin *et al.* 2003).

This paper proposes a novel method for predicting transient instability of power systems based on reachability analysis. The stability of power systems is a well-established subject with a long history of research (Kimbark 1947). Several researchers have recently worked on the intersection of power system stability and hybrid system theory (Hiskens and Pai 2000, DeMarco 2001, Geyer *et al.* 2003, Fourlas *et al.* 2004, Kwatny *et al.* 2005). The authors (Hikihara 2005, Susuki *et al.* 2005) also propose a general framework for modeling and stability analysis of power systems based on nonlinear hybrid automata. This paper focuses on transient stability problems and proposes a novel method for the prediction of transient stability of power systems based on reachability analysis. A key idea in the proposed method is to compute backward reachable sets of unsafe sets for given power system models. This paper demonstrates the proposed method through an analysis of single machine-infinite bus (SMIB) system. For a preliminary discussion of the work reported here, see (Ebina *et al.* 2005).

2. PREDICTING TRANSIENT INSTABILITY: A NOVEL METHOD

This section proposes a method for predicting transient instability based on reachability analysis of nonlinear hybrid automata. Note that the authors (Hikihara 2005, Susuki *et al.* 2005) show a general framework for power system stability analysis based on hybrid system theory.

2.1 Nonlinear hybrid automaton H

A nonlinear hybrid automaton H (Tomlin 1998) is defined to be the collection

$$H = (Q \times X, U \times D, \Sigma_u \times \Sigma_d, f, E, Inv, I, \Omega), \quad (1)$$

with

- $Q \times X$ is the state space, with $Q \triangleq \{q_1, q_1, \cdots, q_m\}$ a finite set of discrete states and X a n-dimensional manifold. A state of the system is a pair $(q_i, x) \in Q \times X$;
- $U \times D \subset \mathbb{R}^u \times \mathbb{R}^d$ is the product of the set of continuous control inputs and the set of continuous disturbances. The space of acceptable control and disturbance trajectories are denoted by $\mathcal{U} \triangleq \{u(\cdot) \in PC^0 \mid u(\tau) \in U \, \forall \tau \in \mathbb{R}\}$ and $\mathcal{D} \triangleq \{d(\cdot) \in PC^0 \mid d(\tau) \in D \, \forall \tau \in \mathbb{R}\}$. PC^0 denotes the space of piecewise continuous functions over $\mathbb{R}$;
- $\Sigma_u \times \Sigma_d$ is the product of the finite set of discrete control actions and the finite set of discrete disturbance actions;
- $f : Q \times X \times U \times D \to TX$ is the vector field which associates a nonlinear control system $f(q, x, u, d)$ with each discrete state $q \in Q$;
- $E : Q \times X \times \Sigma_u \times \Sigma_d \to 2^{Q \times X}$ is the discrete transition function;
- $Inv \subseteq Q \times X$ is the invariant associated with each discrete state, meaning that the system evolves according to $\dot{x} = f(q, x, u, d)$ only if $(q, x) \in Inv$;
- $I \subseteq Q \times X$ is the set of initial states;
- Ω is the trajectory acceptance condition [2].

2.2 Modeling transient dynamics via H

First of all conventional problem setting for transient stability analysis is reviewed. The readers can refer to (Chiang *et al.* 1987, Chiang *et al.* 1995). For the transient stability analysis, the following system described by a set of three differential equations is examined:

$$\begin{cases} \dot{x} = f_{\rm pre}(x) & \text{for } t < t_{\rm f}, \\ \dot{x} = f_{\rm on}(x) & \text{for } t_{\rm f} \le t < t_{\rm c}, \\ \dot{x} = f_{\rm post}(x) & \text{for } t_{\rm c} \le t, \end{cases} \quad (2)$$

where x stands for the state vector that includes rotor angles, rotor speed deviation, and so on. The differential equations $f_{\rm pre}$, $f_{\rm on}$, and $f_{\rm post}$ represent electro-mechanical dynamics of generators. At some time $t_{\rm f}$ a power system undergoes a large fault, that is, there is a topological change of the constitute transmission network. This is represented by the change of the differential equations from $f_{\rm pre}$ to $f_{\rm on}$. Before the fault, it is assumed that we have the pre-fault differential equation $f_{\rm pre}$. The topological change is caused by controlled and uncontrolled line switching: relay operations and accidental faults such as line trip and plant outage. At $t = t_{\rm c} (> t_{\rm f})$ the fault is cleared. The dynamics are then governed by the post-fault differential equation $f_{\rm post}$. In addition, by re-closing operation, at $t = t_{\rm r} (> t_{\rm c})$ the network topology returns to the pre-fault one. The differential equation is then fixed at $f_{\rm pre}$.

