THE ABE LABORATORY

Head: Prof. Dr. Kiyoshi Abe

$BaTiO_3$ Ceramics as Electrostrictive Vibrator

Kiyoshi Abe, Tetsuro Tanaka, Isao Saito and **Kiyoshi Okazaki**

There are three modes of vibration in $BaTiO_3$ ceramic plate which can be excited by electrostrictive effect, namely the longitudinal length mode of rectangular plate, the radial mode of circular plate and the thickness mode of plate in arbitrary shape. The plate with special treatment can be used to excite shearing vibration. With regard to the frequency constants of these modes of vibration or the mechanical constants of the material, the reader may be refered to the reports previously published.[1] In the present report the authors intend to discuss the snbjects which are necessary in applying $BaTiO_3$ ceramics to the electro-acoustic transducer.

1. Electrostrictive Vibration of $BaTiO_3$ Ceramics

If we apply A. C. voltage between the two surfaces of a ceramic plate represented in Fig. 1, there take place three kinds of vibration, namely the thickness mode, the logitudinal mode in direction l and another longitudinal mode in direction b. We shall begin with the longitudinal mode in direction l. Near the resonant frequency of the longitudinal mode in direction l, the equivalent mass constant m, stiffness constant s and the force factor A of the ceramic plate as electro-acoustic transducer are given by:

$$\left.\begin{aligned} m &= \frac{btl\rho}{2} \\ s &= \frac{\pi^2 btl}{8l} \\ A &= \frac{b\lambda}{4\pi} \end{aligned}\right\} \qquad (1)$$

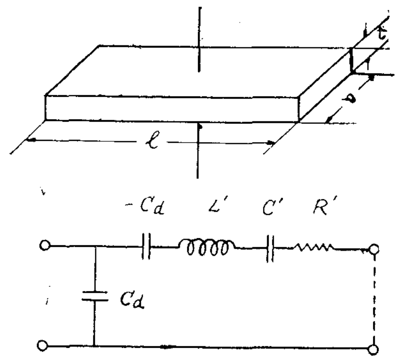

Fig. 1

where ρ is the density of material, E–the Young's modulus and λ–the electrostrctive constant. Besides these, there must be introduced the friction coefficient r due to modulus the loss of energy which includes also the radiation of energy, if any acoustic radiation takes place. As is well known from the theory of electro-acoustic transformation, the equivalent electric circuit of these vibrator can be represented as in Fig. 1 and the constants of the circuit are given by:

$$\left.\begin{aligned} C_d &= \frac{\varepsilon bl}{4\pi t} \\ R' &= r/A^2 \\ L' &= m/A^2 \\ 1/C' &= s/A^2 \end{aligned}\right\} \qquad (2)$$

in which ε represents the dielectric constant of $BaTiO_3$.

Then, if we measure the electrical impedance of the ceramic plate by some suitable method, we are able to calculate the numerical values of C', R', L' and C_d in formula (2) as will be explained in the following. And furthermore, if we know one of the three constants m, s and A in formula (1), for example, in case of known m which is just half the net weight of the plate we can calculate the the other constant from formula (2).

To measure the electrical impedance of the ceramic plate, the ordinary Wien-Bridge circuit was employed, and as A. C. voltage source a C–R oscillator of the Yokogawa Electric Works was used, the frequency of which could be changed by 10 cycle step precisely. Fig. 2 shows a result of impedance measurement of a ceramic plate 3.33 cm×3.18 cm×0.225 cm. From the figure, we are able to know the values of the resonant frequency f_r, Δf (f_2-f_1), damped impedance Z_d or damped admittance Y_d, motional impedance Zm_0 or motional admittance Ym_0, and the damping constant Δ. And between these values and the constants C', R', L', and C_d in formula (2) there exsist the following relations;