The nonlinear hybrid automaton can combine the transient dynamics with line switching operations. In H the vector field f is described by the differential equations $f_{\rm pre}$, $f_{\rm on}$, and $f_{\rm post}$ in (2). On the other hand, discrete variables $\{q_i\}$ are assigned to the above system states: pre-fault, fault-on, and post-fault ones. The discrete transition E can then describe the topological change of the constitute network, that is, the change of the differential equations such as the one from $f_{\rm pre}$ to $f_{\rm on}$. The transition is driven by discrete control and disturbance actions $(\sigma_u[\cdot], \sigma_d[\cdot]) \in \Sigma_u \times \Sigma_d$. They include relay operations and accidental faults. Additionally, H can model continuous controller of

[2] $\Omega = \Box F$ for $F \subseteq Q \times X$. $\Box$ denotes a map, called property, from the set of all executions of H to {True,False} (Lygeros *et al.* 1999).

power systems such as dc links by $u(\cdot) \in U$ and unregulated power flow due to electricity trading as $d(\cdot) \in D$. Hence the nonlinear hybrid automaton H is applicable to modeling of the transient dynamics with taking the relay operations into account.

2.3 Predicting transient instability using reachable sets

The present subsection introduces a novel method for predicting the transient instability. Now define an unsafe set $G \subset Q \times X$ for the hybrid automaton H. The unsafe set is interpreted as a subset of the system states in which a power system shows unacceptable operations: for examples, occurrence of large rotor speed deviation and stepping-out of generators. A reachable set $R_t(G)$ for the time $t(< 0)$ in the hybrid automaton H is then defined by a subset of $Q \times X$ in which any system state reaches the boundary ∂G of G in $|t|$ time despite of any control $(u(\cdot), \sigma_u[\cdot])$. Fig. 1 shows the concept of reachable sets in continuous state space. The concept of reachable sets is much important for estimating the transient instability of power system. If a system state exists in $R_t(G)$, then we can evaluate that the power system will reach unacceptable operation in $|t|$ time. The estimation is possible at any onset of discrete transitions such as accidental faults, clearing and re-closing operations. Namely, by evaluating the reachable sets, we can discuss at the onset of accidental faults whether the power system goes to unacceptable operations or not. The reachability analysis thus makes it possible to predict the transient instability of power system.

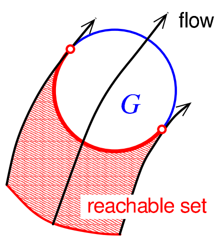


Fig. 1. Concept of reachable set for continuous state space. G is an unsafe set in which a power system shows unacceptable operations.

3. APPLICATION TO SINGLE MACHINE-INFINITE BUS SYSTEM

This section applies the proposed method to an analysis of single machine-infinite bus (SMIB) system in Fig. 2. The SMIB system consists of a synchronous machine, an infinite bus, and two parallel transmission lines. An infinite bus is a source of voltage constant in phase, magnitude, and frequency, and is not affected by the amount of current withdrawn from it (Kimbark 1947).

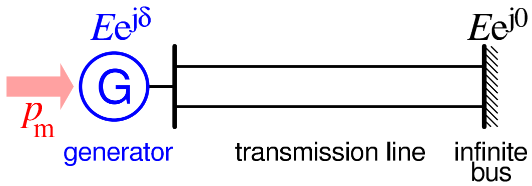


Fig. 2. Single machine-infinite bus (SMIB) system

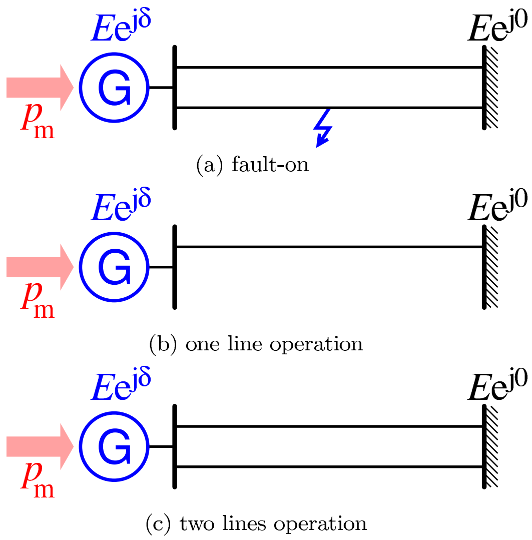


Fig. 3. Fault condition following control sequence of relay devices

3.1 Fault condition

The subsection gives us a fault condition for the analysis of SMIB system and model it via the hybrid automaton H. Fig. 3 shows the fault condition following control sequence of relay devices. The three modes in the figure are represented as follows:

- *fault-on* (a) is the state during a sustained fault on one line. Then the generator cannot supply its electric power output to the ac transmission;
- *one line operation* (b) is the state after clearing the fault line by protective relay operation; and
- *two lines operation* (c) is the state after re-closing the fault line.

The fault-clearing time $t_{\rm c}$ and the re-closing time $t_{\rm r}$ are used as control parameters in the following discussion. The onset of accidental fault is fixed at zero in this paper.

3.2 Description of H

The fault condition and associated transient dynamics are now modeled via the following nonlinear hybrid automaton H:

$$
\begin{cases}
Q \times X & = \{q_1, q_2, q_3\} \times (S^1 \times \mathbb{R}^2), \\
 & \quad (\delta, \omega, z) \in X, \\
U \times D & = \emptyset \times \emptyset, \\
\Sigma_u \times \Sigma_d & = \{\sigma^1, \sigma^2\} \times \emptyset, \\
f(q, (\delta, \omega, z)^{\mathrm{T}}) & = \begin{pmatrix} \omega \\ -k\omega - p_{\mathrm{m}} - \alpha b \sin\delta \\ 1 \end{pmatrix} \\
 & \quad \text{at} \begin{cases} \alpha = 0 & \text{if } q = q_1, \\ \alpha = 0.5 & \text{if } q = q_2, \\ \alpha = 1 & \text{if } q = q_3, \end{cases} \\
E(q_1, (\delta, \omega, t_{\mathrm{c}})^{\mathrm{T}}, \sigma^1) & = (q_2, (\delta, \omega, t_{\mathrm{c}})^{\mathrm{T}}), \\
E(q_2, (\delta, \omega, t_{\mathrm{r}})^{\mathrm{T}}, \sigma^2) & = (q_3, (\delta, \omega, t_{\mathrm{r}})^{\mathrm{T}}), \\
Inv & = \bigcup_{i=1}^{3} (q_i, X),
\end{cases}
\tag{3}
$$

The above description is based on (Tomlin 1998). Table 1 shows the physical meaning of variables and parameters in H. The variables and parameters are in per unit system. In H the discrete variable q_1 is assigned to the fault-on state, q_2 to the one line operation, and q_3 to the two lines operation. The clearing and re-closing operations are also regarded as control actions σ^1 and σ^2. The continuous vector field f is the well-known swing equation system and is parameterized by α that depends on every discrete variable. The discrete transitions E are driven by the control actions σ^1 and σ^2. Fig. 4 describes the hybrid automaton H including the two control actions.

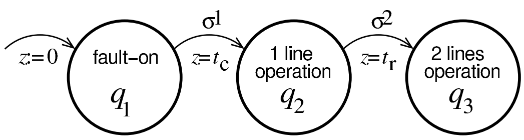


Fig. 4. Hybrid automaton H. H includes the two discrete control actions which represent the clearing and re-closing operations.