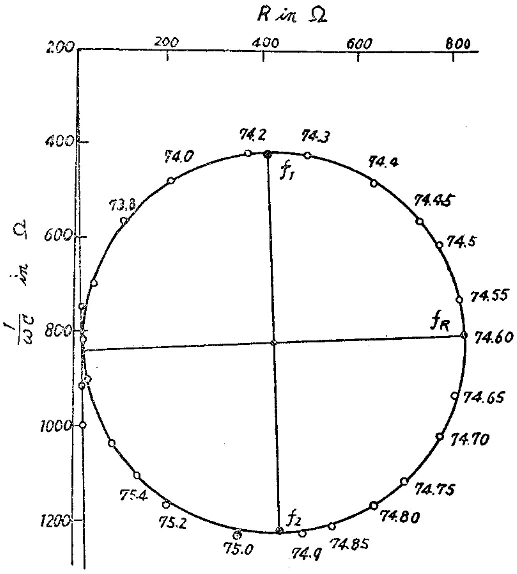

Fig. 2

$$\left.\begin{aligned} |Ym_0| &= \frac{1}{R'} \\ Y_d &= \omega_r C_d = 2\pi f_r C_d \\ \omega_r^2 &= (2\pi f_r)^2 = \frac{1}{L'}\left(\frac{1}{C'}-\frac{1}{C_d}\right) \\ \Delta &= \pi\cdot\Delta f = \frac{R'}{2L'} \end{aligned}\right\} \qquad (3)$$

And so, we can calculate the values of C', R', L' and C_d from the impedance measurement of the ceramic plate.

In the following we show the result of calculation from Fig. 2.

$Ym_0 = 1167\mu U$	$K' = 858\Omega$	$A = 1.9\times10^4$ dynes/volt
$Y_d = 1227\mu U$	$L' = 177$ mH	$r = 30720$ dynes/kine
$\omega_R = 467{,}000$ rad/sec	$C' = 25.7$ pF	$\lambda = 7.5\times10^4$ dynes/volt/cm
$\Delta = 2420$ rad/sec	$C_d = 2630$ pF	

In the $BaTiO_3$ ceramic plate, electrostrictive vibration is very much influenced by the D. C. bias-voltage which is to be applied between two electrodes. To illustrate this, a circular disc of ceramics of 0.12 cm thick and 2.05 cm in diameter was employed, and ascending the bias-voltage, the impedance measurement was carried out, the result of which is shown in Fig. 3. In this figure, nothing is shown about the case when the bias-voltage was descending. But it may be said that there took place an evident phenomenon of hysteresis, which the authors intend to report elsewhere.

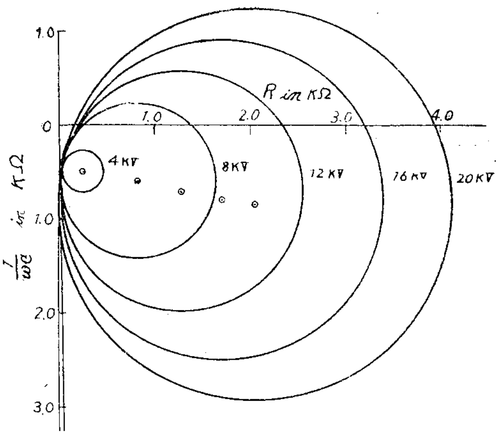

Fig. 3.

2. Rectangular Plate[2) 3)]

In the foregoing paragraph, we treated the longitudinal vibration in direction l in Fig. 1. Now we are going to discuss the relations between the two longitudinal vibrations in directions l and b. In the course of experiments, we found that the frequency constants of these two vibrations differed from each other considerably, and that the motional impedance circle was always large in the vibration of the shorter side as compared with that of the longer side. Table 1 shows the result of preliminary tests about several samples.

Table 1

Sample	in cm		f_R in kc	$f_R \times l$ in kc-cm
1	l_1	3.85	54.25	208.86
	l_2	3.01	77.31	232.86
2	l_1	3.05	67.30	205.27
	l_2	2.09	108.08	225.88
3	l_1	3.80	53.83	204.55
	l_2	2.65	83.86	222.23
4	l_1	3.49	58.63	204.62
	l_2	2.12	102.25	216.77

With a view to obtaining enough data to explain the above results, we carried out the following experiment. A rectangular plate of 4.395 cm long and 0.225 cm thick was coated with thin silver coating on both surfaces, and between these silver electrodes D.C. voltage corresponding to 20 KV/cm had been applied for one hour. After these treatments, the longer side of 4.395 cm long was shortened step by step, while the side of 3.18 cm wide was kept constant, and at every step the impedance was measured precisely. Fig. 4 (a) shows one of the results in which $(f_r)_l$ and $(f_r)_s$ are the resonant frequencies, and $(f_r \times l)_l$ and $(f_r \times l)_s$ are the frequency constants, of the longer and shorter side, respectively. For the abscissa of Fig. 4 was taken ratio l/l_0 in which l is the length of the variable side and l_0 is the length of the constant side. When ratio l/l_0 became nearer to 1, the vibration corresponding to the longer side became very weak, while the vibration corresponding to the shorter side became very strong. Fig. 4 (b) is another result in which motional admittance $(Y_{m0})_l$, $(Y_{m0})_s$ and damping constant Δ_l, Δ_s are plotted. From the value of Y_{m0} and Δ shown in Fig. 4(b), we can calculate force factor A and electrostrictive constant λ by formulae (1), (2), and (3), and the calculated results are plotted in Fig. 4 (c).