The unsafe set G and the set I of initial conditions are defined for the present analysis by

Table 1. Physical meaning of variables and parameters in H

rotor position with respect to synchronous reference axis	δ	
rotor speed deviation relative to system angular frequency	ω	
damping coefficient	k	0.05
mechanical input power	p_{m}	0.2
critical power of two lines operation	b	0.7
onset time of fault	t_{f}	$(0\ \mathrm{s})/t_{\mathrm{b}}$
onset time of clearing operation	t_{c}	
onset time of re-closing operation	t_{r}	
base quantity of time	t_{b}	

$$
\begin{cases}
G^{\mathrm{o}} & = \{q_1, q_2, q_3\} \times \{(\delta, \omega, z) \in X;\ |\omega| > \omega_{\mathrm{c}}\}, \\
\partial G & = \{q_1, q_2, q_3\} \times \{(\delta, \omega, z) \in X;\ |\omega| = \omega_{\mathrm{c}}\}, \\
I & = \{q_1\} \times \{(\delta, \omega, z) \in X;\ |\omega| < \omega_{\mathrm{c}}, z = 0\},
\end{cases}
\tag{4}
$$

where $\omega_{\mathrm{c}} = 2.0$. Any state in G physically implies unacceptable operations of the SMIB system because of the occurrence of large rotor speed deviation and stepping-out of generator.

Note that the definition of unsafe sets is crucial for the proposed method in this paper. The estimation of instability strongly depends on how we fix unacceptable states of power systems. In the present automaton H, continuous dynamics are described by the swing equation system. The system represents the stepping-out state of generator as a stable limit cycle of the second kind (Minorsky 1947). The location of limit cycle is $\omega \approx p_{\mathrm{m}}/k$ (Hasegawa and Ueda 1999). Therefore, to avoid the large rotor speed deviation and stepping-out, in this paper, ω_{c} is fixed at the above value which does not exceed the location.

3.3 Hybrid reachable set and predicting transient instability

The present section gives us a numerical result of hybrid reachable set. Fig. 5 shows the reachable set of the hybrid automaton H under $t_{\mathrm{c}} = (0.1\ \mathrm{s})/t_{\mathrm{b}}$ and $t_{\mathrm{r}} = (0.6\ \mathrm{s})/t_{\mathrm{b}}$. The numerical integration is performed in sufficiently large time at the discrete state q_3. This figure shows the set I of initial conditions. The reachable set is now decomposed into the three subsets R_1, R_2, and R_3. R_1 is the subset of I from which any trajectory reaches ∂G before the discrete transition from q_1 to q_2. R_2 is the subset of I which goes to ∂G between the discrete transitions from q_1 to q_2 and from q_2 to q_3. R_3 is also the subset of I after the discrete transition from q_2 to q_3. The white region in Fig. 5 therefore corresponds to transient stability region (Chiang *et al.* 1987) with taking the discrete transitions into account. Fig. 6 shows

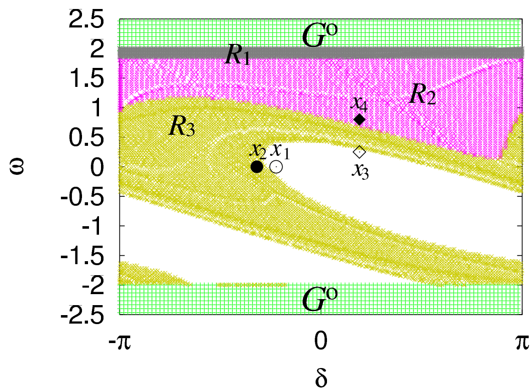
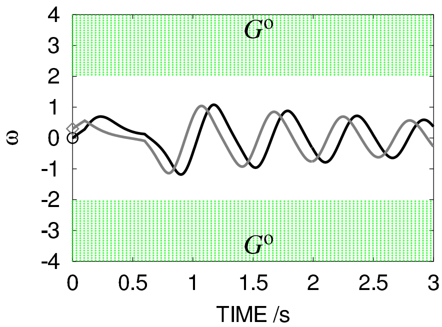
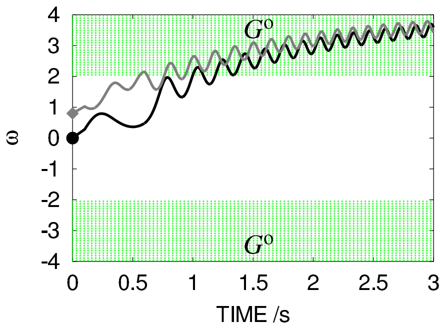


Fig. 5. Hybrid reachable set under $t_{\mathrm{c}} = (0.1\ \mathrm{s})/t_{\mathrm{b}}$ and $t_{\mathrm{r}} = (0.6\ \mathrm{s})/t_{\mathrm{b}}$. The reachable set is decomposed into three subsets R_1, R_2, and R_3 based on transient behavior.