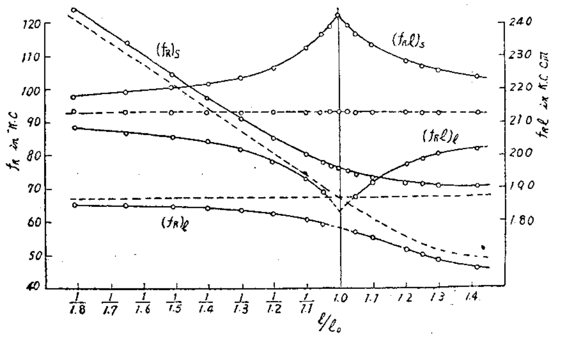

Fig. 4 (a)

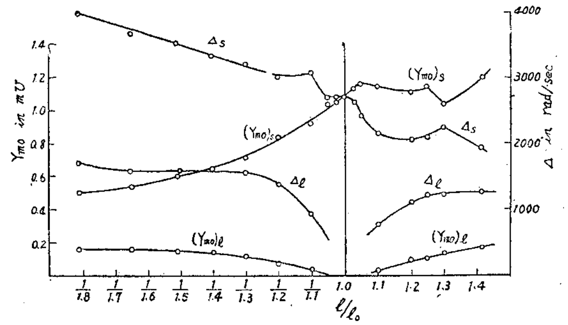

Fig. 4 (b)

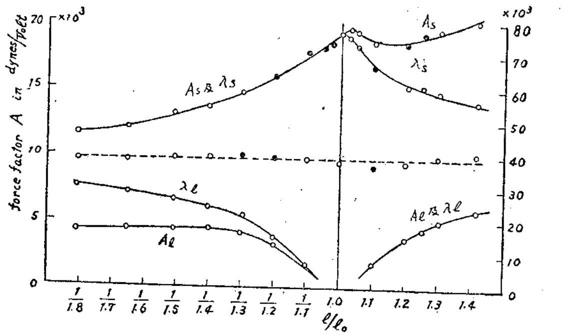

Fig. 4 (c)

Conclusions we get from these experiments are as follows;

(1) The frequency constants of two vibrations corresponding to the longer side and the shorter side of a ceramic plate are much effected by the ratio of the length of both sides.

(2) The frequency constant corresponding to the shorter side is larger as compared with that of the longer side. The frequency constant of the shorter

side becomes larger, while that of the longer side becomes smaller, when the ratio of the length of both sides becomes near to 1.

(3) The apparent electrostrictive constant corresponding to both sides calculated from the measured valus also differ from each other, and have the same tendency as the frequency constants.

When the ratio of both sides of a rectangular plate becomes near to 1, the frequencies of the two vibrations become almost the same, and from the interference between the two vibrations, the above mentioned abnormal phenomena result.

3. Underwater Sound Transmittor and Receiver of Langevin Type[4) 5)]

As was already reported, when an electric field is applied to a $BaTiO_3$ ceramics, the elongation in the direction of electric field is theoretically twice the contraction in the direction perpendicular to the field. Moreover, the contraction is an indirect effect of the elongation, and, therefore, is very much influenced by the internal condition of the substance, so that practically the contraction is approximately equal or less than 1/3 of the elongation. From these reasons, when ceramic substance is to be used as vibrators or vibrator elements, it is desireable to utilize the mode of vibration in the direction of electric field, i. e., the thickness vibration of the plate.

But, as an underwater sound transmittor, the resonant frequency must be restricted under 50 kc from the view point of attenuation of supersonic wave, when it propagates in the water, and therefore, if we want a ceramic plate with the resonant frequency of 50 kc, about 5 cm thick is required. These plates are not only difficult to obtain, but inconvenient to use because higher voltage should be applied between electrodes.