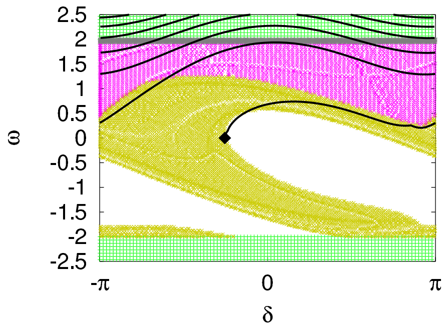


(a) x_1 (○), x_3 (◇)

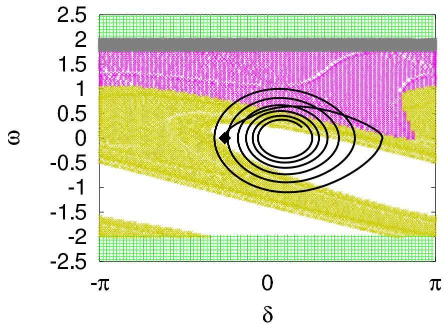


(b) x_2 (●), x_4 (filled ◇ with black)

Fig. 6. Transient behavior with four initial points in Fig. 5



(a) $t_c = (0.05 \text{ s})/t_b$ and $t_r = (0.55 \text{ s})/t_b$



(b) $t_c = (0.15 \text{ s})/t_b$ and $t_r = (0.65 \text{ s})/t_b$

Fig. 7. Reachable sets and projected trajectories onto the set I of initial conditions

the transient behavior with four initial points in Fig. 5. Fig. 6 implies that the solution from the initial point x_2 in R_3 actually reaches ∂G after the re-closing time $t_r = (0.6 \text{ s})/t_b$, and that the solution from the white region remains in the supplement of G for any time. The prediction of transient instability is hence possible *at the onset of the accidental fault* based on the hybrid reachable set.

3.4 Clarifying the effect of relay control to tran -sient stabilization

The reachability analysis can also clarify the effect of clearing operation to transient stabilization of power systems. Fig. 7 describes the reachable sets and projected trajectories onto the set I of initial conditions under $(t_c, t_r) = ((0.05 \text{ s})/t_b, (0.55 \text{ s})/t_b)$ and $(t_c, t_r) = ((0.15 \text{ s})/t_b, (0.65 \text{ s})/t_b)$. The figure shows how the control action σ^1 affects the reachable set. The trajectory in Fig. 7(b) starts from the same initial condition as the one in Fig. 7(a). The trajectory in Fig. 7(a) reaches ∂G after the discrete transition from q_2 to q_3. On the other hand, the trajectory in Fig. 7(b) converges to a stable equilibrium of the vector field on the invariant $\{q_3\} \times X$. This implies that the power system can survive from the fault by the *slow* clearing not the *fast* one. Such effects of transient stabilization are not fully clarified using conventional methods of transient stability analysis. The reachability analysis thus makes it possible to confirm the effect of relay control to transient stabilization.

4. SUMMARY AND FUTURE DIRECTIONS

This paper showed a method for predicting transient instability of power systems based on hybrid system reachability analysis. The nonlinear hybrid automata can represent both transient dynamics and discrete transitions caused by transmission line switching. The hybrid model also makes it possible to analyze continuous and discrete controlled systems by dc links and transmission switching. It can therefore contribute to the synthesis of stabilizing controllers for power systems. On the basis of the hybrid model, this paper proposed a novel method for the prediction of transient instability at the onset of accidental faults based on reachable sets of hybrid automata. Of course, the proposed method is valid at the onset of relay operations or control. This paper shows its effectiveness for not only predicting the transient instability but also clarifying the effect of relay control to transient stabilization of power systems.

Future directions are in (i) reachability analysis using level set methods (Mitchell and Tomlin 2000, Tomlin *et al.* 2003); (ii) application of the proposed method to practical system analysis (Sakiyama *et al.* 2006); (iii) voltage stability analysis based on hybrid system reachability analysis (Susuki and Hikihara 2006); and (iv) hybrid controller synthesis via power apparatuses such as HVDC systems and FACTS.

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