We tried, for these reasons, to develop the Langevin type of transducer using a $BaTiO_3$ ceramic plate in place of quartz crystal plates. On both surfaces of the ceramic disc of 3~5 mm thick and of 55~70 mm in diameter, iron or brass cylinders were fixed by a suitable method, and between the two metal cylinders was applied D.C. voltage corresponding to 20 KV/cm for several hours.

Fig. 5 shows the impedance diagram of one of these vibrators. The outer circle indicates the motional impedance in the air,

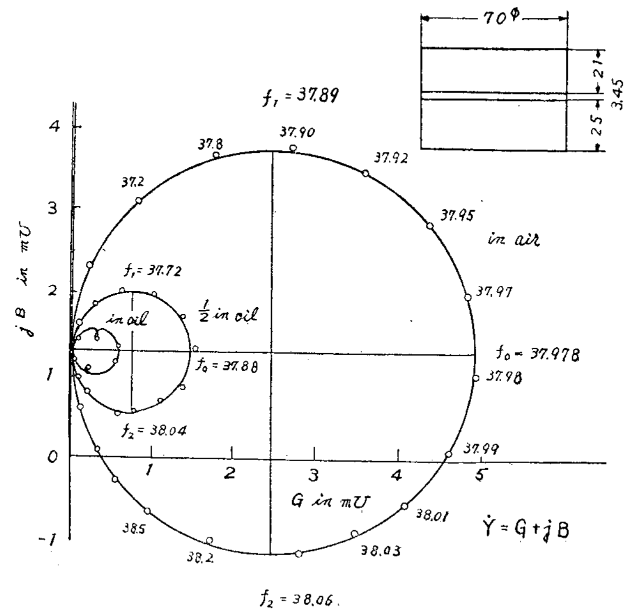

Fig. 5.

the inner circle that in an oil vessal, while the middle circle indicates the motional impedance, when only one surface of the vibrator is immersed in the oil vessel. The inner circle is not a perfect circle due to the effect of standing wave in the small oil vessel.

As the dielectric loss angle of the ordinary $BaTiO_3$ ceramics is about 0.02, very good efficiency can be expected of these vibrators, and by calculating from the data in Fig. 5, we got a value of 95% of electro-mechanical efficiency and 88% of mechano-acoustic efficiency. Thus the total efficiency is about 84%.

The resonant frequency of the Langevin type vibrators which were constructed in our laboratory was very much lower than expected from the material constants of $BaTiO_3$ ceramics and the metal fixed to the ceramics, and it was conspicuously low especially, when the ratio of diameter to the length of the vibrator was large. Fig. 6 (a) shows the experimental data about resonant frequencies of vibrators fixed with steel cylinders, where three kinds of vibrator were examined; one kind with cylinders of 1.0 cm radius, the other two with cylinders 2.0 and 2.8 cm in radius. The detted line in Fig. 6 (a) is the expected value of natural frequency, when l is varied, but the experimental curve was far from the expected value, especially when the radius of the cylinder was large. Fig. 6 (b) shows the same kind of experiments of vibrators with brass cylinders.

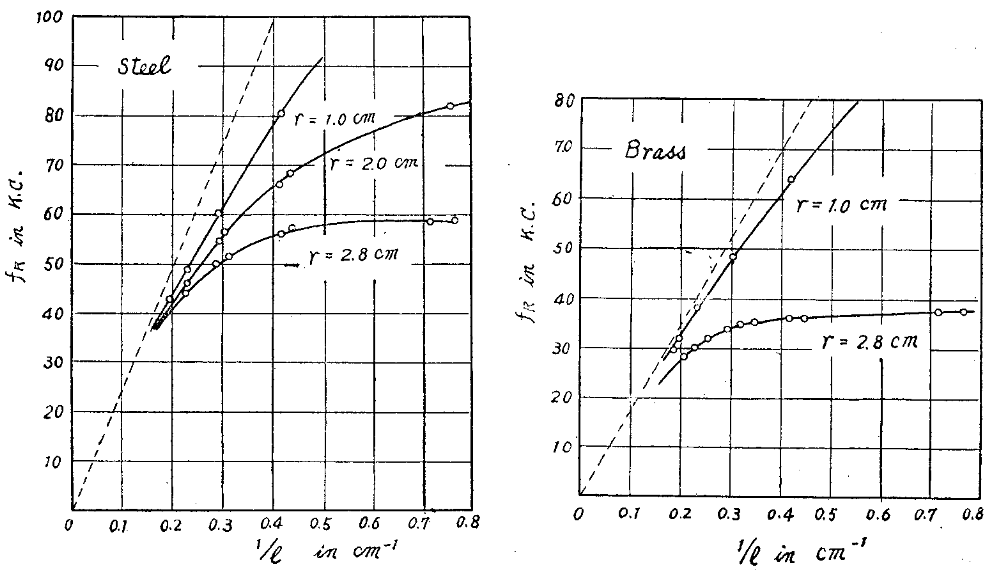

Fig. 6 (a) Fig. 6 (b)

In general, when the radius of cylinder becomes large to a certain point, the natural frequency of cylinder is lowered and the theoretical value considering the Poisson's ratio is given by:

$$\frac{f}{f_0}=\frac{1}{1+\frac{\sigma^2\pi^2}{4}\cdot\frac{r^2}{l^2}} \tag{4}$$

where f......natural frequency in case of considering the Poisson's ratio
f_0......natural frequency in case of neglecting the Poisson's ratio
σ......Poisson's ratio
r......radius of cylinder
l......length of cylinder.

But the natural frequency obtained from experiments was far low from the value calculated from the above formula. So we assumed, therefore, that lowering of natural frequency was due to coupling the fundamental mode with another mode of vibration, as in the case of the rectangular plate. To confirm this idea, the impedance measurement was carried out at frequencies of wide range. There were, in general, more than one resonant frequency in the Langevin type vibrator, as are plotted in Figs. 7(a) and (b). It may be concluded from these figures that there existed a clear phenomenon of coupling the fundamental mode with another mode.

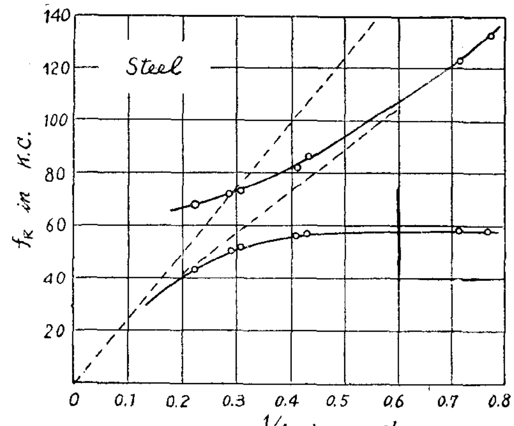

Fig 7 (a)

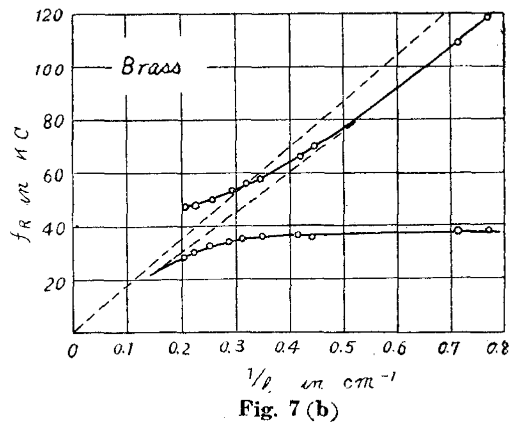

Fig. 7 (b)

To find the mode which is coupled with the fundamental mode, we arranged many data as represented in Fig. 7(a) in one figure such as Fig. 8(a). Fig. 8(a) is of steel vibrators and Fig. 8(b) of brass. In these figures, the ratio of radius to length r/l is taken as abscissa and frequency constant $f_R\times l$ as ordinate. The figures show two modes of vibration coupled togather. Frequency constant $f_R\times l$ of one mode has a nearly constant value, when r/l is varied, which shows that the resonant frequency f_R is inversely proportional to l. This is the fundamental mode of vibration.

In the other mode of vibration, frequency constant $f_R\times l$ changes on a hyperbolic line, when r/l is varied. Hence;

$$[f_R\times l]\times[r/l]=\text{constant},$$
$$f_R\times r=\text{constant}.$$

In this mode, resonant frequency

f_R is inversely proportional to r. This is the radial mode of vibration. As mentioned above, lowering of the frequency of the Langevin type of vibrator is due to coupling the fundamental mode with the radial mode of vibration. These data are applicable in the design of the Langevin type vibrator.

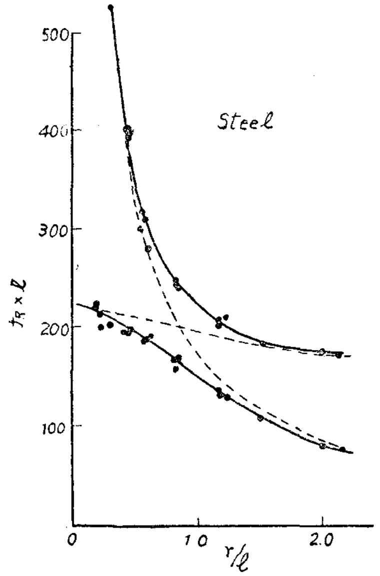

Fig. 8 (a)

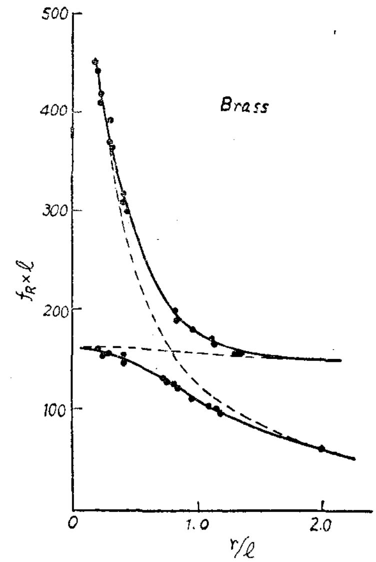

Fig. 8 (b)

Photo. 1 shows an example of vibrator actually constructed, and the slits upon the surface are made in order to prevent coupling of the radial mode with the fundamental mode of vibration. Photo. 2 (a) shows the figure on Braun tube oscillograph of a simple depth-sounder with 35 kc $BaTiO_3$ Langevin type vibrators as transmittor and receiver, when both are placed confronting in the air 0.4 meter apart. In the photograph we can observe clearly 2nd and 3rd echoes, etc. In Photo. 2 (b) the transmittor and receiver are placed 1 meter apart.

These vibrators were examined on the spot and proved successful in practical use as the transmittor and receiver of a depth-sounder. In this case the vibrators were of brass and of 35 kc natural frequency, and the transmittor was always biased with 1000 V D. C., and excited by a damped electrical oscillation of 1000 V peak and of 35 kc, and the receiver had no bias voltage but was previously polarlized with high voltage. As a result of the unsuitable of mounting of vibrators to the waterproof cases the sensitivity was considerably lowered but the depth-sounder caught echoes from the bottom of the sea about 150 meter deep and also those from fish on Braun tube oscillograph.

1) K. Abe and T. Tanaka: Jour. of the Denki Hyoron (Electrical Review), **37**, No. 4, 2 (1949).
2) K. Abe, T. Tanaka, I. Saito and K. Okazaki: Advance Copy of the General Meeting of the Electrical Engineering Institutes (I). May, 144 (1951).
3) K. Abe, T. Tanaka, I. Saito and S. Miura: *Ibid.* (II) May, 177 (1951).
4) K. Abe, T. Tanaka, I. Saito, M. Hirano and K. Okazaki: Jour. of the Acoustic Soc. of Japan **7**, No. 1. 20 (1951).
5) K. Abe, T. Tanaka, M. Hirano and A. Murata: *Ibid.* 16.

Photo. 1

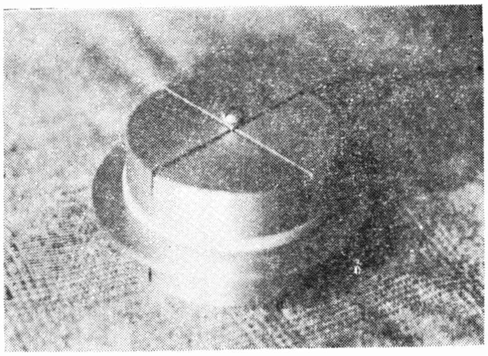

Photo. 2 (a)

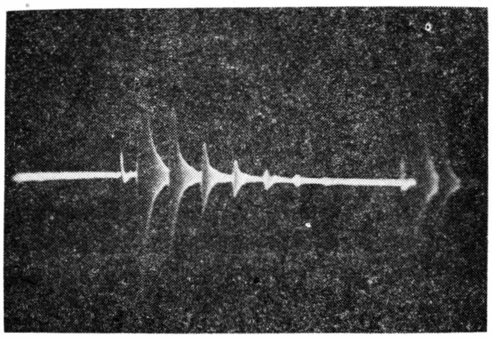

Photo. 2 (b)